

## MATHEMATICS

## OBJECTIVE

TIME: 30 MINUTES

GROUP : FIRST

DGL-11-1-23

MARKS: 20

NOTE : You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

## QUESTION NO. 1

- 1 A.M between  $\frac{1}{2}$  and  $\frac{1}{4}$  is  
(A)  $-\frac{1}{8}$  (B)  $\frac{1}{8}$  (C)  $\frac{3}{4}$  (D)  $\frac{3}{8}$
- 2 If  $r = n$  then  ${}^nC_r$  is equal to  
(A) 0 (B) 1 (C)  $n!$  (D)  $(n-1)!$
- 3 For mutually exclusive events A and B  
(A)  $A \cup B = \emptyset$  (B)  $A - B = \emptyset$  (C)  $A \cap B = \emptyset$  (D)  $A \cup B = A \cap B$
- 4 The in - equality  $n^2 > n + 3$  is valid if  
(A)  $n \geq 2$  (B)  $n \geq 0$  (C)  $n \geq 1$  (D)  $n \geq 3$
- 5 Sum of even coefficient in expansion of  $(a + b)^4$  is  
(A) 18 (B) 10 (C) 12 (D) 16
- 6 The angle  $\frac{\pi}{12}$  in degree measure is  
(A)  $30^\circ$  (B)  $20^\circ$  (C)  $45^\circ$  (D)  $15^\circ$
- 7  $\sin 390^\circ$  is equal to  
(A)  $\cos 30^\circ$  (B) Zero (C)  $\sin 30^\circ$  (D)  $\sin 60^\circ$
- 8 Smallest positive number 'p' for which  $f(x + p) = f(x)$  is called  
(A) Domain (B) Range (C) Co - domain (D) Period
- 9 Radius of e - circle opposite to vertex B of triangle ABC is  
(A)  $\frac{\Delta}{s-a}$  (B)  $\frac{\Delta}{s-b}$  (C)  $\frac{\Delta}{s-c}$  (D)  $\frac{\Delta}{s}$
- 10 In an equilateral Triangle ABC  $r_1 : r_2 : r_3$  is equal to  
(A) 1 : 2 : 3 (B) 1 : 3 : 3 (C) 3 : 3 : 3 (D) 2 : 3 : 3
- 11  $\cos^{-1}(-x) = ?$   
(A)  $\pi - \cos^{-1}x$  (B)  $\cos^{-1}x$  (C)  $\pi + \cos^{-1}x$  (D)  $\sin^{-1}x$
- 12 Solution of  $\tan 2x = 1, x \in [0, 2\pi]$  is  
(A)  $\left\{\frac{\pi}{8}, \frac{5\pi}{8}\right\}$  (B)  $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$  (C)  $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$  (D)  $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$
- 13  $(-i)^{19}$  is equal to  
(A)  $-i$  (B) 1 (C)  $-1$  (D)  $i$
- 14 A function  $f : A \rightarrow B$  is surjective if  
(A) Range of  $f = A$  (B) Range of  $f = B$  (C) Range of  $f \neq B$  (D) Both A and B
- 15 A matrix  $m \times 1$  is called  
(A) Scalar Matrix (B) Row Matrix (C) Column Matrix (D) Null Matrix
- 16 If 'A' is a square Matrix of order  $2 \times 2$  then  $|KA|$  is equal to  
(A)  $2K|A|$  (B)  $K^3|A|$  (C)  $K|A|$  (D)  $K^2|A|$
- 17 If one solution of equation  $x^2 - ax + 2 = 0$  is  $x = 1$  the 'a' is equal to  
(A) 0 (B)  $-7$  (C) 7 (D) 3
- 18 A quadratic equation  $ax^2 + bx + c = 0$  becomes linear if  
(A)  $a = 0, b \neq 0$  (B)  $a \neq 0$  (C)  $b = 0$  (D)  $b \neq 0$
- 19  $\frac{A}{x-1} + \frac{B}{x+1}$  are partial fractions of  
(A)  $\frac{1}{x^3-1}$  (B)  $\frac{1}{x^2-1}$  (C)  $\frac{1}{x^2+1}$  (D)  $\frac{1}{x^3+1}$
- 20  $\sum_{k=1}^n k$  is equal to  
(A)  $\frac{n^2(n+1)^2}{4}$  (B)  $\frac{n(n+1)(2n+1)}{6}$  (C)  $\frac{n(n+1)}{2}$  (D)  $\frac{n^2(n+1)}{4}$

