Exercise 1.1

1. **Identify each of the following as a rational or irrational numbers:**

(i)	2.353535	(ii)	$0.\bar{6}$	(iii)	2.236067…
(iv)	$\sqrt{7}$	(v)	e	(vi)	π
(vii)	$5+\sqrt{11}$	(viii)	$\sqrt{3}+\sqrt{13}$		
(ix)	$\frac{15}{4}$	(x)	$(2-\sqrt{2})(2+\sqrt{2})$		

Let's go through each number and determine whether it is rational or irrational.

(i) 2.353535

Solution: This is a repeating decimal $(2.\overline{35})$, which can be written as a fraction. Therefore, it is a **rational number.**

(ii) $0.\bar{6}$

Solution: This is a repeating decimal $(0.\bar{6})$, which can be written as a fraction. Therefore, it is a **rational number.**

(iii) 2.236067...

Solution: This is the decimal expansion of $\sqrt{5}$, which is non-repeating and non-termirating. Therefore, it is an **irrational number.**

(iv) $\sqrt{7}$

Solution: The square root of 7 is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number.**

(v) e

Solution: $e = 2.71828182845904\ldots$

The mathematical constant e is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number**.

(vi) π

Solution: $\pi = 3.141592653589793 2\ldots$

The mathematical constant π is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number**.

(vii) $5 + \sqrt{11}$

Solution: Since $\sqrt{11}$ is irrational and adding a rational number (5) to it results in an irrational number, this is an **irrational number**.

(viii) $\sqrt{3} + \sqrt{13}$

Solution: Both $\sqrt{3}$ and $\sqrt{13}$ are irrational, and their sum is also irrational. Therefore, it is an **irrational number**.

(ix) $\frac{15}{4}$

Solution: This is a fraction, which is a rational number. Therefore, it is a **rational number**.

(x) $(2 - \sqrt{2})(2 + \sqrt{2})$

Solution: This expression simplifies to $2^2 - (\sqrt{2})^2 = 4 - 2 = 2$, which is a rational number. Therefore, it is a **rational number**.

2. **Represent the following numbers on number line:**

(i)	$\sqrt{2}$	(ii)	$\sqrt{3}$	(iii)	$4\frac{1}{3}$
(iv)	$-2\frac{1}{7}$	(v)	$\frac{5}{8}$	(vi)	$2\frac{3}{4}$

(i) $\sqrt{2}$

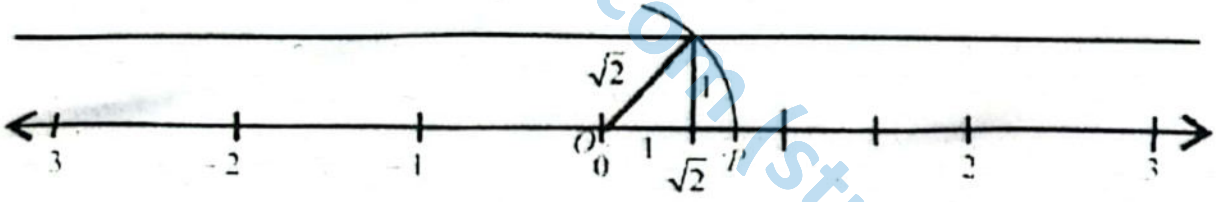
Solution: To represent $\sqrt{2}$ on a number line, we can follow a similar approach to the one outlined in your example:

- **Approximate Value**: $\sqrt{2} \approx 1.414$, which is a value slightly greater than 1.
- **Constructing the Number Line**:
    - Draw a line and mark points for 0 and 1.
    - Measure the distance from 0 to 1 as 1 unit.
    - Construct a right-angle triangle with one leg as 1 unit (the segment from 0 to 1) and the other leg also 1 unit.
    - Use the Pythagorean theorem to find the hypotenuse:

$$(mOB)^2 = (mOA)^2 + (mAB)^2 = 1^2 + 1^2 = 2 \Rightarrow mOB = \sqrt{2}.$$

    - Draw the arc with radius $\sqrt{2}$ from the point O (which is the origin) on the number line.
- **Locating $\sqrt{2}$ on the Number Line**: The point P where the arc intersects the line represents the value $\sqrt{2}$.

