Exercise 1.3

1. The sum of three consecutive integers is forty-two, find three integers.

Solution: Let the three consecutive integers be:

$$x, x+1, x+2$$

The sum of these integers is 42:

$$x + (x+1) + (x+2) = 42$$

Simplifying the equation:

$$3x + 3 = 42$$

Now, subtract 3 from both sides:

$$3x = 39$$

Now, divide by 3:

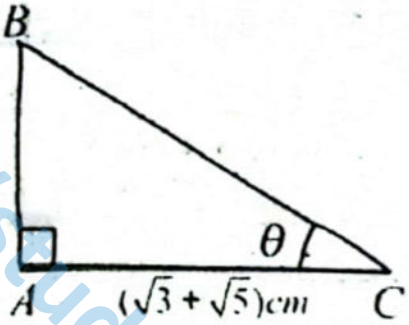
$$x = 13$$

So, the three consecutive integers are:

$$13, 14, 15$$

Thus, the three integers are **13, 14, and 15.**

2. The diagram shows right angled ΔABC in which the length of AC is $(\sqrt{3}+\sqrt{5})$cm. The area of ΔABC is $(1+\sqrt{15})$cm^2; Find the length AB in the form $(a\sqrt{3}+b\sqrt{5})$cm where a and b are integers.



Solution:

Given:

- Triangle ΔABC, where $AC = \sqrt{3} + \sqrt{5}$ cm.
- Area of $\Delta ABC = 1 + \sqrt{15}$ cm^2.

- AB is to be expressed as $AB = a\sqrt{3} + b\sqrt{5}$ cm, where a and b are integers.

Step-by-Step Solution:

1. Formula for the area of the triangle:

The area of a triangle can be written as:

$$\frac{1}{2}AC.AB$$

Angle $A = 90o$

Angle $A = \theta$

$\overline{AC} = \sqrt{3} + \sqrt{5}$

$AB = a\sqrt{3} + b\sqrt{5}$

Area $= 1 + \sqrt{15}$

a and b are to determined.

$$\text{Area of } \Delta ABC = \frac{1}{2}AC \times AB$$

$$\frac{1}{2}AC \times AB = 1 + \sqrt{15}$$

$$\frac{1}{2}\left(\sqrt{3} + \sqrt{5}\right)a\sqrt{3} + b\sqrt{5} = 1 + \sqrt{15}$$

$$a\sqrt{3} + b\sqrt{5} = \frac{2\left(1 + \sqrt{15}\right)}{\sqrt{3} + \sqrt{5}}$$

Rationalizing the R.H.S

$$a\sqrt{3} + b\sqrt{5} = \frac{2\left(1 + \sqrt{15}\right)\left(\sqrt{3} - \sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{5}\right)\left(\sqrt{3} - \sqrt{5}\right)}$$

$$= \frac{2\left(1 + \sqrt{15}\right)\left(\sqrt{3} - \sqrt{5}\right)}{3 - 5}$$

$$= \frac{2\left(1 + \sqrt{15}\right)\left(\sqrt{3} - \sqrt{5}\right)}{-2}$$

$$a\sqrt{3}+b\sqrt{5}=-\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{15}\right)$$

$$a\sqrt{3}+b\sqrt{5}=-\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{5}\right)$$

$$=-\left[\sqrt{3}-\sqrt{5}+3\sqrt{5}-5\sqrt{3}\right]$$

$$=-\left[2\sqrt{5}-4\sqrt{3}\right]$$

$$=-2\sqrt{5}+4\sqrt{3}$$

$$a\sqrt{3}+b\sqrt{5}=4\sqrt{3}-2\sqrt{5}$$

Comparing the coefficients of $\sqrt{3}$ and $\sqrt{5}$ we get $a=4$; $b=-2$

3. A rectangle has sides of length $(2+\sqrt{18})m$ and $(5-\frac{4}{\sqrt{2}})m$.

Expressthe area of the rectangle in the form $a+b\sqrt{2}$ where *a*and*b* are integers.

