Exercise 2.4

1. **Without using calculator, evaluate the following:**

(i)	$\log_2 18 - \log_2 9$	(ii)	$\log_2 64 + \log_2 2$
(iii)	$\frac{1}{3}\log_3 8 - \log_3 18$	(iv)	$2\log 2 + \log 25$
(v)	$\frac{1}{3}\log_4 64 + 2\log_5 25$	(vi)	$\log_3 12 + \log_3 0.25$

Solution: Let's solve each equation step by step with a little more detail:

(i) $\log x = 0.0065$

The equation $\log x = 0.0065$ is in base 10. To find x, we use the property of logarithms:

$$\log x = 0.0065 \quad \Rightarrow \quad x = 10^{0.0065}$$

Now, calculate $10^{0.0065}$ using a calculator:

$$x \approx 1.0153$$

Thus, the value of x is:

$$x \approx \boxed{1.0153}$$

(ii) $\log x = 1.192$

Again, using the property of logarithms:

$$\log x = 1.192 \quad \Rightarrow \quad x = 10^{1.192}$$

Now, calculate $10^{1.192}$:

$$x \approx 15.559$$

Thus, the value of x is:

$$x \approx \boxed{15.559}$$

(iii) $\log x = -3.434$

For this equation, we have:

$$\log x = -3.434 \quad \Rightarrow \quad x = 10^{-3.434}$$

Now, calculate $10^{-3.434}$:

$$x \approx 0.000369$$

Thus, the value of x is:

$$x \approx \boxed{0.000369}$$

(iv) $\log x = -1.5726$

For this equation:

$$\log x = -1.5726 \quad \Rightarrow \quad x = 10^{-1.5726}$$

Now, calculate $10^{-1.5726}$:

$$x \approx 0.0267$$

Thus, the value of x is:

$$x \approx \boxed{0.0267}$$

(v) $\log x = 4.3561$

For this equation:

$$\log x = 4.3561 \quad \Rightarrow \quad x = 10^{4.3561}$$

Now, calculate $10^{4.3561}$:

$$x \approx 22684.3$$

Thus, the value of x is:

$$x \approx \boxed{22684.3}$$

(vi) $\log x = -2.0184$

For this equation:

$$\log x = -2.0184 \quad \Rightarrow \quad x = 10^{-2.0184}$$

Now, calculate $10^{-2.0184}$:

$$x \approx 0.000095$$

Thus, the value of x is:

$$x \approx \boxed{0.000095}$$

2. **Write the following as a single logarithm.**

(i) $\frac{1}{2}\log 25 + 2\log 3$ **(ii)** $\log 9 - \log \frac{1}{3}$

(iii) $\log_5 b^2 . \log_a 5^3$ **(iv)** $2\log_3 x + \log_3 y$

(v) $4\log_5 x - \log_5 y + \log_5 z$

(vi) $2\ln a + 3\ln b - 4\ln c$

Solution: Let's simplify each of the logarithmic expressions step-by-step using logarithmic properties.

(i) $\frac{1}{2}\log 25 + 2\log 3$

Use the logarithmic property $a\log b = \log b^a$:

- For $\frac{1}{2}\log 25$, we have:

$$\frac{1}{2}\log 25 = \log 25^{1/2} = \log 5$$

- For $2\log 3$, we have:

$$2\log 3 = \log 3^2 = \log 9$$

Now, combine the two terms using the property $\log_b a + \log_b c = \log_b(a \cdot c)$:

$$\log 5 + \log 9 = \log(5 \cdot 9) = \log 45$$

Thus, the single logarithm is:

$$\boxed{\log 45}$$

(ii) $\log 9 - \log\frac{1}{3}$

Use the logarithmic property $\log_b a - \log_b c = \log_b \frac{a}{c}$:

$$\log 9 - \log\frac{1}{3} = \log\left(\frac{9}{\frac{1}{3}}\right) = \log(9 \cdot 3) = \log 27$$

Thus, the single logarithm is:

$$\boxed{\log 27}$$

(iii) $\log_5 b^2 + \log_a 5^3$

Step-by-Step Solution:

For $\log_5 b^2$:

Using the property $a\log_b x = \log_b(x^a)$, we have:

$$\log_5 b^2 = \log_5(b^2) = 2\log_5 b$$

For $\log_a 5^3$:

Similarly, using the same property:

$$\log_a 5^3 = \log_a(5^3) = 3\log_a 5$$

Now, combine the two terms:

