Exercise 2.2

1. **Express each of the following in logarithmic form:**

(i) $10^3 = 1000$ (ii) $2^8 = 256$

(iii) $3^{-3} = \frac{1}{27}$ (iv) $20^2 = 400$

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$ (vi) $11^2 = 121$

(vii) $p = q^r$ (viii) $(32)^{-\frac{1}{5}} = \frac{1}{2}$

Solution: To express the given exponential equations in logarithmic form, we follow the basic rule of logarithms:

$$a^b = c \quad \text{can be rewritten as} \quad \log_a(c) = b$$

Let's solve each part:

(i) $10^3 = 1000$

We can express this as:

$$\log_{10}(1000) = 3$$

This is the logarithmic form where the base is 10, the result is 1000, and the exponent is 3.

(ii) $\mathbf{2^8 = 256}$

To express this in logarithmic form:

$$\log_2(256) = 8$$

Here, the base is 2, the result is 256, and the exponent is 8.

(iii) $\mathbf{3^{-3} = \frac{1}{27}}$

This can be expressed as:

$$\log_3\left(\frac{1}{27}\right) = -3$$

The base is 3, the result is $\frac{1}{27}$, and the exponent is -3.

(iv) $\mathbf{20^2 = 400}$

In logarithmic form, this is:

$$\log_{20}(400) = 2$$

Here, the base is 20, the result is 400, and the exponent is 2.

(v) $\mathbf{16^{-\frac{1}{4}} = \frac{1}{2}}$

To express this in logarithmic form:

$$\log_{16}\left(\frac{1}{2}\right) = -\frac{1}{4}$$

The base is 16, the result is $\frac{1}{2}$, and the exponent is $-\frac{1}{4}$.

(vi) $\mathbf{11^2 = 121}$

Step 1: Exponential Form

$$11^2 = 121$$

Step 2: Logarithmic Form

$$\log_{11}(121) = 2$$

Explanation:

- The **base** is 11.
- The **result** is 121.
- The **exponent** is 2.

(vii) $p = q^r$

Expressing this in logarithmic form:

$$\log_q(p) = r$$

The base is q, the result is p, and the exponent is r.

(viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$

To express this in logarithmic form:

$$\log_{32}\left(\frac{1}{2}\right) = -\frac{1}{5}$$

Here, the base is 32, the result is $\frac{1}{2}$, and the exponent is $-\frac{1}{5}$.

2. Express each of the following in exponential form:

(i)	$\log_5 125 = 3$	**(ii)**	$\log_2 16 = 4$
(iii)	$\log_{23} 1 = 0$	**(iv)**	$\log_5 5 = 1$
(v)	$\log_2 \frac{1}{8} = -3$	**(vi)**	$\frac{1}{2} = \log_9 3$
(vii)	$5 = \log_{10} 100000$	**(viii)**	$\log_4 \frac{1}{16} = -2$

Solution: To convert the given logarithmic expressions into exponential form, we use the basic rule of logarithms:

$$\log_b(a) = c \quad \text{can be rewritten as} \quad b^c = a$$

Let's solve each part separately:

(i) $\log_5(125) = 3$

We can express this in exponential form as:

$$5^3 = 125$$

This means 5 raised to the power of 3 equals 125.

(ii) $\log_2(16) = 4$

In exponential form, this is:

$$2^4 = 16$$

This means 2 raised to the power of 4 equals 16.

(iii) $\log_{23}(1) = 0$

We express this as:

$$23^0 = 1$$

Any number raised to the power of 0 equals 1.

(iv) $\log_5(5) = 1$

This can be expressed as:

$$5^1 = 5$$

This means 5 raised to the power of 1 equals 5.

(v) $\log_2\left(\frac{1}{8}\right) = -3$

In exponential form:

$$2^{-3} = \frac{1}{8}$$

This means 2 raised to the power of -3 equals $\frac{1}{8}$.

(vi) $\frac{1}{2} = \log_9(3)$

We express this in exponential form as:

$$9^{\frac{1}{2}} = 3$$

This means 9 raised to the power of $\frac{1}{2}$ (the square root of 9) equals 3.

(vii) $5 = \log_{10}(100000)$

In exponential form:

$$10^5 = 100000$$

This means 10 raised to the power of 5 equals 100000.

(viii) $\log_4\left(\frac{1}{16}\right) = -2$

We express this as:

$$4^{-2} = \frac{1}{16}$$

This means 4 raised to the power of -2 equals $\frac{1}{16}$.

3. *Find the value of x in each of the following:*

(i) $\log_x 64 = 3$ **(ii)** $\log_5 1 = x$

(iii) $\log_x 8 = 1$ **(iv)** $\log_{10} x = -3$

(v) $\log_4 x = \frac{3}{2}$ **(vi)** $\log_2 1024 = x$

Solutions: We will solve each logarithmic equation separately and find the value of x.

(i) $\log_x(64) = 3$

The equation $\log_x(64) = 3$ can be rewritten in exponential form as:

$$x^3 = 64$$

Now, to find x, we take the cube root of both sides:

$$x = \sqrt[3]{64} = 4$$

Thus, the value of x is:

$$\boxed{x = 4}$$

(ii) $\log_5(1) = x$

We know that any logarithm of 1 is equal to 0, because:

$$\log_b(1) = 0 \quad \text{for any base } b$$

Thus:

$$x = 0$$

So the value of x is:

$$\boxed{x = 0}$$

(iii) $\log_x(8) = 1$

This equation can be rewritten in exponential form as:

$$x^1 = 8$$

Thus, $x = 8$.

So the value of x is:

$$\boxed{x = 8}$$

(iv) $\log_{10}(x) = -3$

Rewriting the equation in exponential form:

$$10^{-3} = x$$

This gives:

$$x = \frac{1}{10^3} = \frac{1}{1000}$$

$$x = 0.001$$

So the value of x is:

$$\boxed{x = 0.001}$$

(v) $\log_4(x) = \frac{3}{2}$

Rewriting this in exponential form:

$$x = 4^{\frac{3}{2}}$$

We can simplify $4^{\frac{3}{2}}$ as follows:

$$x = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$$

Thus:

$$x = 8$$

So the value of x is:

$$\boxed{x = 8}$$

(vi) $\log_2(1024) = x$

Rewriting this in exponential form

We get $2^x = 1024$

$2^x = 2^{10}$

Thus:

$$\therefore x = 10$$

So the value of x is:

$$\boxed{x = 10}$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1