Exercise 1.2

1. **Rationalize the denominator of following:**

(i) $\frac{13}{4+\sqrt{3}}$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solutions:

(i) $\frac{13}{4+\sqrt{3}}$

To rationalize the denominator, multiply both the numerator and denominator by the conjugate of the denominator. The conjugate of $4+\sqrt{3}$ is $4-\sqrt{3}$.

$$\frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{13(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$$

First, simplify the denominator using the difference of squares formula:

$$(4+\sqrt{3})(4-\sqrt{3}) = 4^2 - \left(\sqrt{3}\right)^2 = 16 - 3 = 13$$

Now simplify the numerator:

$$13(4-\sqrt{3}) = 52 - 13\sqrt{3}$$

So the rationalized expression is:

$$\frac{52-13\sqrt{3}}{13}$$

Simplifying:

$$= \frac{52}{13} - \frac{13\sqrt{3}}{13} = 4 - \sqrt{3}$$

Thus, the rationalized form is:

$$4 - \sqrt{3}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

To rationalize the denominator, multiply both the numerator and denominator by $\sqrt{3}$:

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{5}) \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

Simplify the denominator:

$$\sqrt{3} \cdot \sqrt{3} = 3$$

Now simplify the numerator:

$$\left(\sqrt{2}+\sqrt{5}\right) \cdot \sqrt{3} = \sqrt{6} + \sqrt{15}$$

Thus, the rationalized expression is:

$$\frac{\sqrt{6}+\sqrt{15}}{3}$$

iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

To rationalize the denominator, multiply both the numerator and denominator by $\sqrt{5}$:

$$\frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{(\sqrt{2}-1)\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}$$

Simplify the denominator:

$$\sqrt{5}\cdot\sqrt{5} = 5$$

Now simplify the numerator:

$$(\sqrt{2}-1)\cdot\sqrt{5} = \sqrt{10}-\sqrt{5}$$

Thus, the rationalized expression is:

$$\frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

To rationalize the denominator, multiply both the numerator and denominator by the conjugate of the denominator, $6-4\sqrt{2}$:

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} = \frac{(6-4\sqrt{2})^2}{(6+4\sqrt{2})(6-4\sqrt{2})}$$

First, simplify the denominator using the difference of squares formula:

$$(6+4\sqrt{2})(6-4\sqrt{2}) = 6^2-(4\sqrt{2})^2 = 36-32 = 4$$

Now simplify the numerator:

$$(6-4\sqrt{2})^2 = 6^2 - 2\cdot 6\cdot 4\sqrt{2} + (4\sqrt{2})^2 = 36-48\sqrt{2}+32$$
$$= 68-48\sqrt{2}$$

Thus, the rationalized expression is:

$$\frac{68-48\sqrt{2}}{4} = \frac{68}{4} - \frac{48\sqrt{2}}{4} = 17-12\sqrt{2}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

To rationalize the denominator, multiply both the numerator and denominator by the conjugate of the denominator, $\sqrt{3}-\sqrt{2}$:

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

First, simplify the denominator using the difference of squares formula:

$$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3-2 = 1$$

Now simplify the numerator:

$$(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2 - 2\cdot\sqrt{3}\cdot\sqrt{2} + (\sqrt{2})^2 = 3 - 2\sqrt{6} + 2$$
$$= 5 - 2\sqrt{6}$$

Thus, the rationalized expression is:

$$5 - 2\sqrt{6}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Step 1: Multiply numerator and denominator by the *conjugate* of the denominator

The conjugate of $\sqrt{7}+\sqrt{5}$ is $\sqrt{7}-\sqrt{5}$. Multiplying the numerator and denominator by the conjugate will help eliminate the square roots in the denominator.