SUBJECTIVE  
SECTION-I

DGR-11-1-23

QUESTION NO. 2 Write short answers of any Eight (8) parts of the following

16

i	State commutative law of addition and associative law of multiplication of real numbers.
ii	Separate into real and imaginary parts $\frac{i}{1+i}$
iii	Write the set $\{x/x \in \mathcal{R} \wedge x \neq x\}$ in the descriptive and tabular form
iv	Write converse and inverse of the conditional $\sim p \rightarrow q$
v	Show that the statement $(p \wedge q) \rightarrow p$ is tautology.
vi	If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find the values of "a" and "b"
vii	If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{bmatrix}$ , then find $A_{32}$
viii	If the matrices A and B are symmetric and $AB = BA$ , show that AB is symmetric
ix	Define reciprocal equation.
x	Evaluate $(1 + \omega - \omega^2)^8$
xi	Prove that sum of four 4th roots of unity is zero.
xii	Use remainder theorem to find the remainder when $x^2 + 3x + 7$ is divided by $x + 1$

QUESTION NO. 3 Write short answers of any Eight (8) parts of the following

16

i	What are partial fractions ?
ii	Find the 13th term of the sequence $x, 1, 2-x, 3-2x, \dots$
iii	Find three A.Ms between 3 and 11.
iv	The sum of $S_9$ and $S_7$ is 203 and $S_9 - S_7 = 49$ , $S_7$ and $S_9$ being the sums of the first 7 and 9 terms of an A.P respectively. Determine the series.
v	If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in G.P show that the common ratio is $\pm \sqrt{\frac{a}{c}}$
vi	Find the Geometric means between 4 and 16.
vii	Find the value of n when ${}^n P_4 : {}^{n-1} P_3 = 9 : 1$
viii	In how many ways can 4 keys be arranged on a circular key ring?
ix	A natural number is chosen out of first fifty natural numbers. What is the probability that the chosen number is a multiple of 3 or of 5?
x	Prove the formula for $n = -1, 0, 3+5+7+\dots+(2n+5) = (n+2)(n+4)$
xi	Expand $(a - \sqrt{2}x)^4$
xii	Expand the following up to 4 terms $(2 - 3x)^{-2}$

QUESTION NO. 4 Write short answers of any Nine (9) parts of the following

18

i	Show that the area of a sector of a circular region of radius r is $\frac{1}{2} r^2 \theta$ , where $\theta$ is the circular measure of central angle of the sector.
ii	If $\tan \theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in III quad, find the value of $\frac{\operatorname{Cosec}^2 \theta - \sec^2 \theta}{\operatorname{Cosec}^2 \theta + \sec^2 \theta}$
iii	Prove the identity $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$
iv	Prove that $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$
v	Prove $\frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{\cos(\alpha+\beta) + \cos(\alpha-\beta)} = \tan \alpha$
vi	Prove the identity $\cot \alpha - \tan \alpha = 2 \cot 2\alpha$
vii	Find the period of $\sec 9x$
viii	Find the area of $\Delta ABC$ , given three sides $a=18, b=24, c=30$
ix	Show that the $r_3 = s \tan \frac{Y}{2}$
x	Prove that $\tan \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
xi	Without using Calculator show that $\operatorname{Cos}^{-1} \frac{4}{5} = \operatorname{Cot}^{-1} \frac{4}{3}$
xii	Solve $\sec^2 \theta = \frac{4}{3}$ , $\theta \in [0, 2\pi]$
xiii	Find the value of $\theta$ $2 \sin^2 \theta - \sin \theta = 0$ , $\theta \in [0, 2\pi]$

**SECTION-II**

*DGK-11-1-23*

Note: Attempt any Three questions from this section

10 x 3 = 30

Q. 5-(A)	Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x + 3)(x - 1)^3$
(B)	Solve the equation $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$
Q. 6 -(A)	Resolve $\frac{1}{(x-1)^2(x+1)}$ into partial fraction
(B)	Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$
Q. 7-(A)	If $y = \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \dots$ and if $0 < x < 2$ then prove that $x = \frac{2y}{1+y}$
(B)	Identify the series : $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \dots$ as a binomial expansion and find its sum.
Q. 8 -(A)	If $\cot \theta = \frac{5}{2}$ and the terminal arm of the angle is in 1st quadrant. Find the values of $\frac{3 \sin \theta + 4 \cos \theta}{\cos \theta - \sin \theta}$
(B)	Prove that $\frac{2 \sin \theta \sin 2 \theta}{\cos \theta + \cos 3 \theta} = \tan 2 \theta \tan \theta$
Q. 9 -(A)	Prove that $r_1 + r_2 + r_3 - r = 4R$
(B)	Prove that $\cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5} = \sin^{-1} \frac{3}{5}$