Thus, $\sqrt{2}$ is located at approximately 1.414 units from 0, slightly more than 1 but less than 2, on the number line.



(ii) $\sqrt{3}$

Solution: To represent $\sqrt{3}$ on a number line, follow these steps:

- **Estimate the value**:
  $\sqrt{3} \approx 1.732$, which is between 1 and 2 on the number line.
- **Draw a line segment from 0 to 2**:
  Mark points 0, 1, and 2 on the line.
- **Construct a right-angled triangle**:
    - Draw a horizontal line segment of length 1 from point 0 to point A.

- From point A, draw a vertical line segment of length $\sqrt{3}$ (approximately 1.732) to point B, forming a right triangle with side lengths 1 and $\sqrt{3}$.

- **Use Pythagoras' theorem:**
    - The hypotenuse of the triangle will be $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2.$

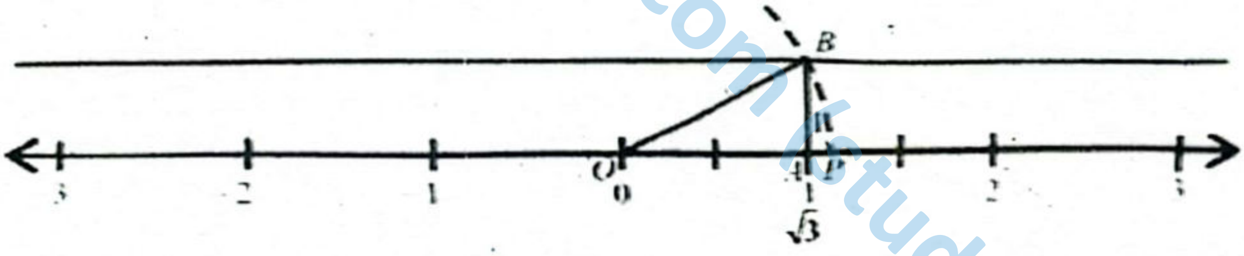
- **Draw the arc:**

  Draw an arc with a radius of length $\sqrt{3}$ (from the origin) on the number line, intersecting the line at the point representing $\sqrt{3}$.

- **Mark the point:**

  The intersection point represents $\sqrt{3}$ on the number line.

Thus, the point corresponding to $\sqrt{3}$ is located slightly before 2 and closer to 1.



(iii) $4\frac{1}{3}$

Solution: To represent $4\frac{1}{3}$ on the number line, follow these steps:

- **Convert the mixed fraction to an improper fraction:**

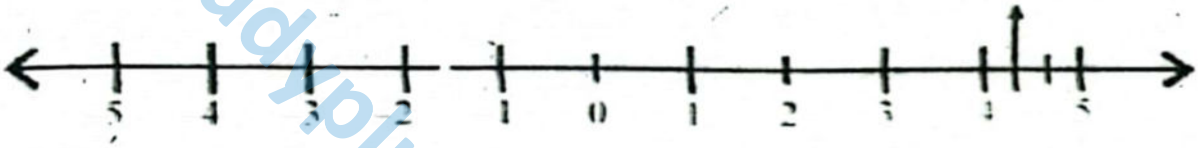
  $4\frac{1}{3} = \frac{13}{3}.$

- **Estimate the value:**

  $\frac{13}{3} = 4.333 ...$, which is slightly more than 4 but less than 5.

- **Draw a number line:**
  Mark the points 0, 1, 2, 3, 4, and 5 on the number line.
- **Locate $4\frac{1}{3}$:**
  - Since $4\frac{1}{3}$ is just slightly more than 4, you will mark a point between 4 and 5.
  - Divide the segment between 4 and 5 into 3 equal parts, as the denominator is 3.
  - $4\frac{1}{3}$ will be at the first mark after 4.