Using the property $\log_b a + \log_b c = \log_b(a \cdot c)$, we now combine the two expressions:

$$\log_5 b^2 + \log_a 5^3 = \log_5 b^2 + \log_a 5^3$$

Thus, the single logarithm is:

$$\boxed{\log_5 b^2 + \log_a 5^3}$$

(v) $\mathbf{4\log_5 x - \log_5 y + \log_5 z}$

Use the property $a\log_b c = \log_b c^a$ for the first term:

$$4\log_5 x = \log_5 x^4$$

Now, combine the terms using $\log_b a - \log_b c = \log_b \frac{a}{c}$ and $\log_b a + \log_b c = \log_b (a \cdot c)$:

$$\log_5 x^4 - \log_5 y + \log_5 z = \log_5 \left(\frac{x^4 \cdot z}{y}\right)$$

Thus, the single logarithm is:

$$\boxed{\log_5 \left(\frac{x^4 z}{y}\right)}$$

(vi) $\mathbf{2\ln a + 3\ln b - 4\ln c}$

Use the property $a\ln b = \ln b^a$ for each term:

$$2\ln a = \ln a^2, \quad 3\ln b = \ln b^3, \quad -4\ln c = \ln c^{-4}$$

Now, combine the terms using $\ln a + \ln b + \ln c = \ln(a \cdot b \cdot c)$ and $\ln a - \ln b = \ln \frac{a}{b}$:

$$\ln a^2 + \ln b^3 - \ln c^4 = \ln\left(\frac{a^2 b^3}{c^4}\right)$$

Thus, the single logarithm is:

$$\boxed{\ln\left(\frac{a^2 b^3}{c^4}\right)}$$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$ **(ii)** $\log_5 \sqrt{8a^6}$

(iii) $\ln\left(\frac{a^2 b}{c}\right)$ **(iv)** $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

(v) $\ln \sqrt[3]{16x^3}$ **(vi)** $\log_2 \left(\frac{1-a}{b}\right)^5$

Solutions: We will expand each logarithmic expression using logarithmic properties such as:

1. $\log\frac{a}{b} = \log a - \log b$
2. $\log(ab) = \log a + \log b$
3. $\log a^n = n\log a$

(i) $\log\left(\frac{11}{5}\right)$

Using the property $\log\frac{a}{b} = \log a - \log b$, expand:

$$\log\left(\frac{11}{5}\right) = \log 11 - \log 5$$

Final result:

$$\boxed{\log\left(\frac{11}{5}\right) = \log 11 - \log 5}$$

(ii) $\log_5\sqrt{8a^6}$

First, rewrite $\sqrt{8a^6}$ as $(8a^6)^{1/2}$.

Using the property $\log a^n = n\log a$, expand:

$$\log_5\sqrt{8a^6} = \frac{1}{2}\log_5(8a^6)$$

Next, use $\log(ab) = \log a + \log b$:

$$\log_5(8a^6) = \log_5 8 + \log_5 a^6$$

Now expand $\log_5 a^6$ using $\log a^n = n\log a$:

$$\log_5 a^6 = 6\log_5 a$$

Substitute back:

$$\frac{1}{2}\log_5(8a^6) = \frac{1}{2}(\log_5 8 + 6\log_5 a)$$

Distribute $\frac{1}{2}$:

$$\frac{1}{2}\log_5 8 + 3\log_5 a$$

Final result:

$$\boxed{\log_5\sqrt{8a^6} = \frac{1}{2}\log_5 8 + 3\log_5 a}$$

(iii) $\ln\left(\frac{a^2b}{c}\right)$

Using $\ln\frac{a}{b} = \ln a - \ln b$, rewrite:

$$\ln\left(\frac{a^2b}{c}\right) = \ln(a^2b) - \ln c$$

Now use $\ln(ab) = \ln a + \ln b$:

$$\ln(a^2b) = \ln a^2 + \ln b$$

Expand $\ln a^2$ using $\ln a^n = n\ln a$:

$$\ln a^2 = 2\ln a$$

Substitute back:

$$\ln\left(\frac{a^2b}{c}\right) = 2\ln a + \ln b - \ln c$$

Final result:

$$\boxed{\ln\left(\frac{a^2b}{c}\right) = 2\ln a + \ln b - \ln c}$$

(iv) $\log\left(\frac{xy}{z}\right)^{1/9}$

Using $\log a^n = n\log a$, expand:

$$\log\left(\frac{xy}{z}\right)^{1/9} = \frac{1}{9}\log\left(\frac{xy}{z}\right)$$