**MATHEMATICS**  
**GROUP : SECOND**

**OBJECTIVE**

**TIME: 30 MINUTES**

**MARKS: 20**

**NOTE :** You have four choices for each objective type question as A , B , C and D . The choice which you think is correct , fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

**QUESTION NO. 1**

- |    |   |
|----|---|
| 1  | If $x^{1/4} = -2$ then $x =$ _____<br>(A) 8 (B) -8 (C) 16 (D) -16   |
| 2  | If $w$ is the cube root of unity. Then $(1 + w - w^2)^8 =$ _____<br>(A) 256 (B) -256 (C) -256 $w$ (D) 256 $w$   |
| 3  | Degree of a constant polynomial is<br>(A) 1 (B) 0 (C) 2 (D) 3   |
| 4  | A.M between $1-x+x^2$ and $1+x+x^2$ is<br>(A) $x^2 + 1$ (B) $x+1$ (C) $\frac{x+1}{2}$ (D) $\frac{x^2+1}{2}$   |
| 5  | If $a_n = (-1)^n (2n - 3)$ Then $a_5 =$ _____<br>(A) 7 (B) -7 (C) 13 (D) -13  |
| 6  | If $n$ is a negative integer. Then $n!$ is<br>(A) 1 (B) Not defined (C) Zero (D) $n$  |
| 7  | Number of ways of writing the letters of the "WORD" taken all at a time.<br>(A) 24 (B) 4 (C) 6 (D) 25   |
| 8  | Francesco Maurolico devised the method of<br>(A) Partial fraction (B) Logarithm (C) Induction (D) Binomial expansion  |
| 9  | The middle term in the expansion of $(x - y)^{12}$ is<br>(A) 5th (B) 6th (C) 8th (D) 7th  |
| 10 | One radian is equal to<br>(A) $57.296^\circ$ (B) $57^\circ$ (C) $56^\circ$ (D) $0.01875^\circ$  |
| 11 | $\sin 8\theta - \sin 4\theta =$ _____<br>(A) $2 \sin 6\theta \sin 4\theta$ (B) $2 \cos 2\theta \sin 6\theta$ (C) $2 \cos 6\theta \sin 2\theta$ (D) $-2 \sin 6\theta \cos 2\theta$                   |
| 12 | Period of $\tan \frac{x}{3}$ is<br>(A) $\pi$ (B) $2\pi$ (C) $3\pi$ (D) $\frac{\pi}{2}$  |
| 13 | Radius of Escribed circle apposite to the vertex B is equal to<br>(A) $\frac{\Delta}{s}$ (B) $\frac{\Delta}{s-c}$ (C) $\frac{\Delta}{s-a}$ (D) $\frac{\Delta}{s-b}$                                 |
| 14 | With usual notation $\frac{abc}{4\Delta}$ is equal to<br>(A) $r$ (B) $2r$ (C) $R$ (D) $r_1$   |
| 15 | The domain of $y = \sin^{-1} x$ is<br>(A) $-1 \leq x < 1$ (B) $-1 < x < 1$ (C) $-\pi/2 \leq x \leq \pi/2$ (D) $-\pi/2 < x < \pi/2$  |
| 16 | If $\sin x = \cos x$ then $x =$ _____<br>(A) $30^\circ$ (B) $45^\circ$ (C) $0^\circ$ (D) $60^\circ$   |
| 17 | $ a + ib $ is equal to<br>(A) $a^2 + b^2$ (B) $\sqrt{a^2 + b^2}$ (C) $a^2 - b^2$ (D) $\sqrt{a^2 - b^2}$   |
| 18 | If $A^c$ is complement of set A. Then $A \cap A^c =$ _____<br>(A) $A$ (B) $A^c$ (C) $\cup$ (D) $\emptyset$  |
| 19 | If a system of linear equation has a unique solution or infinitely many solutions. Then it can be known as<br>(A) Consistent System (B) Inconsistent System (C) Non linear System (D) Unique System |
| 20 | Transpose of Matrix $A = [a_{ij}]_{m \times n}$ is equal to<br>(A) $[a_{ij}]_{r \times m}$ (B) $[a_{ij}]_{m \times n}$ (C) $[a_{ij}]_{n \times m}$ (D) $[a_{ij}]_{n \times n}$                      |