Thus, the point representing $4\frac{1}{3}$ will be just past 4 on the number line, about one-third of the way toward 5.



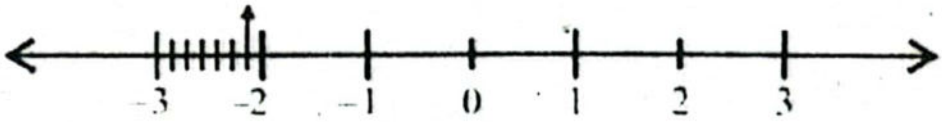
(iv) $-2\frac{1}{7}$

Solution: To represent $-2\frac{1}{7}$ on the number line, follow these steps:

- **Convert the mixed fraction to an improper fraction:**
  $-2\frac{1}{7} = -\frac{15}{7}$.
- **Estimate the value:**
  $-\frac{15}{7} \approx -2.142857$, which is slightly more than -2 but less than -3.
- **Draw a number line:**
  Mark the points $-3, -2, -1, 0, 1, 2, 3$ on the number line.
- **Locate $-2\frac{1}{7}$:**
  - Since $-2\frac{1}{7}$ is slightly more than -2, it will be located just past -2 but before -3.
  - Divide the segment between -2 and -3 into 7 equal parts (since the denominator is 7).

- $-2\frac{1}{7}$ will be at the first mark after -2, which is one-seventh of the way toward -3.

Thus, the point representing $-2\frac{1}{7}$ will be just past -2 on the number line.

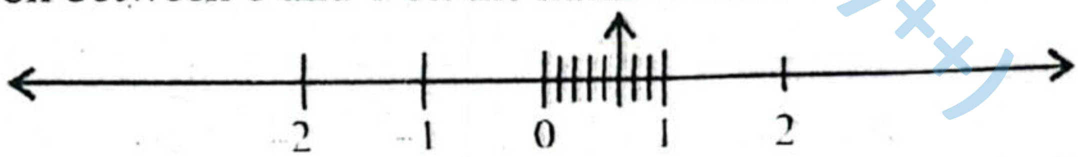


(v) $\frac{5}{8}$

Solution: To represent $\frac{5}{8}$ on the number line, follow these steps:

1. **Estimate the value:**
   $\frac{5}{8} = 0.625.$
2. **Draw a number line:**
   Mark the points 0, 1 on the number line.
3. **Divide the segment between 0 and 1 into 8 equal parts:**
   Since the denominator is 8, divide the space between 0 and 1 into 8 equal parts.
4. **Locate** $\frac{5}{8}$:Since $\frac{5}{8} = 0.625$, it will be 5 parts out of 8 between 0 and 1.

Thus, the point representing $\frac{5}{8}$ will be located after the 5th division between 0 and 1 on the number line.



(vi) $2\frac{3}{4}$

***Solution:* To represent** $2\frac{3}{4}$ **on the number line, follow these steps:**

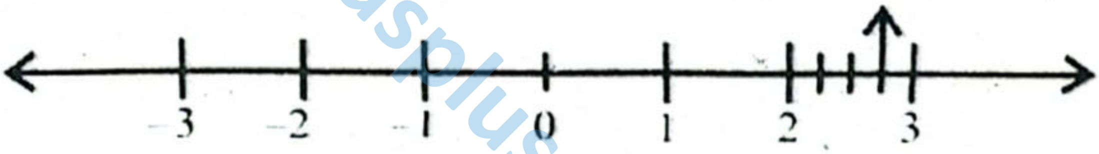
- **Convert the mixed fraction:**
  $2\frac{3}{4}$ can be written as the improper fraction:

$$2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}.$$

So, $2\frac{3}{4} = 2.75$.

- **Draw a number line:**
  Mark the integers 0, 1, 2, and 3 on the number line.
- **Divide the segment between 2 and 3 into 4 equal parts:**
  Since the denominator is 4, divide the space between 2 and 3 into 4 equal parts.
- **Locate $2\frac{3}{4}$:**
  $2\frac{3}{4} = 2.75$ is 3 parts out of 4 between 2 and 3.

