Exercise 3.1

1. Write the following sets in set builder notation:

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

(ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$

(iv) $\{6, 12, 18, \ldots, 120\}$

(v) $\{100, 102, 104, \ldots, 400\}$

(vi) $\{1, 3, 9, 27, 81, \ldots\}$

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

(viii) $\{5, 10, 15, \ldots, 100\}$

(ix) **The set of all integers between –100 and 1000.**

Solution: Here are the sets written in **set-builder notation**:

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

This is the set of all perfect squares up to 484.

$$A = \{x \mid x = n^2, n \in \mathbb{N}, 1 \leq n \leq 22\}$$

(ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$

This is the set of all powers of 2 less than or equal to 150.

$$B = \{x \mid x = 2^n, n \in \mathbb{N}, 1 \leq 2^n \leq 150\}$$

(iii) $\{0, +1, +2, \ldots, +1000\}$

This is the set of all integers from 0 to 1000.

$$C = \{x \mid x \in \mathbb{Z}, 0 \leq x \leq 1000\}$$

(iv) $\{6, 12, 18, \dots, 120\}$

This is the set of all multiples of 6 up to 120.

$$D = \{x \mid x = 6n,\ n \in \mathbb{N},\ 1 \le n \le 20\}$$

(v) $\{100, 102, 104, \dots, 400\}$

This is the set of all even numbers between 100 and 400, inclusive.

$$E = \{x \mid x = 100 + 2n,\ n \in \mathbb{N},\ 0 \le n \le 150\}$$

(vi) $\{1, 3, 9, 27, 81, \dots\}$

This is the set of all powers of 3.

$$F = \{x \mid x = 3^n,\ n \in W\}$$

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

This is the set of all divisors of 100.

$$G = \{x \mid x \text{ divides } 100,\ x \in \mathbb{N}\}$$

(viii) $\{5, 10, 15, \dots, 100\}$

This is the set of all multiples of 5 up to 100.

$$H = \{x \mid x = 5n,\ n \in \mathbb{N},\ 1 \le n \le 20\}$$

(ix) **The set of all integers between -100 and 1000**

$$I = \{x \mid x \in \mathbb{Z},\ -100 \le x \le 1000\}$$

Explanation:

- $x \in \mathbb{Z}$: Specifies that x is an integer.
- $-100 \le x \le 1000$: Specifies the range of the integers.

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x \text{ is a multiple of } 3 \wedge x \le 35\}$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

(iii) $\{x \mid x \in P \wedge x < 12\}$

(iv) $\{x \mid x \text{ is a divisor of } 128\}$

(v) $\{x \mid x = 2^n, n \in N \wedge n < 8\}$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

(vii) $\{x \mid xN \wedge x = x\}$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

Let's address each set in tabular form based on the given conditions:

(i) $\{x \mid x \text{ is a multiple of } 3 \wedge x < 35\}$

This is the set of multiples of 3 less than 35:

$$\{0,3,6,9,12,15,18,21,24,27,30,33\}$$

(ii) $\{x \mid x \in \mathbb{R} \wedge 2x + 1 = 0\}$

Solve for x from the equation:

$$2x + 1 = 0 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$$

Thus, the set is:

$$\left\{-\frac{1}{2}\right\}$$

(iii) $\{x \mid x \in \mathbb{P} \wedge x < 12\}$

This is the set of prime numbers less than 12:

$$\{2,3,5,7,11\}$$

(iv) $\{x \mid x \text{ is a divisor of } 128\}$

The divisors of 128 are:

$$\{1,2,4,8,16,32,64,128\}$$

(v) $\{x \mid x = 2^n, n \in \mathbb{N} \wedge n < 8\}$

This is the set of powers of 2 where n is a natural number less than 8:

$$\{2,4,8,16,32,64,128\}$$

(vi) $\{x \mid x \in \mathbb{N} \wedge x + 4 = 0\}$

This is an impossible equation for natural numbers, as no natural number x satisfies $x + 4 = 0$. $x = -4$, $-4 \notin N$. Hence, the set is: $\varnothing$

(vii) $\{x \mid x \in \mathbb{N} \wedge x = x\}$

This set is simply the set of all natural numbers:

$$\mathbb{N} = \{1,2,3,4,5, \dots\}$$

(viii) $\{x \mid x \in \mathbb{Z} \wedge 3x + 1 = 0\}$

Solve for x from the equation:

$$3x + 1 = 0 \Rightarrow 3x = -1 \Rightarrow x = -\frac{1}{3}$$

Since $x = -\frac{1}{3}$ is not an integer $x \notin \mathbb{Z}$, the set is. ϕ

3. Write two proper subsets of each of the following sets:

(i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N

(iv) Z (v) Q (vi) R

(vii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Solution: Here are two proper subsets for each of the given sets:

(i) $\{a, b, c\}$

Proper subsets are subsets of the given set that are not equal to the original set. Two proper subsets:

$$\{a, b\}, \{b, c\}$$

(ii) $\{0, 1\}$

Two proper subsets:

$$\{0\}, \{1\}$$

(iii) **$\mathbb{N}$ (Set of natural numbers)**

Two proper subsets:

$$\{1,2,3\}, \{5,10,15\}$$

(iv) **$\mathbb{Z}$ (Set of integers)**

Two proper subsets:

$$\{-1,0,1\}, \{-10,-5,5\}$$

(v) **$\mathbb{Q}$ (Set of rational numbers)**

Two proper subsets:

$$\left\{\frac{1}{2}, \frac{2}{3}, 1\right\}, \left\{-1, 0, \frac{3}{4}\right\}$$

(vi) **$\mathbb{R}$ (Set of real numbers)**

Two proper subsets:

$$\{x \in \mathbb{R} \mid x > 0\}, \{x \in \mathbb{R} \mid x < 1\}$$

(vii) $\{x \mid x \in \mathbb{Q}, 0 < x \leq 2\}$ **(Set of rational numbers between 0 and 2, inclusive)**

Two proper subsets:

$$\left\{\frac{1}{2}, 1\right\}, \left\{\frac{3}{4}, 2\right\}$$

4. Is there any set which has no proper subset? If so, name that set.

Solution: Yes, there is a set that has no proper subset: the **empty set**, denoted by ⌀ or {}.