**MATHEMATICS**  
**GROUP : SECOND**

**SUBJECTIVE**

**TIME : 2.30 HOURS**

**SECTION-I**

**MARKS : 80**

*DGK-11-2-23*

**QUESTION NO. 2 Write short answers of any Eight (8) parts of the following** **16**

i	Whether closed or not with respect to addition and multiplication is {1}
ii	Simplify $(-1)^{-21}$
iii	Write down power set of $\{\emptyset\}$
iv	Verify De - Morgan's laws for sets $U = \{1,2,3, \dots, 20\}$ $A = \{2,4,6, \dots, 20\}$ , $B = \{1,3,5, \dots, 19\}$
v	Construct truth table for statement $(p \wedge \sim p) \rightarrow q$
vi	If $A = \begin{bmatrix} 1 & 0 \\ 1 & -i \end{bmatrix}$ show that $A^4 = I_2$
vii	Without expansion show that $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$
viii	Define Hermitian Matrix
ix	Evaluate $(-1 + \sqrt{-3})^5 + (-1 - \sqrt{3})^5$
x	When the polynomial $x^3 + 2x^2 + kx + 4$ is divided by $x - 2$ , remainder is 14. Find the value of 'x'
xi	Solve the system of equations $x + y = 5$ , $\frac{2}{x} + \frac{3}{y} = 2$ , $x \neq 0$ , $y \neq 0$
xii	Sum of positive number and its square is 380. Find the number.

**QUESTION NO. 3 Write short answers of any Eight (8) parts of the following** **16**

i	Define improper rational fraction and give one example.
ii	Determine whether 2 is a term of the A.P 17,13,9,.....
iii	If 5, 8 are two A.Ms between "a" and "b", find a and b
iv	Sum the series $(x - a) + (x + a) + (x + 3a) + \dots$ to n terms
v	Find the 5th term of the G.P : 3, 6, 12, .....
vi	If the numbers $\frac{1}{k}$ , $\frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k.
vii	Find the value of n when ${}^n P_2 = 30$
viii	How many arrangements of the letters of word PAKISTAN, taken all together, can be made.
ix	What is the probability that a slip of numbers divisible by 4 is picked from the slips bearing numbers 1,2,3,.....,10 ?
x	Prove that $n! > n^2$ for $n = 4,5$
xi	Find the term independent of x in the expansion of $(x - \frac{2}{x})^{10}$
xii	Expand upto 3 terms $(4 - 3x)^{1/2}$

**QUESTION NO. 4 Write short answers of any Nine (9) parts of the following** **18**

i	What is the circular measure of the angle between the hands of a watch at 4 O' Clock ?
ii	Find the value of $\sin \theta$ and $\cos \theta$ if $\tan \theta = -\frac{1}{3}$ and the terminal arm of the angle is in quadrant II
iii	Prove that $\sec^2 A + \operatorname{Cosec}^2 A = \sec^2 A \operatorname{Cosec}^2 A$ (Where $A \neq \frac{n\pi}{2}$ , $n \in Z$ )
iv	Prove that $\sin(180^\circ + \alpha) \sin(90^\circ - \alpha) = -\sin \alpha \cos \alpha$
v	Find the value of $\tan 105^\circ$
vi	Express $\cos(2x + 30^\circ) \cos(2x - 30^\circ)$ as sum or differences.
vii	Find the period of $3 \cos \frac{x}{5}$
viii	Solve the triangle ABC if $\beta = 60^\circ$ , $\gamma = 15^\circ$ , $b = \sqrt{6}$
ix	Find the area of the triangle ABC $b=37$ , $c=45$ , $\alpha = 30^\circ 50'$
x	Prove that $R = \frac{abc}{4\Delta}$
xi	Find the value of $\sec \left[ \sin^{-1} \left( -\frac{1}{2} \right) \right]$
xii	Find the solution of equation which lies in $[0, 2\pi]$ $\sec x = -2$
xiii	Find the value of $\theta$ satisfying the following equation $2 \sin^2 \theta - \sin \theta = 0$ , $\theta \in [0, 2\pi]$