Problem Statement:

We are given a rectangle with sides of length:

- $(2+\sqrt{18})$ meters and
- $\left(5-\frac{4}{\sqrt{2}}\right)$ meters.

We are asked to find the area of the rectangle in the form $a+b\sqrt{2}$, where a and b are integers.

Step 1: Simplify the lengths of the sides

FIRST SIDE: $2+\sqrt{18}$

We can simplify $\sqrt{18}$ as:

$$\sqrt{18}=\sqrt{9\times 2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$$

So, the first side becomes:

$$2+\sqrt{18}=2+3\sqrt{2}$$

SECOND SIDE: $5-\frac{4}{\sqrt{2}}$

To simplify $\frac{4}{\sqrt{2}}$, multiply both the numerator and denominator by $\sqrt{2}$:

$$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

So, the second side becomes:

$$5 - \frac{4}{\sqrt{2}} = 5 - 2\sqrt{2}$$

Step 2: Find the area of the rectangle

The area A of the rectangle is the product of its two sides:

$$A = (2 + 3\sqrt{2}) \times (5 - 2\sqrt{2})$$

We now expand this expression using the distributive property (FOIL method):

$$A = 2 \times 5 + 2 \times (-2\sqrt{2}) + 3\sqrt{2} \times 5 + 3\sqrt{2} \times (-2\sqrt{2})$$

Let's simplify each term:

- $2 \times 5 = 10$
- $2 \times (-2\sqrt{2}) = -4\sqrt{2}$
- $3\sqrt{2} \times 5 = 15\sqrt{2}$
- $3\sqrt{2} \times (-2\sqrt{2}) = -6 \times 2 = -12$

Now combine all the terms:

$$A = 10 - 4\sqrt{2} + 15\sqrt{2} - 12$$

Step 3: Combine like terms

- The constant terms: $10 - 12 = -2$
- The terms with $\sqrt{2}$: $-4\sqrt{2} + 15\sqrt{2} = 11\sqrt{2}$

Thus, the area is:

$$A = -2 + 11\sqrt{2}$$

4. Find two numbers whose sum is 68 and difference is 22.

Problem Statement:

We need to find two numbers whose sum is 68 and whose difference is 22. Let's denote these two numbers by x and y. We have the following two equations:

- $x + y = 68$ (Equation 1: The sum of the numbers is 68)
- $x - y = 22$ (Equation 2: The difference of the numbers is 22)

Step 1: Add the two equations

To eliminate y, we can add Equation 1 and Equation 2 together. This will help us solve for x.

$$(x + y) + (x - y) = 68 + 22$$

Simplifying both sides:

$$x + x = 90$$
$$2x = 90$$

Now, solve for x:

$$x = \frac{90}{2} = 45$$

Step 2: Substitute $x = 45$ into Equation 1

Now that we have $x = 45$, we can substitute this value into Equation 1 to solve for y.

$$x + y = 68$$

Substituting $x = 45$:

$$45 + y = 68$$

Solve for y:

$$y = 68 - 45 = 23$$

5. The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperatures as high as 48°C. By using the formula, $(F = \frac{9}{5}C + 32)$ find the temperature as Fahrenheit scale.

To convert a temperature from Celsius to Fahrenheit, we can use the formula:

$$F = \frac{9}{5}C + 32$$

Where:

- F is the temperature in Fahrenheit.
- C is the temperature in Celsius.

Given:

The temperature in Lahore during the summer of 2024 is 48°C.

Step 1: Plug the given value of $C = 48$ into the formula

$$F = \frac{9}{5}(48) + 32$$

Step 2: Calculate the value of $\frac{9}{5} \times 48$

$$\frac{9}{5} \times 48 = \frac{432}{5} = 86.4$$

Step 3: Add 32 to 86.4

$$F = 86.4 + 32 = 118.4$$

6. The sum of the ages of the father and son is 72. Six years ago the father's age was 2 times the age of the son. What was Son's age six years ago?