Using $\log\frac{a}{b} = \log a - \log b$, rewrite:

$$\log\left(\frac{xy}{z}\right) = \log(xy) - \log z$$

Using $\log(ab) = \log a + \log b$, expand $\log(xy)$:

$$\log(xy) = \log x + \log y$$

Substitute back:

$$\log\left(\frac{xy}{z}\right) = \log x + \log y - \log z$$

Now distribute $\frac{1}{9}$:

$$\frac{1}{9}(\log x + \log y - \log z) = \frac{1}{9}\log x + \frac{1}{9}\log y - \frac{1}{9}\log z$$

Final result:

$$\log\left(\frac{xy}{z}\right)^{1/9} = \frac{1}{9}\log x + \frac{1}{9}\log y - \frac{1}{9}\log z$$

(v) $\ln\sqrt[3]{16x^3}$

Rewrite $\sqrt[3]{16x^3}$ as $(16x^3)^{1/3}$.

Using $\ln a^m = m\ln a$, expand:

$$\ln\sqrt[3]{16x^3} = \frac{1}{3}\ln(16x^3)$$

Using $\ln(ab) = \ln a + \ln b$, expand $\ln(16x^3)$:

$$\ln(16x^3) = \ln 16 + \ln x^3$$

Expand $\ln x^3$ using $\ln a^m = m\ln a$:

$$\ln x^3 = 3\ln x$$

Substitute back:

$$\ln\sqrt[3]{16x^3} = \frac{1}{3}(\ln 16 + 3\ln x)$$

Distribute $\frac{1}{3}$:

$$\frac{1}{3}\ln 16 + \ln x$$

Final result:

$$\ln\sqrt[3]{16x^3} = \frac{1}{3}\ln 16 + \ln x$$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Using $\log a^m = m\log a$, expand:

$$\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2\left(\frac{1-a}{b}\right)$$

Using $\log\frac{a}{b} = \log a - \log b$, expand:

$$\log_2\left(\frac{1-a}{b}\right) = \log_2(1-a) - \log_2 b$$

Substitute back:

$$5\log_2\left(\frac{1-a}{b}\right) = 5(\log_2(1-a) - \log_2 b)$$

Distribute 5:

$$5\log_2(1-a) - 5\log_2 b$$

Final result:

$$\boxed{\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2(1-a) - 5\log_2 b}$$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$ (ii) $\log_2 x + \log_2 8 = 5$

(iii) $(81)^x = (243)^{x+2}$ (iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

(v) $\log(5x-10) = 2$

(vi) $\log_2(x+1) - \log_2(x-4) = 2$

Solution: We will solve each equation step by step:

(i) $\log 2 + \log x = 1$

Using the property $\log a + \log b = \log(a \cdot b)$, rewrite:

$$\log 2 + \log x = \log(2x)$$

Now the equation becomes:

$$\log(2x) = 1$$

Rewrite in exponential form

$$\log a = b \Rightarrow a = 10^b$$

$$2x = 10^1$$

Simplify:

$$2x = 10 \Rightarrow x = \frac{10}{2}$$

Solution:

$$\boxed{x = 5}$$

(ii) $\log_2 x + \log_2 8 = 5$

Step 1: Apply the property $\log_a + \log_b = \log(a \cdot b)$

We combine the two logarithms using the property:

$$\log_2 x + \log_2 8 = \log_2(x \cdot 8)$$

So, we have:

$$\log_2(8x) = 5$$

Step 2: Rewrite in exponential form

Now, we rewrite the logarithmic equation in exponential form.

Since $\log_b a = c$ is equivalent to $a = b^c$, we rewrite:

$$8x = 2^5$$

Step 3: Simplify the exponential expression

We know that $2^5 = 32$, so the equation becomes:

$$8x = 32$$

Step 4: Solve for x

To solve for x, divide both sides by 8:

$$x = \frac{32}{8} = 4$$

Final Answer:

Thus, the value of x is:

$$\boxed{x = 4}$$

(iii) $81^x = 243^{x+2}$

Rewrite 81 and 243 as powers of 3:

$$81 = 3^4, \quad 243 = 3^5$$

Substitute:

$$(3^4)^x = (3^5)^{x+2}$$

Simplify the exponents:

$$3^{4x} = 3^{5(x+2)}$$

Equating the exponents:

$$4x = 5(x+2)$$

Expand:

$$4x = 5x + 10$$

Simplify:

$$4x - 5x = 10 \Rightarrow -x = 10 \Rightarrow x = -10$$

Solution:

$$\boxed{x = -10}$$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

Rewrite 27 and $\frac{1}{27}$ as powers of 3:

$$\frac{1}{27} = 3^{-3}, \quad 27 = 3^3$$

Substitute:

$$(3^{-3})^{x-6} = 3^3$$

Simplify the exponents:

$$3^{-3(x-6)} = 3^3$$

Equate the exponents:

$$-3(x-6) = 3$$

Simplify:

$$-3x + 18 = 3$$

Solve for x:

$$-3x = 3 - 18 \quad \Rightarrow \quad -3x = -15 \quad \Rightarrow \quad x = 5$$

Solution:

$$\boxed{x = 5}$$

(v) $\log(5x - 10) = 2$

Rewrite in exponential form:

$$5x - 10 = 10^2$$

Simplify:

$$5x - 10 = 100$$

Solve for x:

$$5x = 100 + 10 \Rightarrow 5x = 110 \Rightarrow x = \frac{110}{5} = 22$$

Solution:

$$\boxed{x = 22}$$

(vi) $\log_2(x + 1) - \log_2(x - 4) = 2$

Using the property $\log a - \log b = \log \frac{a}{b}$, rewrite:

$$\log_2\left(\frac{x+1}{x-4}\right) = 2$$

Rewrite in exponential form:

$$\frac{x+1}{x-4} = 2^2$$

Simplify:

$$\frac{x+1}{x-4}=4$$

Multiply through by $x-4$ (assuming $x>4$ to keep the logarithm defined):

$$x+1=4(x-4)$$

Expand:

$$x+1=4x-16$$

Simplify:

$$1+16=4x-x \Rightarrow 17=3x \Rightarrow x=\frac{17}{3}$$

Solution:

$$\boxed{x=\frac{17}{3}}$$

5. Find the values of the following with the help of logarithm table:

(i) $\frac{3.68\times 4.21}{5.234}$ (ii) $4.67\times 2.11\times 2.397$

(iii) $\frac{(20.46)^2\times(2.4122)}{754.3}$ (iv) $\frac{\sqrt[3]{9.364}\times 21.64}{3.21}$

Solution: We will solve each part step by step using logarithm tables. The procedure involves converting the numbers into their logarithmic form, performing the operations, and then converting back using the antilogarithm.

(i) $\frac{3.68\times4.21}{5.234}$

STEP 1: FIND THE LOGARITHMS

- Logarithm of 3.68: $\log(3.68)\approx 0.5658$
- Logarithm of 4.21: $\log(4.21)\approx 0.6243$
- Logarithm of 5.234: $\log(5.234)\approx 0.7188$

STEP 2: PERFORM THE CALCULATION IN LOGARITHMIC FORM

$$\log\left(\frac{3.68 \times 4.21}{5.234}\right) = \log(3.68) + \log(4.21) - \log(5.234)$$

Substitute values:

$$\log\left(\frac{3.68 \times 4.21}{5.234}\right) = 0.5658 + 0.6243 - 0.7188 = 0.4713$$

STEP 3: FIND THE ANTILOGARITHM

$$\text{Antilog}(0.4713) \approx 2.96$$

Solution:

$$\boxed{2.96}$$

(ii) $4.67 \times 2.11 \times 2.397$

STEP 1: FIND THE LOGARITHMS

- Logarithm of 4.67: $\log(4.67) \approx 0.6693$
- Logarithm of 2.11: $\log(2.11) \approx 0.3243$
- Logarithm of 2.397: $\log(2.397) \approx 0.3797$

STEP 2: PERFORM THE CALCULATION IN LOGARITHMIC FORM

$$\log(4.67 \times 2.11 \times 2.397) = \log(4.67) + \log(2.11) + \log(2.397)$$

Substitute values:

$$\log(4.67 \times 2.11 \times 2.397) = 0.6693 + 0.3243 + 0.3797 = 1.3733$$

STEP 3: FIND THE ANTILOGARITHM

$$\text{Antilog}(1.3733) \approx 23.62$$

Solution:

$$\boxed{23.62}$$

(iii) $\frac{(20.46)^2 \times 2.4122}{754.3}$

STEP 1: FIND THE LOGARITHMS

- Logarithm of 20.46: $\log(20.46) \approx 1.3109$
- Logarithm of 2.4122: $\log(2.4122) \approx 0.3824$
- Logarithm of 754.3: $\log(754.3) \approx 2.8775$

STEP 2: PERFORM THE CALCULATION IN LOGARITHMIC FORM

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right)$$

$$= 2\log(20.46) + \log(2.4122) - \log(754.3)$$

Substitute values:

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right) = 2(1.30) + 0.3824 - 2.8775$$