**SECTION-II**

Note: Attempt any Three questions from this section

DGK-11-2-23

10 x 3 = 30

Q. 5-(A)	<p>Show that <math>\begin{vmatrix} x &amp; 1 &amp; 1 &amp; 1 \\ 1 &amp; x &amp; 1 &amp; 1 \\ 1 &amp; 1 &amp; x &amp; 1 \\ 1 &amp; 1 &amp; 1 &amp; x \end{vmatrix} = (x+3)(x-1)^3</math></p> <p>(B) Solve the equation simultaneously <math>\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1</math></p>
Q. 6 -(A)	<p>Resolve <math>\frac{x^2+x-1}{(x+2)^3}</math> into partial fraction</p> <p>(B) There are 20 chits marked 1,2,3,....., 20 in a bag. Find the probability of picking a chit, the number written on which is a multiple of 4 or a multiple of 7</p>
Q. 7-(A)	<p>If <math>\ell</math>, <math>m</math>, <math>n</math> are the <math>p</math>th, <math>q</math>th and <math>r</math>th terms of A.P, show that <math>\ell(q-r) + m(r-p) + n(p-q) = 0</math></p> <p>(B) Find the term involving <math>x^5</math> in the expansion of <math>\left(\frac{3x}{2} - \frac{1}{3x}\right)^{11}</math></p>
Q. 8 -(A)	<p>If <math>\operatorname{cosec} \theta = \frac{m^2+1}{2m}</math> and <math>m &gt; 0</math> (<math>0 &lt; \theta &lt; \frac{\pi}{2}</math>), find the values of the remaining trigonometric ratios.</p> <p>(B) If <math>\alpha, \beta, \gamma</math> are angles of <math>\Delta ABC</math>, prove that <math>\tan \alpha + \tan \beta + \tan \gamma = \tan \gamma \tan \beta \tan \alpha</math></p>
Q. 9 -(A)	<p>Prove that <math>r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2</math></p> <p>(B) Prove that <math>\sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} = \cos^{-1} \frac{253}{325}</math></p>

OBJECTIVE

NOTE: You have four choices for each objective type question as A , B , C and D . The choice which you think is correct , fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

**QUESTION NO. 1**

- (1) Multiplicative identity in complex numbers is  
 (A) (0,0) (B) (0,1) (C) (1,1) (D) (1,0)
- (2) Set  $\{1, w, w^2\}$  is closed w.r.t  
 (A) Addition (+) (B) Multiplication ( $\times$ ) (C) Both A and B (D) Division ( $\div$ )
- (3) Let A be not a square matrix , then  $|A^t| =$   
 (A)  $A^{-1}$  (B)  $|A|^t$  (C)  $|A|$  (D) Not defined
- (4) If A is a matrix of order  $3 \times 1$  , then the order of  $AA^t$  is  
 (A)  $1 \times 3$  (B)  $1 \times 1$  (C)  $3 \times 3$  (D)  $3 \times 1$
- (5) If  $x^{1/4} = -2$  then  $x =$   
 (A) 8 (B) -8 (C) 16 (D) -16
- (6) Remainder is = 11 if  $x^2 + 3x + 7$  is divided by  
 (A)  $x+1$  (B)  $x+2$  (C)  $x+3$  (D)  $x-1$
- (7) The number of co-efficients in the partial fraction of  $\frac{1}{(x-1)^2(x^2+16)}$  are  
 (A) 2 (B) 3 (C) 4 (D) 5
- (8) 26<sup>th</sup> term of  $a_n = (-1)^{n+1}$  is  
 (A) 1 (B) -1 (C) 26 (D) -26
- (9) Relation between A , G , H , is  
 (A)  $A > G > H$  (B)  $A < G < H$  (C) Both A and B (D)  $A > G < H$
- (10) Reciprocal of the sequence  $1/3, 1/5, 1/7, \dots$  forms  
 (A) Geometric sequence (B) Arithmetic sequence (C) Harmonic sequence (D) Null sequence
- (11)  ${}^{n+1}C_r + {}^{n+1}C_{r-1} =$   
 (A)  ${}^{n+1}C_r$  (B)  ${}^{n+2}C_{r-1}$  (C)  ${}^{n+1}C_{r+1}$  (D)  ${}^{n+2}C_r$
- (12) In the middle term  $T_{r+1}$  of the binomial expansion of  $(a+b)^{12}$  ,  $\gamma =$   
 (A) 6 (B) 7 (C) 5 (D) 12
- (13) Which of the following is quadrantal Angle  
 (A)  $350^\circ$  (B)  $-390^\circ$  (C)  $-360^\circ$  (D)  $410^\circ$
- (14)  $\frac{-9\pi}{2}$  coincides with  
 (A) OX (B) OY (C) OX' (D) OY'
- (15)  $\sin(-300^\circ) =$   
 (A)  $\frac{-\sqrt{3}}{2}$  (B)  $\frac{\sqrt{3}}{2}$  (C)  $\frac{2}{\sqrt{3}}$  (D)  $\frac{1}{\sqrt{2}}$
- (16) The period of  $3 \sin \frac{x}{3}$  is  
 (A)  $6\pi$  (B)  $2\pi$  (C)  $3\pi$  (D)  $4\pi$
- (17) The radius of inscribed circle is  
 (A)  $\frac{abc}{4\Delta}$  (B)  $\frac{\Delta}{s}$  (C)  $\frac{\Delta}{s-a}$  (D)  $\frac{\Delta}{s-b}$
- (18)  $\frac{c^2 \sin \alpha \sin \beta}{\sin \gamma} =$   
 (A)  $\Delta$  (B)  $\frac{\Delta}{2}$  (C)  $2\Delta$  (D)  $\Delta S$
- (19)  $\cos(\tan^{-1}(0)) =$   
 (A) 0 (B) -1 (C) 1 (D)  $\infty$
- (20) If  $\cos x = 0$  then number of solutions are  
 (A) 2 (B) 4 (C) 6 (D) Infinite