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}$$

Step 2: Simplify the denominator using the difference of squares formula

The denominator is now in the form of $(a+b)(a-b) = a^2 - b^2$. So:

$$(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5}) = 7-5 = 2$$

Step 3: Simplify the numerator

Now, distribute $4\sqrt{3}$ across $(\sqrt{7}-\sqrt{5})$:

$$4\sqrt{3}(\sqrt{7}-\sqrt{5}) = 4\sqrt{21} - 4\sqrt{15}$$

Step 4: Combine the results

Now, the expression becomes:

$$\frac{4\sqrt{21}-4\sqrt{15}}{2}$$

Step 5: Simplify the fraction

Finally, simplify the fraction by dividing both terms in the numerator by 2:

$$\frac{4\sqrt{21}}{2} - \frac{4\sqrt{15}}{2} = 2\sqrt{21} - 2\sqrt{15}$$

Final Answer:

The rationalized form of the expression is:

$$2\sqrt{21} - 2\sqrt{15}$$

2. **Simplify the following**

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

(iii) $(0.027)^{-\frac{1}{3}}$

(iv) $\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}}$

(v) $\frac{5\cdot(25)^{n-1} - 25\cdot(5)^{2n}}{5\cdot(25)^{2n+3} - (25)^{n+1}}$

(vi) $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

(viii) $\frac{3^{n} \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

(ix) $\frac{5^{n+3} - 6.5^{n+1}}{9\times 5^{n} - 2^{n} \times 5^{n}}$

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

We will start by applying the negative exponent rule, which states that $a^{-b} = \frac{1}{a^b}$:

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \frac{1}{\left(\frac{81}{16}\right)^{\frac{3}{4}}}$$

Next, simplify the expression $\left(\frac{81}{16}\right)^{\frac{3}{4}}$:

$$\left(\frac{81}{16}\right)^{\frac{3}{4}} = \frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}}$$

Now calculate the powers:

$$81 = 3^4 \quad \Rightarrow \quad 81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^3 = 27$$

$$16 = 2^4 \quad \Rightarrow \quad 16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$$

Thus:

$$\frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}} = \frac{27}{8}$$

Therefore, the simplified expression is:

$$\frac{1}{\frac{27}{8}} = \frac{8}{27}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

First, simplify each part step by step.

$$\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^{2}} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$$

$$\left(\frac{4}{9}\right)^{3} = \frac{4^3}{9^3} = \frac{64}{729}$$

Now combine all the terms:

$$\frac{16}{9} \div \frac{64}{729} \times \frac{16}{27}$$

To divide by a fraction, multiply by its reciprocal:

$$\frac{16}{9} \times \frac{729}{64} \times \frac{16}{27}$$

Multiply the numerators and denominators:

$$\frac{16 \times 729 \times 16}{9 \times 64 \times 27} = \frac{16^2 \times 729}{9 \times 64 \times 27}$$

Simplify the expression step by step:

$$16^2 = 256 \quad \text{and} \quad 64 \times 27 = 1728$$

Now simplify:

$$\frac{256 \times 729}{9 \times 1728}$$

Factor the denominator:

$$9 \times 1728 = 15552$$

Thus, the simplified result is:

$$\frac{256 \times 729}{15552} = \frac{186624}{15552} = 12$$

(iii) $(0.027)^{-\frac{1}{3}}$

Rewrite 0.027 as a power of 3:

$$0.027 = 3^{-3}$$

Now apply the negative exponent:

$$(0.027)^{-\frac{1}{3}} = (3^{-3})^{-\frac{1}{3}} = 3^1 = 3$$

Thus, the simplified result is:

$$3$$

(iv) $\sqrt[7]{\frac{x^{14}y^{21}z^{35}}{y^{14}z^7}}$

First, simplify the expression inside the radical:

$$\frac{x^{14}y^{21}z^{35}}{y^{14}z^7} = x^{14} \times y^{21-14} \times z^{35-7} = x^{14} \times y^7 \times z^{28}$$

Now take the 7th root of each term:

$$\sqrt[7]{x^{14}} = x^2, \quad \sqrt[7]{y^7} = y, \quad \sqrt[7]{z^{28}} = z^4$$

Thus, the simplified result is:

$$x^2yz^4$$

(v) $$\frac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(25)^{2n+3} - (25)^{n+1}}$$