Let the present age of the father be F and the present age of the son be S.

Given:

1. The sum of their ages is 72:

$$F + S = 72 \quad \text{(Equation 1)}$$

2. Six years ago, the father's age was twice the son's age:

$$F - 6 = 2(S - 6) \quad \text{(Equation 2)}$$

Step 1: Solve the system of equations

FROM EQUATION 1:

$$F + S = .72$$

Solving for F:

$$F = 72 - S$$

SUBSTITUTE F = 72 – S INTO EQUATION 2:

$$72 - S - 6 = 2(S - 6)$$

Simplifying:

$$66 - S = 2(S - 6)$$
$$66 - S = 2S - 12$$

Now, solve for S:

$$66 + 12 = 2S + S$$
$$78 = 3S$$
$$S = \frac{78}{3} = 26$$

Step 2: Find the Father's current age

Now that we know the son's age, we can substitute $S = 26$ into Equation 1:

$$F + 26 = 72$$

$$F = 72 - 26 = 46$$

Step 3: Find the Son's age 6 years ago

The son's current age is 26, so 6 years ago:

$$S - 6 = 26 - 6 = 20$$

7. Mirha bought a tory for Rs. 1500 and sold for Rs. 1520. What was her profit percentage?

Sol: **Part I:**

C. P = 1500

S. P = 1520

Profit % = 20

$$\text{Profit \%} = \frac{20 \times 100}{1500} = \frac{20}{15} = \frac{4}{3}$$

$$= 1.33\%$$

Part II:

C. P = 1500

Profit = 15%

$$\text{S.P} = \frac{115 \times 1500}{100} = 172.5$$

8. The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have *to* pay at the rate of 0.75%.

To find out how much tax Tayyab would have to pay, we need to follow these steps:

Given:

- Annual income of Tayyab = Rs. 9,60,000
- Exempted amount = Rs. 1,30,000
- Tax rate = 0.75%

Step 1: Calculate the taxable income

Taxable income is calculated by subtracting the exempted amount from the total income.

$$\text{Taxable Income} = \text{Annual Income} - \text{Exempted Amount}$$

$$\text{Taxable Income} = 9{,}60{,}000 - 1{,}30{,}000$$

$$\text{Taxable Income} = 8{,}30{,}000$$

Step 2: Calculate the tax to be paid

The tax is calculated at a rate of 0.75% on the taxable income.

$$\text{Tax} = \frac{0.75}{100} \times \text{Taxable Income}$$

$$\text{Tax} = \frac{0.75}{100} \times 8{,}30{,}000$$

$$\text{Tax} = 0.0075 \times 8{,}30{,}000$$

$$\text{Tax} = 6{,}225$$

9. Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

To calculate the compound markup on Rs. 3,75,000 at a rate of 14% compounded annually for one year, we can use the formula for compound interest:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Where:

- A is the amount after interest
- P is the principal (initial amount)
- r is the rate of interest
- n is the number of years

Given:

- $P = 3{,}75{,}000$ (principal amount)
- $r = 14\%$ (interest rate)
- $n = 1$ year (since the interest is compounded for one year)

Step 1: Substitute the values into the compound interest formula:

$$A = 3{,}75{,}000\left(1 + \frac{14}{100}\right)^1$$

$$A = 3{,}75{,}000(1 + 0.14)$$

$$A = 3{,}75{,}000 \times 1.14$$

Step 2: Calculate the amount after one year:

$$A = 3{,}75{,}000 \times 1.14 = 4{,}27{,}500$$

Step 3: Calculate the compound markup:

The compound markup is the difference between the final amount and the principal:

$$\text{Compound Markup} = A - P$$

$$\text{Compound Markup} = 4{,}27{,}500 - 3{,}75{,}000$$

$$\text{Compound Markup} = 52{,}500$$