QUESTION NO. 2 Write short answers of any Eight (8) parts of the following

16

1	Check the closure property in the set $\{0, -1\}$ w.r.t addition and multiplication
2	Find the multiplicative inverse of the number $(\sqrt{2}, -\sqrt{5})$
3	If $Z$ is any complex number, then prove that $Z\bar{Z} =  Z ^2$
4	Write the descriptive form and tabular form of the set $\{x x \in O \wedge 5 \leq x \leq 7\}$
5	Show that the statement $(p \wedge q) \rightarrow P$ is a tautology
6	Show that the set of natural numbers $N$ is non-commutative and non-associative w.r.t subtraction
7	Find the values of $x$ and $y$ if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$
8	Find the matrix $X$ , if $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$
9	Find the value of $\lambda$ if matrix $A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ is singular
10	Find the roots of the equation $5x^2 - 13x + 6 = 0$
11	Find four fourth roots of unity
12	When the polynomial $x^4 + 2x^3 + kx^2 + 3$ is divided by $x - 2$ , the remainder is 1. Find the value of $k$

QUESTION NO. 3 Write short answers of any Eight (8) parts of the following

16

1	Resolve $\frac{9}{(x+2)^2(x-1)}$ into partial fraction without finding the constants A, B and C
2	Resolve $\frac{3x+7}{(x^2+4)(x+3)}$ into partial fraction without finding the constants A, B and C.
3	Which term of the A.P $-2, 4, 10, \dots$ is 148 ?
4	Find the 5 <sup>th</sup> term of the G.P $3, 6, 12, \dots$
5	Find the sum of the infinite G.P $2, \sqrt{2}, 1, \dots$
6	Find A, G, H if $a = \frac{-2}{5}$ , $b = \frac{-8}{5}$
7	Evaluate ${}^9P_8$
8	How many arrangements of the letters of the word "ATTACKED" can be made if each arrangement begins with C and ends with K ?
9	Find the value of $n$ when ${}^nC_{12} = {}^nC_6$
10	Show that the inequality $4^n > 3^n + 4$ is true for $n = 2, 3$
11	Calculate $(9.98)^4$ by using binomial theorem.
12	Expand $(8-2x)^{-1}$ up to 4 terms by using binomial theorem

QUESTION NO. 4 Write short answers of any Nine (9) parts of the following

18

1	Express the sexagesimal measure of angle $120'40''$ in radian
2	Verify $\sin 2\theta = 2\sin\theta \cos\theta$ , when $\theta = 30^\circ, 45^\circ$
3	Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ , where $\theta$ is not an odd multiple of $\frac{\pi}{2}$
4	Without using the tables, Find the value of $\cot(-855^\circ)$
5	Prove that $\frac{1-\tan\theta \tan\phi}{1+\tan\theta \tan\phi} = \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$
6	Express the difference $\sin 8\theta - \sin 4\theta$ as product
7	Find the period of $3 \cos \frac{x}{5}$
8	A vertical pole is 8m high and length of its shadow is 6m. What is the angle of elevation of the sun at that moment?
9	Find the smallest angle of the triangle ABC, when $a = 37.34$ , $b = 3.24$ , $c = 35.06$
10	Find the area of a triangle ABC, when $b = 37$ , $c = 45$ , $\alpha = 30^\circ 50'$
11	Without using tables/calculator, Find $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
12	Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$ which lie in $[0, 2\pi]$
13	Solve the trigonometric equation $\tan^2\theta = \frac{1}{3}$ in $[0, 2\pi]$