First, rewrite 25 as 5^2:

$$\frac{5\cdot(5^2)^{n+1} - 25\cdot 5^{2n}}{5\cdot(5^2)^{2n+3} - (5^2)^{n+1}}$$

Simplify the powers of 5^2:

$$= \frac{5\cdot 5^{2(n+1)} - 25\cdot 5^{2n}}{5\cdot 5^{2(2n+3)} - 5^{2(n+1)}}$$

Simplify the exponents:

$$= \frac{5^{2n+2+1} - 5^{2n+2}}{5^{4n+7} - 5^{2n+2}}$$

Factor out the common terms:

$$= \frac{5^{2n+3}(5^2 - 1)}{5^{2n+2}(5^{4n+5} - 1)}$$

Simplify:

$$= \frac{5^{2n+3} \times 24}{5^{2n+2} \times (5^{4n+5} - 1)}$$

Cancel out 5^{2n+2} from both the numerator and denominator:

$$= \frac{5 \times 24}{5^{4n+5} - 1}$$

Thus, the simplified expression is:

$$\frac{120}{5^{4n+5} - 1}$$

(vi) $$\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$$

Simplify the powers of 16 and 4 in terms of powers of 2:

$$16 = 2^4, \quad 4 = 2^2, \quad 8 = 2^3$$

So:

$$\frac{(2^4)^{x+1} + 20(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}}$$

Simplify the powers:

$$= \frac{2^{4(x+1)} + 20 \cdot 2^{4x}}{2^{x-3} \times 2^{3(x+2)}}$$

Simplify:

$$= \frac{2^{4x+4} + 20 \cdot 2^{4x}}{2^{x-3} \times 2^{3x+6}}$$

Now factor out 2^{4x} from the numerator:

$$= \frac{2^{4x}(2^4 + 20)}{2^{x-3} \times 2^{3x+6}}$$

Simplify further:

$$= \frac{2^{4x} \times 336}{2^{x-3} \times 2^{3x+6}} = \frac{336 \times 2^{4x}}{2^{4x+x+3}}$$

Cancel out the 2^{4x}:

$$= \frac{336}{2^{x+3}} = 336 \times 2^{-(x+3)}$$

Thus, the simplified expression is:

$$336 \times 2^{-(x+3)}$$

(vii) $\quad (64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

We begin by rewriting 64 and 9 as powers of 2 and 3, respectively:

$$64 = 2^6 \quad \text{and} \quad 9 = 3^2$$

Now apply the negative exponents:

$$= (2^6)^{-\frac{2}{3}} = 2^{-4}$$

$$= (3^2)^{-\frac{3}{2}} = 3^{-3}$$

$$= 2^{-4} \times 3^3 = \frac{3^3}{2^4}$$

$$\text{and} \quad 2^4 = 16$$

Thus, the simplified result is:

$$\frac{27}{16}$$

(viii) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

First, rewrite 9 as 3^2:

$$9^{n+1} = (3^2)^{n+1} = 3^{2(n+1)} = 3^{2n+2}$$
$$9^{n-1} = (3^2)^{n-1} = 3^{2(n-1)} = 3^{2n-2}$$

Now simplify the expression:

$$\frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$$

Use the property of exponents $\frac{a^m}{a^n} = a^{m-n}$:

$$= \frac{3^{n+2n+2}}{3^{n-1+2n-2}} = \frac{3^{3n+2}}{3^{3n-3}}$$

Now subtract the exponents:

$$= 3^{(3n+2)-(3n-3)} = 3^5$$

Thus, the simplified result is:

$$3^5 = 243$$

(ix) $\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^n \times 5^n}$

First, factor the denominator:

$$9 \times 5^n - 2^n \times 5^n = 5^n(9 - 2^n)$$

Thus, the expression becomes:

$$\frac{5^{n+3} - 6.5^{n+1}}{5^n(9 - 2^n)}$$

Now factor out 5^n from the numerator:

$$5^{n+3} = 5^n \times 5^3 = 5^n \times 125$$

So, the expression becomes:

$$= \frac{5^n(125 - 6.5^{n+1})}{5^n(9 - 2^n)}$$
$$= \frac{125 - 6.5^{n+1}}{9 - 2^n}$$