Explanation:

- A **proper subset** of a set A is any subset of A that is not equal to A itself.
- The empty set ⌀ is a subset of every set, including itself.
- However, ⌀ does not contain any elements, so it cannot have a subset that is different from itself.
- Thus, the empty set has no proper subsets.

5. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Solution: The sets $\{a, b\}$ and $\{\{a, b\}\}$ are fundamentally different because they contain different elements.

Explanation:

- **Set $\{a, b\}$:**
  - This set contains two elements: a and b.
  - Example: If $a = 1$ and $b = 2$, then $\{a, b\} = \{1,2\}$.
- **Set $\{\{a, b\}\}$:**
  - This set contains only one element, which is itself a set: $\{a, b\}$.
  - Example: If $a = 1$ and $b = 2$, then $\{\{a, b\}\} = \{\{1,2\}\}$.

Key Differences:

Aspect	$\{a, b\}$	$\{\{a, b\}\}$
Number of Elements	2 (a and b)	1 ($\{a, b\}$)
Type of Elements	Basic elements (a, b)	A single set ($\{a, b\}$)
Nested Structure	Not nested	Nested (contains a set)

Examples:

- $\{1,2\}$ is a set of two elements: 1 and 2.
- $\{\{1,2\}\}$ is a set containing one element: the set $\{1,2\}$.

In summary, $\{a, b\}$ is a set of elements a and b, while $\{\{a, b\}\}$ is a set containing a single set, $\{a, b\}$.

6. What is the number of elements of the power set of each of the following sets?

(i)	**{ }**	**(ii)**	**{0, 1}**
(iii)	**{1, 2, 3, 4, 5, 6, 7}**	**(iv)**	**{0, 1, 2, 3, 4, 5, 6, 7}**
(v)	**{a, {*b*, *c*}}**	**(vi)**	**{{*a*, *b*}, {*b*, *c*}, {*d*, *e*}}**

Solution: The number of elements in the **power set** of a set is given by 2^n, where n is the number of elements in the set. Here's the solution for each case:

(i) $\{\}$:

- The empty set has $n = 0$ elements.
- The power set has $2^0 = 1$ element.
- Power set: P (1).

Answer: 1 element.

(ii) $\{0, 1\}$:

- The set has $n = 2$ elements.
- The power set has $2^2 = 4$ elements.
- Power set: $\{\{\}, \{0\}, \{1\}, \{0,1\}\}$.

Answer: 4 elements.

(iii) $\{1, 2, 3, 4, 5, 6, 7\}$:

- The set has $n = 7$ elements.
- The power set has $2^7 = 128$ elements.

Answer: 128 elements.

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$:

- The set has $n = 8$ elements.
- The power set has $2^8 = 256$ elements.

Answer: 256 elements.

(v) $\{a, \{b, c\}\}$:

- The set has $n = 2$ elements (a and $\{b, c\}$ are distinct elements).
- The power set has $2^2 = 4$ elements.
- Power set: $\{\{\}, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$.

(vi) $\{\{a,b\},\{b,c\},\{d,e\}\}$:

- The set has $n = 3$ elements ($\{a,b\}$, $\{b,c\}$, and $\{d,e\}$ are distinct elements).
- The power set has $2^3 = 8$ elements.
- Power set:

$\{\{\},\{\{a,b\}\},\{\{b,c\}\},\{\{d,e\}\},\{\{a,b\},\{b,c\}\},\{\{a,b\},\{d,e\}\},$

- $\{\{b,c\},\{d,e\}\},\{\{a,b\},\{b,c\},\{d,e\}\}\}$.

Answer: 8 elements.

7. Write down the power set of each of the following sets:

(i) {9, 11} **(ii)** {+, − ×, ÷}

(iii) {ϕ} **(iv)** {a, {b, c}}

Solution: Here are the power sets for the given sets:

(i) $\{9, 11\}$:

The set has 2 elements: 9 and 11. The power set contains $2^2 = 4$ subsets.

Power set:

$$\{\{\},\{9\},\{11\},\{9,11\}\}$$

(ii) $\{+,-,\times,\div\}$:

The set has 4 elements: $+,-,\times,\div$. The power set contains $2^4 = 16$ subsets.

Power set:

$\{\{\},\{+\},\{-\},\{\times\},\{\div\},\{+,-\},\{+,\times\},\{+,\div\},\{-,\times\},\{-,\div\},\{\times\div\},$
$\{+,-,\times\},\{+,-,\div\},\{+,\times,\div\},\{-,\times,\div\},\{+,-,\times,\div\}\}$

(iii) {} **(empty set):**

The set has no elements. The power set contains $2^0 = 1$ subset.

Power set:

$$\{\{\}\}$$

(iv) $\{a,\{b,c\}\}$:

The set has 2 elements: a and $\{b,c\}$. The power set contains $2^2 = 4$ subsets.

Power set: $\{\{\},\{a\},\{\{b,c\}\},\{a,\{b,c\}\}\}$