Thus, the point representing $2\frac{3}{4}$ will be located after the 3rd division between 2 and 3 on the number line.



3. **Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$**

(i) $0.\overline{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

Let's solve $0.\overline{4}$ step-by-step using the same method:

Step 1: Represent the repeating decimal mathematically.

Let:

$$x = 0.\overline{4}$$

This means:

$$x = 0.4444\ldots$$

Step 2: Multiply by a power of 10 to align the repeating part.

Since the repeating part ("4") has one digit, multiply x by 10:

$$10x = 4.4444\ldots$$

Step 3: Subtract the original equation from the new one.

$$10x - x = 4.4444\ldots - 0.4444\ldots$$

$$9x = 4$$

Step 4: Solve for x.

$$x = \frac{4}{9}$$

Final Answer for $0.\bar{4}$:

$$0.\bar{4} = \frac{4}{9}$$

Let's solve each problem step by step to express the given recurring decimals as rational numbers:

For $0.\overline{37}$:

Step 1: Represent the repeating decimal mathematically.

Let $x = 0.\overline{37}$.

This means:

$$x = 0.373737\,...$$

Step 2: Multiply by a power of 10 to align the repeating part.

Since the repeating part ("37") has two digits, multiply x by 100:

$$100x = 37.373737\,...$$

Step 3: Subtract the original equation from the new one.

$$100x - x = 37.373737\,... - 0.373737\,...$$

$$99x = 37$$

Step 4: Solve for x.

$$x = \frac{37}{99}$$

Final Answer for $0.\overline{37}$:

$$0.\overline{37} = \frac{37}{99}$$

For $0.\overline{21}$:

Step 1: Represent the repeating decimal mathematically.

Let $x = 0.\overline{21}$.

This means:

$$x = 0.212121\,...$$

Step 2: Multiply by a power of 10 to align the repeating part.

Since the repeating part ("21") has two digits, multiply x by 100:

$$100x = 21.212121\,...$$

Step 3: Subtract the original equation from the new one.

$$100x - x = 21.212121\ldots - 0.212121\ldots$$

$$99x = 21$$

Step 4: Solve for x.

$$x = \frac{21}{99}$$

Step 5: Simplify the fraction.

Simplify $\frac{21}{99}$ by dividing the numerator and denominator by their greatest common divisor (GCD), which is 3:

$$\frac{21}{99} = \frac{7}{33}$$

Final Answer for $0.\overline{21}$:

$$0.\overline{21} = \frac{7}{33}$$

4. **Name the property used in the following.**

(i) $(a+4)+b=a+(4+b)$

(ii) $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$

(iii) $x-x=0$ (iv) $a(b+c)=ab+ac$

(v) $16+0=16$ (vi) $100\times1=100$

(vii) $4\times(5\times8)=(4\times5)\times8$ (viii) $ab=ba$

Here are the properties used in each equation:

(i) $(a+4)+b=a+(4+b)$

Solution:

Property: *Associative Property of Addition*

Explanation: The grouping of numbers in addition does not affect the result.

(ii) $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$

Solution:

Property: *Commutative Property of Addition*

Explanation: The order of numbers in addition does not affect the result.

(iii) $x - x = 0$

Solution:

Property: *Additive Inverse Property*

Explanation: For any number x, subtracting itself results in zero.

(iv) $a(b + c) = ab + ac$

Solution:

Property: *Distributive Property of Multiplication over Addition*

Explanation: A number multiplied by a sum is equal to the sum of the products of the number with each term.

(v) $16 + 0 = 16$

Solution:

Property: *Additive Identity Property*

Explanation: Adding zero to a number leaves the number unchanged.

(vi) $100 \times 1 = 100$

Solution:

Property: *Multiplicative Identity Property*

Explanation: Multiplying a number by one leaves the number unchanged.