3. If $x = 3 + \sqrt{8}$ then find the value of

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$

(iii) $x^2 + \frac{1}{x^2}$ (iv) $x^2 - \frac{1}{x^2}$

(v) $x^4 + \frac{1}{x^4}$ (vi) $\left(x - \frac{1}{x}\right)^2$

Given $x = 3 + \sqrt{8}$

(i) $x + \frac{1}{x}$

We are given $x = 3 + \sqrt{8}$. To find $\frac{1}{x}$, we rationalize the denominator.

First, express x as:

$$x = 3 + \sqrt{8}$$

To find $\frac{1}{x}$, multiply the numerator and denominator by the conjugate of x, which is $3 - \sqrt{8}$:

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} = \frac{3 - \sqrt{8}}{(3 + \sqrt{8})(3 - \sqrt{8})}$$

Now, simplify the denominator using the difference of squares formula $(a + b)(a - b) = a^2 - b^2$:

$$(3 + \sqrt{8})(3 - \sqrt{8}) = 3^2 - (\sqrt{8})^2 = 9 - 8 = 1$$

So,

$$\frac{1}{x} = 3 - \sqrt{8}$$

Now, calculate $x + \frac{1}{x}$:

$$x + \frac{1}{x} = (3 + \sqrt{8}) + (3 - \sqrt{8}) = 3 + 3 = 6$$

Thus, the value of $x + \frac{1}{x}$ is:

6

(ii) $x - \frac{1}{x}$

Using the expressions for x and $\frac{1}{x}$ from above:

$$x = 3 + \sqrt{8} \quad \text{and} \quad \frac{1}{x} = 3 - \sqrt{8}$$

Now, calculate $x - \frac{1}{x}$:

$$x - \frac{1}{x} = (3 + \sqrt{8}) - (3 - \sqrt{8}) = 3 + \sqrt{8} - 3 + \sqrt{8} = 2\sqrt{8}$$

Since $\sqrt{8} = 2\sqrt{2}$, we have:

$$x - \frac{1}{x} = 2 \times 2\sqrt{2} = 4\sqrt{2}$$

Thus, the value of $x - \frac{1}{x}$ is:

$$\boxed{4\sqrt{2}}$$

(iii) $x^2 + \frac{1}{x^2}$

We can use the identity for $x^2 + \frac{1}{x^2}$ in terms of $x + \frac{1}{x}$:

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

From part (i), we found that $x + \frac{1}{x} = 6$. So:

$$x^2 + \frac{1}{x^2} = 6^2 - 2 = 36 - 2 = 34$$

Thus, the value of $x^2 + \frac{1}{x^2}$ is:

$$\boxed{34}$$

(iv) $x^2 - \frac{1}{x^2}$

Step 1: Find x^2

First, we square $x = 3 + \sqrt{8}$:

$$x^2 = (3 + \sqrt{8})^2$$

Expanding:

$$x^2 = 3^2 + 2(3)(\sqrt{8}) + (\sqrt{8})^2$$

$$x^2 = 9 + 6\sqrt{8} + 8$$

$$x^2 = 17 + 6\sqrt{8}$$

Step 2: Find $\frac{1}{x^2}$

We need to find $\frac{1}{x^2}$, so let's rationalize $\frac{1}{x}$:

Since $x = 3 + \sqrt{8}$, the conjugate of $3 + \sqrt{8}$ is $3 - \sqrt{8}$. We multiply both the numerator and denominator by $3 - \sqrt{8}$:

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

Now, we simplify the denominator:

$$(3 + \sqrt{8})(3 - \sqrt{8}) = 3^2 - (\sqrt{8})^2 = 9 - 8 = 1$$

So, $\frac{1}{x} = 3 - \sqrt{8}$.

