Solve the following:

(a) $y' + (x+2)y^2 = 0 \qquad y(1) = 1$

$$\frac{dy}{dx} = -(x+2)y^2$$

$$\int \frac{dy}{y^2} = \int -(x+2)\,dx$$

$$-\frac{1}{y} = -\left(\frac{x^2}{2} + 2x\right) + c \quad \rightarrow \quad \boxed{y = \frac{2}{x^2+4x-3}}$$

$$y(1) = -1 = -\left(\frac{1}{2} + 2\right) + c \rightarrow c = \frac{3}{2}$$

(b) $yy' + 4x = 0 \qquad y(0) = 3$

$$\int y\,dy = \int -4x\,dx$$

$$\frac{y^2}{2} = -2x^2 + c \quad \rightarrow \quad y^2 = 9 - 4x^2$$

$$y = \pm\sqrt{9-4x^2} \quad \rightarrow \quad \boxed{y = \sqrt{9-4x^2}}$$

$$y(0) = \frac{9}{2} = c$$

(- does not meet initial condition)

(c) $y' = \frac{x-1}{y} e^{-y^2} \qquad y(0) = 1$

$$\int y e^{y^2}\,dy = \int (x-1)\,dx$$

$$\frac{1}{2}e^{y^2} = \frac{x^2}{2} - x + c \quad \rightarrow \quad e^{y^2} = x^2 - 2x + e$$

$$y(0) = 1 \rightarrow \frac{1}{2}e^{1^2} = c \qquad y^2 = \ln(x^2 - 2x + e)$$

$$\boxed{y = \sqrt{\ln(x^2-2x+e)}}$$

(+ only by IC)