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$

Solution:

Property: *Associative Property of Multiplication*

Explanation: The grouping of numbers in multiplication does not affect the result.

(viii) $ab = ba$

Solution:

Property: *Commutative Property of Multiplication*

Explanation: The order of numbers in multiplication does not affect the result.

5. **Name the properties used in the following:**

(i) $-3 < -1 \Rightarrow 0 < 2$

(ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

(iii) If $a < b$ Then $a + c < b + c$

(iv) If $ac < bc$ and $c > 0$ then $a < b$

(v) If $ac < bc$ and $c < 0$ then $a > b$

(vi) Either $a > b$ or $a = b$ or $a < b$

(i) $\boldsymbol{-3 < -1 \Rightarrow < 2}$

Property: *Order Property of Real Numbers*

Explanation: This property states that for any two real numbers, a smaller number remains smaller when compared.

(ii) ***If*** $\boldsymbol{a < b}$***, then*** $\boldsymbol{\frac{1}{a} > \frac{1}{b}}$

Property: *Reciprocal Property for Inequalities with Positive Numbers*

Explanation: When $a, b > 0$, taking reciprocals reverses the inequality.

(iii) ***If*** $\boldsymbol{a < b}$***, then*** $\boldsymbol{a + c < b + c}$

Property: *Additive Property of Inequalities*

Explanation: Adding the same number to both sides of an inequality preserves the inequality.

(iv) ***If*** $\boldsymbol{ac < bc}$ ***and*** $\boldsymbol{c > 0}$***, then*** $\boldsymbol{a < b}$

Property: *Multiplicative Property of Inequalities (Positive Factor)*

Explanation: When multiplying an inequality by a positive number, the inequality's direction remains unchanged.

(v) ***If*** $\boldsymbol{ac < bc}$ ***and*** $\boldsymbol{c < 0}$***, then*** $\boldsymbol{a > b}$

Property: *Multiplicative Property of Inequalities (Negative Factor)*

Explanation: When multiplying an inequality by a negative number, the inequality's direction reverses.

(vi) *Either $a > b$, or $a = b$, or $a < b$*

Property: *Trichotomy Property*

Explanation: For any two real numbers a and b, exactly one of these three statements is true.

6. **Insert two rational numbers between**

(i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

Solution:

(i) Insert two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$

First, we need to express both fractions with a common denominator. The least common denominator of 3 and 4 is 12.

$$\frac{1}{3} = \frac{4}{12}, \quad \frac{1}{4} = \frac{3}{12}$$

Now, to insert two rational numbers between $\frac{4}{12}$ and $\frac{3}{12}$, we can simply take fractions with the denominator 12 that lie between these two values:

$$\frac{3.5}{12}, \quad \frac{3.75}{12}$$

Converting them back into proper fractions, we get:

$$\frac{7}{24}, \quad \frac{15}{40}$$

Thus, the two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{15}{40}$.

(ii) Insert two rational numbers between 3 and 4

Step 1: Express both numbers with a common denominator

We begin by expressing the numbers 3 and 4 as fractions. The least common denominator (LCD) of 1 and 1 is 1, but we will choose a larger denominator to insert rational numbers between them.

- $3 = \frac{3}{1}$
- $4 = \frac{4}{1}$

Now, let's convert both fractions into equivalent fractions with a denominator of **10** (for simplicity, as you used a denominator of 12 in your example).

- $3 = \frac{30}{10}$
- $4 = \frac{40}{10}$

(iii) Insert two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Here, the fractions are already in the form of common denominators, so let's find two rational numbers in between:

$$\frac{3}{5} = 0.6, \quad \frac{4}{5} = 0.8$$

We can choose fractions between these two:

$$\frac{7}{10} = 0.7, \quad \frac{8}{10} = 0.8$$

1.2 Radical Expressions

If n is a positive integer greater than 1, and a is a real number, then any real number x such that $x = \sqrt[n]{a}$ is called n^{th} root of a.

