Exercise 2.1

1. **Express the following numbers in scientific notation.**

(i)	**2000000**	**(ii)**	**48900**
(iii)	**0.0042**	**(iv)**	**0.0000009**
(v)	73×10^3	**(vi)**	0.65×10^2

Let's express each number in scientific notation.

Solution:

(i) 2,000,000

Scientific notation expresses numbers as a product of a number between 1 and 10 and a power of 10. To convert 2,000,000 into scientific notation:

$$2{,}000{,}000 = 2 \times 10^6$$

Thus, the scientific notation for 2,000,000 is 2×10^6.

(ii) 48,900

To express 48,900 in scientific notation, we move the decimal point so that only one non-zero digit remains on the left of the decimal point. For 48,900, move the decimal point 4 places to the left.

$$48{,}900 = 4.89 \times 10^4$$

Thus, the scientific notation for 48,900 is 4.89×10^4.

(iii) 0.0042

For numbers less than 1, we move the decimal point to the right so that the first digit is just to the right of the decimal point. For 0.0042, move the decimal point 3 places to the right.

$$0.0042 = 4.2 \times 10^{-3}$$

Thus, the scientific notation for 0.0042 is 4.2×10^{-3}.

(iv) 0.0000009

Similarly, for very small numbers, we move the decimal point to the right. For 0.0000009, we move the decimal 7 places to the right.

$$0.0000009 = 9 \times 10^{-7}$$

Thus, the scientific notation for 0.0000009 is 9×10^{-7}.

(v) 73×10^3

This number is already in a form that is almost in scientific notation, except that the coefficient is not between 1 and 10. We need to adjust the coefficient. To make the coefficient fall between 1 and 10, we can write 73 as 7.3×10^1, and multiply by 10^3:

$$73 \times 10^3 = 7.3 \times 10^1 \times 10^3 = 7.3 \times 10^4$$

Thus, the scientific notation for 73×10^3 is 7.3×10^4.

(vi) 0.65×10^2

Here, we have 0.65×10^2, and we want to adjust the coefficient so that it is between 1 and 10. To do this, move the decimal point in 0.65 one place to the right to make it 6.5, and adjust the power of 10 accordingly:

$$0.65 \times 10^2 = 6.5 \times 10^{-1} \times 10^2 = 6.5 \times 10^1$$

Thus, the scientific notation for 0.65×10^2 is 6.5×10^1.

2. Express the following numbers in ordinary notation.

(i)	8.04×10^2	(ii)	3×10^5
(iii)	1.5×10^{-2}	iv)	1.77×10^7
(v)	5.5×10^{-6}	(vi)	4×10^{-5}

Let's express each number in ordinary (standard) notation.

Solution:

(i) 8.04×10^2

To convert to ordinary notation, we multiply 8.04 by 10^2 (which is 100). This means we move the decimal point two places to the right:

$$8.04 \times 10^2 = 804$$

Thus, 8.04×10^2 in ordinary notation is 804.

(ii) 3×10^5

To convert to ordinary notation, we multiply 3 by 10^5 (which is 100,000). This means we move the decimal point five places to the right:

$$3 \times 10^5 = 300{,}000$$

Thus, 3×10^5 in ordinary notation is 300,000.

(iii) 1.5×10^{-2}

To convert to ordinary notation, we multiply 1.5 by 10^{-2} (which is $\frac{1}{100}$). This means we move the decimal point two places to the left:

$$1.5 \times 10^{-2} = 0.015$$

Thus, 1.5×10^{-2} in ordinary notation is 0.015.

(iv) 1.77×10^7

To convert to ordinary notation, we multiply 1.77 by 10^7 (whic is 10,000,000). This means we move the decimal point seven places to the right:

$$1.77 \times 10^7 = 17{,}700{,}000$$

Thus, 1.77×10^7 in ordinary notation is 17,700,000.

(v) 5.5×10^{-6}

To convert to ordinary notation, we multiply 5.5 by 10^{-6} (which is $\frac{1}{1{,}000{,}000}$). This means we move the decimal poin six places to the left:

$$5.5 \times 10^{-6} = 0.0000055$$

Thus, 5.5×10^{-6} in ordinary notation is 0.0000055.

(vi) 4×10^{-5}

To convert to ordinary notation, we multiply 4 by 10^{-5} (which is $\frac{1}{100{,}000}$). This means we move the decimal point five places to the left:

$$4 \times 10^{-5} = 0.00004$$

Thus, 4×10^{-5} in ordinary notation is 0.00004.