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SECTION-II

Note: Attempt any Three questions from this section

10 x 3 = 30

<p>Q. 5-(A)</p>	<p>Use Cramer's rule to solve</p> $\begin{aligned} 3x_1 + x_2 - x_3 &= -4 \\ x_1 + x_2 - 2x_3 &= -4 \\ -x_1 + 2x_2 - x_3 &= 1 \end{aligned}$ <p>(B) Show that the roots of <math>x^2 + (mx + c)^2 = a^2</math> will be equal if <math>c^2 = a^2(1 + m^2)</math></p>
<p>Q. 6-(A)</p>	<p>Resolve into partial fraction <math>\frac{x^2+1}{x^3+1}</math></p> <p>(B) For what value of <math>n</math>, <math>\frac{a^n+b^n}{a^{n-1}+b^{n-1}}</math>, is the positive geometric mean between <math>a</math> and <math>b</math></p>
<p>Q. 7-(A)</p>	<p>How many numbers greater than 1000,000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4</p> <p>(B) Find the term independent of <math>x</math> in the expansion of <math>\left(x - \frac{2}{x}\right)^{10}</math></p>
<p>Q. 8-(A)</p>	<p>Prove that : <math>\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta \cos^2\theta)</math></p> <p>(B) If <math>\tan \alpha = \frac{3}{4}</math>, <math>\cos \beta = \frac{5}{13}</math> and neither the terminal side of the angle of measure <math>\alpha</math> nor that of <math>\beta</math> is in the I quadrant, Find <math>\sin(\alpha + \beta)</math></p>
<p>Q. 9-(A)</p>	<p>Prove that in an equilateral triangle <math>r : R : r_1 = 1 : 2 : 3</math></p> <p>(B) Prove that <math>2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}</math></p>

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

## QUESTION NO. 1

- (1) If degree of  $P(x)$  is less than degree of  $Q(x)$  then rational fraction  $\frac{P(x)}{Q(x)}$  is called  
(A) Proper rational fraction (B) Improper rational fraction (C) Common fraction  
(D) Rational number
- (2) Next term of the sequence 7, 9, 12, 16, ..... is  
(A) 20 (B) 21 (C) 22 (D) 23
- (3) A.M between  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  is  
(A) 4 (B)  $\sqrt{3}$  (C) 2 (D)  $2\sqrt{3}$
- (4) No term of a Harmonic sequence can be  
(A) 1 (B) -1 (C) 2 (D) 0
- (5) Factorial of 0 i.e  $0!$  is equal to  
(A) 2 (B) 0 (C) Does not exist (D) 1
- (6) The number of terms in the binomial expansion  $(a+x)^6$  are  
(A) 7 (B) 6 (C) 5 (D) 4
- (7) 60<sup>th</sup> part of a minute is called  
(A) Second (B) Minute (C) Degree (D) Hour
- (8)  $\frac{1}{2}$  Rotation in clock wise direction equals to  
(A)  $180^\circ$  (B)  $-180^\circ$  (C)  $90^\circ$  (D)  $-90^\circ$
- (9)  $\sin\left(\frac{\pi}{2} + \alpha\right)$  equals to  
(A)  $-\cos \alpha$  (B)  $\sin \alpha$  (C)  $\cos \alpha$  (D)  $-\sin \alpha$
- (10) Period of Secant Function is  
(A)  $\pi$  (B)  $3\pi$  (C)  $4\pi$  (D)  $2\pi$
- (11) In any triangle ABC with usual notations  $\frac{b^2+c^2-a^2}{2bc}$  equals to  
(A)  $\cos \beta$  (B)  $\cos \alpha$  (C)  $\sin \beta$  (D)  $\sin \alpha$
- (12) If the sides of a triangle are 18, 24, 30 then the value of S is  
(A) 36 (B) 72 (C) 144 (D) 24
- (13) The function  $y = \cos x$  is called principal cosine if  
(A)  $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$  (B)  $\frac{-\pi}{2} < x < \frac{\pi}{2}$  (C)  $0 \leq x \leq \pi$  (D)  $0 < x < \pi$
- (14) If  $\sin x = \frac{-1}{\sqrt{2}}$  then the reference angle is  
(A)  $\frac{\pi}{3}$  (B)  $\frac{-\pi}{4}$  (C)  $\frac{-\pi}{3}$  (D)  $\frac{\pi}{4}$
- (15) "0" is  
(A) Irrational number (B) Positive integer (C) Rational number (D) Negative integer
- (16) The set  $\{x \mid x \in \mathbb{R} \wedge x \neq x\}$  is  
(A) Empty set (B) Infinite set (C) Singleton set (D) Binary set
- (17) Which of the following has no inverse?  
(A) Identity matrix (B) Singular matrix (C) Diagonal matrix (D) Non singular matrix
- (18) If order of the matrix A is  $m \times n$  and order of B is  $n \times p$  then order of AB is equal to  
(A)  $p \times m$  (B)  $m \times m$  (C)  $n \times n$  (D)  $m \times p$
- (19) If 1,  $w$ ,  $w^2$  are cube roots of unity then  $w + w^2 =$   
(A) 1 (B)  $w$  (C) -1 (D) 0
- (20) The degree of the polynomial  $ox^{15} + x^{14} + x^{12} + 5$  is  
(A) 15 (B) 14 (C) 12 (D) 5