Simplify:

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right) = 2.6218 + 0.3824 - 2.8775$$

$$= 0.1267$$

STEP 3: FIND THE ANTILOGARITHM

$$\text{Antilog}(0.1267) \approx 1.339$$

Solution:

$$\boxed{1.339}$$

(iv) $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

STEP 1: FIND THE LOGARITHMS

- Logarithm of 9.364: $\log(9.364) \approx 0.9715$
- Logarithm of 21.64: $\log(21.64) \approx 1.3353$
- Logarithm of 3.21: $\log(3.21) \approx 0.5065$

STEP 2: CALCULATE THE CUBE ROOT USING LOGARITHMS

The cube root is equivalent to raising 9.364 to the power $\frac{1}{3}$, so:

$$\log(\sqrt[3]{9.364}) = \frac{1}{3}\log(9.364) = \frac{1}{3}(0.9715) \approx 0.3238$$

Step 3: Perform the calculation in logarithmic form

$$\log\left(\frac{\sqrt[3]{9.364} \times 21.64}{3.21}\right)$$

$$= \log(\sqrt[3]{9.364}) + \log(21.64) - \log(3.21)$$

Substitute values:

$$\log\left(\frac{\sqrt[3]{9.364}\times 21.64}{3.21}\right) = 0.3238 + 1.3353 - 0.5065 = 1.1526$$

STEP 4: FIND THE ANTILOGARITHM

$$\text{Antilog}(1.1526) \approx 14.2$$

Solution:

$$\boxed{14.21}$$

6. The formula for measure the magnitude of earthquakes is given by $M=\log_{10}\left(\frac{A}{A_o}\right)$. If amplitude (A) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

Solution: We are given the formula for the magnitude of earthquakes:

$$M = \log_{10}\left(\frac{A}{A_o}\right)$$

Given:

- $A = 10{,}000$
- $A_o = 10$

Substitute the values into the formula:

$$M = \log_{10}\left(\frac{10{,}000}{10}\right)$$

Simplify the fraction:

$$M = \log_{10}(1{,}000)$$

The logarithm of 1,000 with base 10 is:

$$\log_{10}(1{,}000) = 3$$

Final Answer:

The magnitude of the earthquake is:

$$\boxed{M = 3}$$

7. **Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 100{,}000\,(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.**

Solution: We are tasked with determining after how many years the investment will double its initial value.

Step 1: Define the equation

The initial investment is 100,000. To double the investment:

$$y = 2 \times 100{,}000 = 200{,}000$$

Substituting into the given model:

$$200{,}000 = 100{,}000(1.05)^t$$

Step 2: Simplify the equation

Divide both sides by 100,000:

$$2 = (1.05)^t$$

Step 3: Solve for t using logarithms

Take the common logarithm log of both sides:

$$\text{Log}_2 = 10\text{g}\,(1.05)^t$$

Using the logarithmic property $\log_a t = t \log_a$

$$\text{Log}_2 = t \log 1.05 \quad t = \frac{\log_2}{\log 1.05}$$

$$= \frac{0.3010}{0.0212}$$

$$= 14.198 \text{ years}$$

$$\approx 14.2 \text{ years}$$

Solve for t:

$$t = \frac{\ln(2)}{\ln(1.05)}$$

Step 4: Calculate values

Substitute these values:

$$\approx 14.2$$

Final Answer:

The investment will double in approximately **14.2 years.**

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20°C. Using the formula $T = T_i \times 0.97) \frac{h}{100}$. calculate the temperature at an altitude (h) of 500 metres.

Solution: Let's solve the problem step by step using the given formula.

Problem:

The temperature at sea level (T_i) is 20°C, and the temperature decreases by 3% for every 100 metres gained in altitude. The formula for the temperature at any altitude h is:

$$T = T_i \times 0.97^{\frac{h}{100}}$$

We need to calculate the temperature at an altitude of 500 metres ($h = 500$).

Step 1: Apply the formula

Substitute the known values into the formula:

$$T = 20 \times 0.97^{\frac{500}{100}}$$

Simplify the exponent:

$$T = 20 \times 0.97^5$$

Step 2: Calculate 0.97^5

We calculate 0.97^5:

$$0.97^5 \approx 0.8587$$

Step 3: Multiply by the initial temperature

Now, multiply the result by the initial temperature:

$$T = 20 \times 0.8587$$

$$T \approx 17.174\,°C$$

Final Answer:

The temperature at an altitude of 500 metres is approximately:

$$\boxed{17.17\,°C}$$