3. The speed of light is approximately 3×10^8 miles per second. Express it in standard form.

Solution: The speed of light in scientific notation is approximately 3×10^8 miles per second.

To express this in standard form:

- Start with the base number 3.
- Multiply it by 10^8, which means moving the decimal point 8 places to the right.

Thus, the speed of light in standard form is:

300,000,000 miles per second.

4. The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.

Solution: To express 40,075,000 meters in scientific notation:

- Place the decimal point after the first non-zero digit: 4.0075.
- Count the number of places the decimal point has moved to get back to the original number. In this case, it moves 7 places to the right.

Thus, the circumference of the Earth in scientific notation is:

$$4.0075 \times 10^7 \text{ meters.}$$

5. The diameter of Mars is 6.7779×10^3km. Express this number in standard form.

Solution: Let's solve 6.7779×10^3 km step by step in detail:

Step 1: Understand the given scientific notation

The number 6.7779×10^3 means that the decimal point in 6.7779 needs to be shifted **3 places to the right**, because the exponent of 10 is +3.

Step 2: Perform the shifting

Start with 6.7779:

- Shift the decimal point **1 place** to the right: 67.779.
- Shift it **2 places** to the right: 677.79.
- Shift it **3 places** to the right: 6777.9.

Step 3: Write the result

After moving the decimal point 3 places to the right, the number becomes:

$$6{,}777.9 \text{ km.}$$

Final Answer:

The diameter of Mars in standard form is:

$$6{,}777.9 \text{ km.}$$

6. **The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.**

To express the diameter of Earth, 1.2763×10^4 km, in standard form, we need to convert it into a regular number.

Step 1: Understand the scientific notation

The expression 1.2763×10^4 means:

- The coefficient is 1.2763.
- The exponent 10^4 means we need to move the decimal point 4 places to the right.

Step 2: Convert to standard form

Starting with the coefficient 1.2763, moving the decimal point 4 places to the right:

$$1.2763 \rightarrow 12763$$

Final Answer:

The diameter of Earth in standard form is:

$$\boxed{12763 \text{ km}}$$

2.2 Logarithm

A logarithm is based on two Greek words: logos and arithmos which means ratio or proportion. John Napier, a Scottish mathematician, invented the word logarithm. It is a way to simplify complex calculations, especially those involving multiplication and division of large numbers. Today, logarithm remain fundamental in mathematics, with applications in science, finance and technology.

2.2.1 Logarithm of a Number

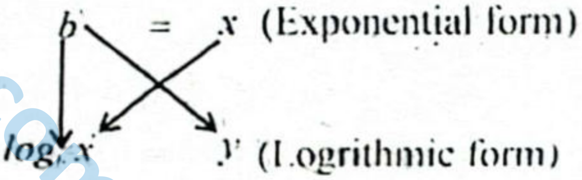
In simple words, the logarithm of a number tells us how many times one number must be multiplied by itself to get another number.

The general form of a logarithm is:

$$log_b(x) = y$$

Where:

- b is the **base**,
- x is the **result** or the number whose logarithm is being taken,
- y is the **exponent** or the logarithm of x to the base b.



This means that:

$$b^y = x$$

In words, **"the logarithm of x to the base b is y,** means that when b is raised to the power y, it equals x. The relationship between logarithmic form and exponential form is given below:

$\log_b(x) = y \quad \Leftrightarrow \quad b^y = x$ where b > 0, $x > 0$ and $b \neq 1$

Example 5: Convert $\log_2 8 = 3$ to exponential form.

Solution: $\log_2 8 = 3$

Its exponential form is: $2^3 = 8$

Example 6: Convert $\log_{10} 100 = 2$ to exponential form.

Solution: $\log_{10} 100 = 2$

Its exponential form is: $10^2 = 100$

Example 7: Find the value of x in each case:

(i) $\log_5 25 = x$ (ii) $\log_2 x = 6$

Solution: (i) $\log_5 25 = x$

Its exponential form is:

$$5^x = 25$$

$$\Rightarrow \quad 5^x = 5^2$$

$$\Rightarrow \quad x = 2$$

(ii) $\log_2 x = 6$

Its exponential form is:

$$2^6 = x$$

$$\Rightarrow \quad x = 64$$

Example 8: Convert the following in logarithmic form:

(i) $3^4 = 81$ (ii) $7^0 = 1$

Solution: (i) $3^4 = 81$

Its logarithmic form is:

$\log_3 81 = 4$

(ii) $7^0 = 1$

Its logarithmic form is:

$\log_7 1 = 0$