Here $\sqrt{\ }$ is called radical, and n is the index of radical. A real number under the radical sign is called a radicand. $\sqrt[3]{5}, \sqrt[5]{7}$ are examples of radical form.

Exponential form of $x = \sqrt[n]{a}$ is $x = (a)^{\frac{1}{n}}$.

1.2.1 Laws of Radicals and Indices

Laws of Radical

(i) $\sqrt[n]{ab} = \sqrt[n]{a}, \sqrt[n]{b}$

(ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Laws of Indices

(i) $a^m . a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(iv) $\left(\sqrt[n]{a}\right)^n = (a^{\frac{1}{n}})^n = a$

(iii) $(ab)^n = a^n b^n$

(iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v) $\frac{a^m}{a^n} = a^{m-n}$

(vi) $a^0 = 1$

Example 7: Simplify the following:

(i) $\sqrt[4]{16x^4y^8}$ (ii) $\sqrt[3]{27x^6y^9z^3}$

(iii) $(64)^{-\frac{4}{3}}$

Solution: (i) $\sqrt[4]{16x^4y^3}$

$= (16x^4y^3)^{\frac{1}{4}}$ $\quad \sqrt[n]{a} = a^{\frac{1}{n}})$

$= (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}(y^3)^{\frac{1}{4}}$ $\quad \because (ab)^m = a^m b^m$

$= 2^{4 \cdot \frac{1}{4}} \times x^{4 \cdot \frac{1}{4}} \times y^{8 \cdot \frac{1}{4}}$ $\quad \because (a^m)^n = a^{mn}$

$= 2xy^2$

(ii) $\sqrt[3]{27x^6y^9z^3}$

$= \left(27x^6y^9z^3\right)^{\frac{1}{3}}$ $\quad \because \sqrt[n]{a} = a^{\frac{1}{n}}$

$= (27)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because (ab)^m = a^m b^m$

$= \left(3^3\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because (ab)^m = a^m b^m$

$= 3^{3\times\frac{1}{3}} \cdot x^{6\times\frac{1}{3}} \cdot y^{9\times\frac{1}{3}} . z^{3\times\frac{1}{3}}$

$= 3x^2y^3z$

(iii) $(64)^{-\frac{4}{3}}$

$$= \frac{1}{(64)^{\frac{4}{3}}} = \frac{1}{(4)^{\frac{4}{3}}} = \frac{1}{4^{3\times\frac{4}{3}}}$$

$$= \frac{1}{4^4} = \frac{1}{256}$$

1.2.2 Surds and their Applications

An irrational radical with rational radicand is called a surd.

For example, if we take the nth root of any rational number a then $\sqrt[n]{a}$ is a surd. $\sqrt{5}$ is a surd because the square root of 5 does not give a whole number but $\sqrt{9}$ is not a surd because it simplifies to a whole number 3 and our result is not an irrational number. Therefore, the radical $\sqrt[n]{a}$ is irrational $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}$ are surds but $\sqrt{\pi}, \sqrt{e}$ are not surds.

> **Remember!**
>
> Every surd is an irrational number but every irrational number is not a surd e.g., $\sqrt{\pi}$ is not a surd.

The different type of surds are as follow:

(i) A surd that contains a single term is called a monomial e.g., $\sqrt{5}, \sqrt{7}$

(ii) A surd that contains the sum of two monomial surds is called a binomial surd e.g., $\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{7}$ etc.

> **Remember**
>
> The product of two conjugate surds is a rational number.

(iii) $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

1.2.3 Rationalization of denominator

To rationalize a denominator of the form $a + b\sqrt{x}$ or $a - b\sqrt{x}$, we multiply both the numerator and denominator by the conjugate factor.

Example 8: Rationalize the denominator of (i) $\frac{3}{\sqrt{5}+\sqrt{2}}$

(ii) $\frac{3}{\sqrt{5}-\sqrt{3}}$

Solution (i):

$$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5}-\sqrt{2}$$

(ii)

$$\frac{3}{\sqrt{5}-\sqrt{3}} = \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$$