Next, square $\frac{1}{x}$:

$$\frac{1}{x^2} = (3 - \sqrt{8})^2$$

Expanding:

$$\frac{1}{x^2} = 3^2 - 2(3)(\sqrt{8}) + (\sqrt{8})^2$$

$$\frac{1}{x^2} = 9 - 6\sqrt{8} + 8$$

$$\frac{1}{x^2} = 17 - 6\sqrt{8}$$

Step 3: Compute $x^2 - \frac{1}{x^2}$

Now, subtract $\frac{1}{x^2}$ from x^2:

$$x^2 - \frac{1}{x^2} = (17 + 6\sqrt{8}) - (17 - 6\sqrt{8})$$

$$x^2 - \frac{1}{x^2} = 17 + 6\sqrt{8} - 17 + 6\sqrt{8}$$

$$x^2 - \frac{1}{x^2} = 12\sqrt{8}$$

We can simplify $\sqrt{8}$ as $2\sqrt{2}$:

$$x^2 - \frac{1}{x^2} = 12 \times 2\sqrt{2} = 24\sqrt{2}$$

(v) $x^4 + \frac{1}{x^4}$

Step 1: Find x^4

We already know that $x^2 = 17 + 6\sqrt{8}$. To find x^4, we square x^2:

$$x^4 = (x^2)^2 = (17 + 6\sqrt{8})^2$$

Expanding:

$$x^4 = 17^2 + 2(17)(6\sqrt{8}) + (6\sqrt{8})^2$$
$$x^4 = 289 + 204\sqrt{8} + 288$$
$$x^4 = 577 + 204\sqrt{8}$$

Step 2: Find $\frac{1}{x^4}$

Now, we find $\frac{1}{x^4}$, using the same method as before:

$$\frac{1}{x^4} = (3 - \sqrt{8})^4$$

We will follow the same squaring method to expand, and after simplifying, we get:

$$\frac{1}{x^4} = 577 - 204\sqrt{8}$$

Step 3: Compute $x^4 + \frac{1}{x^4}$

Now, add x^4 and $\frac{1}{x^4}$:

$$x^4 + \frac{1}{x^4} = (577 + 204\sqrt{8}) + (577 - 204\sqrt{8})$$

$$x^4 + \frac{1}{x^4} = 577 + 577$$

$$x^4 + \frac{1}{x^4} = 1154$$

(vi) $\left(x - \frac{1}{x}\right)^2$

Step 1: Find $x - \frac{1}{x}$

We already know:

$$x = 3 + \sqrt{8}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

Now, subtract $\frac{1}{x}$ from x:

$$x - \frac{1}{x} = (3 + \sqrt{8}) - (3 - \sqrt{8})$$

$$x - \frac{1}{x} = 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$x - \frac{1}{x} = 2\sqrt{8} = 4\sqrt{2}$$

Step 2: Square the result

Now, square $x - \frac{1}{x}$:

$$\left(x - \frac{1}{x}\right)^2 = (4\sqrt{2})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 16 \times 2 = 32$$

4. **Find the rational numbers p and q such that**

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p+q\sqrt{2}$$

We are asked to express $\frac{8-3\sqrt{2}}{4+3\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers.

Step 1: Rationalize the denominator

To simplify the expression and eliminate the square root from the denominator, we rationalize by multiplying both the numerator and denominator by the conjugate of the denominator. The conjugate of $4 + 3\sqrt{2}$ is $4 - 3\sqrt{2}$.

Multiply the expression by $\frac{4-3\sqrt{2}}{4-3\sqrt{2}}$:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(8-3\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$$

Step 2: Simplify the denominator

The denominator is a difference of squares:

$$(4+3\sqrt{2})(4-3\sqrt{2}) = 4^2 - (3\sqrt{2})^2 = 16 - 18 = -2$$

Step 3: Simplify the numerator

Now, expand the numerator $(8-3\sqrt{2})(4-3\sqrt{2})$:

$$(8-3\sqrt{2})(4-3\sqrt{2})$$
$$= 8\cdot 4 + 8\cdot(-3\sqrt{2}) + (-3\sqrt{2})\cdot 4 + (-3\sqrt{2})\cdot(-3\sqrt{2})$$
$$= 32 - 24\sqrt{2} - 12\sqrt{2} + 18$$
$$= 32 + 18 - 36\sqrt{2}$$
$$= 50 - 36\sqrt{2}$$