QUESTION NO. 2 Write short answers of any Eight (8) parts of the following 16

1	Prove that $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
2	Simplify $(2, 6)(3, 7)$
3	Factorize $a^2 + 4b^2$
4	Verify the commutative property of union if $A = \{1, 2, 3, 4, 5\}$ ; $B = \{4, 6, 8, 10\}$
5	Write two proper subsets of $\{a, b, c\}$
6	Find the inverse of the relation $\{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$
7	Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$
8	If $A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$ then find $AA^t$
9	If $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$ Show that $A+(\bar{A})^t$ is hermitian
10	Solve $x^2 + 7x + 12 = 0$ by factorization
11	Show that $x^3 - y^3 = (x - y)(x - wy)(x - w^2y)$
12	Show that the roots of the equation $(P + q)x^2 - Px - q = 0$ will be rational

QUESTION NO. 3 Write short answers of any Eight (8) parts of the following 16

1	Resolve into partial fraction $\frac{x^2+x-1}{(x+2)^3}$ without finding values of unknown constants
2	Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fraction
3	Find the next two terms of 1, 3, 7, 15, 31, .....
4	Find the Arithmetic Mean (A.M) between $x - 3$ and $x + 5$
5	Find the sum of Geometric progression 2, $\sqrt{2}$ , 1, .....
6	Find the 12 <sup>th</sup> term of $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \dots$
7	Find the value of n, when ${}^{11}P_n = 11.10.9$
8	What is the probability that a slip of a number divisible by 4 is picked from the slips bearing numbers 1, 2, 3, ..... 10 ?
9	A die is thrown twice, what is the probability that the sum of the numbers of dots shown 3 or 11
10	Evaluate $\sqrt[3]{30}$ correct to three decimal
11	Use mathematical induction to prove that $1 + 5 + 9 + \dots + (4n - 3) = n(2n - 1)$ is true for $n = 1$ and $n = 2$
12	Determine the middle term of the expansion $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$

QUESTION NO. 4 Write short answers of any Nine (9) parts of the following 18

1	Find the value of $\sin \theta$ and $\cos \theta$ if $\theta = \frac{-9\pi}{2}$
2	Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec \theta - \tan \theta$ , where $\theta$ is not an odd multiple of $\frac{\pi}{2}$
3	Convert $54^\circ 45'$ into radian
4	Prove that $\sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2} \cos 2\theta$
5	Prove that $\cot \alpha - \tan \alpha = 2 \cot 2\alpha$
6	Without using calculator, prove that $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$
7	Find the period of $\cos 2x$
8	The area of triangle is 121.34. If $\alpha = 32^\circ 15'$ , $\beta = 65^\circ 37'$ , then find c and angle $\gamma$
9	Prove that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$
10	Solve the right triangle ABC in which $\gamma = 90^\circ$ and $\alpha = 62^\circ 40'$ , $b = 796$
11	Show that $\sin^{-1}(-x) = -\sin^{-1}x$
12	Solve the equation $\cot \theta = \frac{-1}{\sqrt{3}}$ , $\theta \in [0, 2\pi]$

**SECTION-II**

**Note: Attempt any Three questions from this section**

**10 x 3 = 30**

Q. 5-(A)	Find the rank of matrix $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$
(B)	Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$ , $a \neq 0$ , $b \neq 0$
Q. 6 -(A)	Resolve into partial fractions $\frac{9}{(x+2)^2(x-1)}$
(B)	Find four numbers in A.P. whose sum is 32 and sum of whose squares is 276
Q. 7-(A)	A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen number is a multiple of 3 or of 5
(B)	Use mathematical induction to prove that $1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2(2n^2-1)$ is true for every positive integer 'n'
Q. 8 -(A)	Find the values of all trigonometric functions of the angle $\theta = \frac{-17\pi}{3}$
(B)	Prove without using calculator $\sin 19^\circ \cos 11^\circ + \sin 71^\circ \sin 11^\circ = \frac{1}{2}$
Q. 9 -(A)	Prove that $\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{5}{6} = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$
(B)	Solve the triangle ABC in which $a = \sqrt{3} - 1$ , $b = \sqrt{3} + 1$ and $\gamma = 60^\circ$