Step 4: Combine the numerator and denominator

We now have:

$$\frac{50-36\sqrt{2}}{-2}$$

We can split the fraction into two parts:

$$\frac{50}{-2} - \frac{36\sqrt{2}}{-2} = -25 + 18\sqrt{2}$$

Step 5: Express in the desired form

Now we have the expression:

$$-25 + 18\sqrt{2}$$

This is in the form $p + q\sqrt{2}$, where $p = -25$ and $q = 18$.

5. Simplify the following:

(i) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$ (ii) $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

(i) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Step 1: Simplify the powers

We will simplify each of the numbers in the powers.

- $25 = 5^2$, so $(25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^3 = 125$
- $243 = 3^5$, so $(243)^{\frac{3}{5}} = (3^5)^{\frac{3}{5}} = 3^3 = 27$
- $16 = 2^4$, so $(16)^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^5 = 32$
- $8 = 2^3$, so $(8)^{\frac{4}{3}} = (2^3)^{\frac{4}{3}} = 2^4 = 16$

Step 2: Substitute into the expression

Now substitute the simplified values:

$$\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{125 \times 27}{32 \times 16}$$

Step 3: Simplify the fraction

Multiply the numerators and denominators:

$$\frac{125 \times 27}{32 \times 16} = \frac{3375}{512}$$

Thus, the simplified answer is:

$$\frac{3375}{512}$$

(ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

Step 1: Simplify the cube root term

- $27 = 3^3$, so $\sqrt[3]{(27)^{2x}} = \sqrt[3]{(3^3)^{2x}} = 3^{2x}$

So, the numerator becomes:

$$54 \times 3^{2x}$$

Step 2: Simplify the denominator

- $9 = 3^2$, so $9^{x+1} = (3^2)^{x+1} = 3^{2(x+1)} = 3^{2x+2}$
- $216 = 3^3 \times 2^3$, so $216(3^{2x-1}) = 3^3 \times 2^3 \times 3^{2x-1} = 2^3 \times 3^{2x+2}$

Now the denominator becomes:

$$3^{2x+2} + 2^3 \times 3^{2x+2} = 3^{2x+2}(1+8) = 9 \times 3^{2x+2}$$

Step 3: Substitute into the expression

The original expression is now:

$$\frac{54 \times 3^{2x}}{9 \times 3^{2x+2}}$$

Step 4: Simplify the fraction

We can cancel the common powers of 3:

$$\frac{54 \times 3^{2x}}{9 \times 3^{2x+2}} = \frac{54}{9} \times \frac{1}{3^2} = 6 \times \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

Thus, the simplified answer is:

$$\frac{2}{3}$$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

Step 1: Simplify each term

- $216 = 6^3$, so $(216)^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^2 = 36$
- $25 = 5^2$, so $(25)^{\frac{1}{2}} = 5$
- $0.04 = \frac{1}{25}$, so $(0.04)^{\frac{-3}{2}} = \left(\frac{1}{25}\right)^{\frac{-3}{2}} = 25^{\frac{3}{2}} = 125$

Step 2: Substitute into the expression

Now the expression becomes:

$$\sqrt{\frac{36\times5}{125}} = \sqrt{\frac{180}{125}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$$

Thus, the simplified answer is:

$$\frac{6}{5}$$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Step 1: Recognize the pattern

This is a product of two binomials, which is in the form $(x + y)(x^2 - xy + y^2)$, where:

- $x = a^{\frac{1}{3}}$
- $y = b^{\frac{2}{3}}$

Using the identity $(x + y)(x^2 - xy + y^2) = x^3 + y^3$, we can simplify:

$$a^{\frac{1}{3}} + b^{\frac{2}{3}} \quad \text{and} \quad a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}} \quad \text{becomes} \quad a + b$$

Thus, the product is:

$$a + b$$

The simplified answer is:

$$a + b$$