Exercise 3.2

1. Consider the universal set $U = \{x : x$ is multiple of 2 and $0 < x \leq 30\}$, $A = \{x : x$ is a multiple of 6$\}$ and $B = \{x : x$ is a multiple of 8$\}$

(i) List all elements of sets A and B in tabular form

(ii) Find $A \cap B$ (iii) Draw a Venn diagram

Let's break this down step by step:

Given:

- Universal set $U = \{2,4,6,8,10,12,14,16,18,20,22,24,26,28\}$
- $A = \{x : x$ is a multiple of 6$\}$
- $B = \{x : x$ is a multiple of 8$\}$

(i) List all elements of A and B:

ELEMENTS OF A:

- Multiples of 6 in U: 6,12,18,24.
- $n(A) = \{6,12,18,24\}$

ELEMENTS OF B:

- Multiples of 8 in U: 8,16,24.
- $B = \{8,16,24\}$

TABULAR REPRESENTATION:

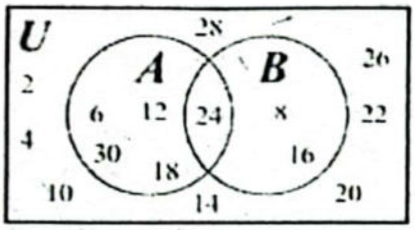
Set	Elements
A	$\{6,12,18,24\}$
B	$\{8,16,24\}$

(ii) Find $A \cap B$:

The intersection of A and B is the set of elements common to both A and B.

$$A \cap B = \{24\}$$

(iii) Draw a Venn Diagram:



The Venn diagram shows:

- $A = \{6,12,18,24\}$
- $B = \{8,16,24\}$
- $A \cap B = \{24\}$

The remaining elements of U, which are not in A or B, belong to the "outside" region of the Venn diagram.

I will generate this diagram for you.

I couldn't generate the Venn diagram image, but here's how you can visualize it:

- **Universal Set U:** Enclose everything in a rectangle labeled U. It contains the elements $\{2,4,6,8,10,12,14,16,18,20,22,24,26,28\}$.
- **Circle A:** Contains the elements $\{6,12,18,24\}$.
- **Circle B:** Contains the elements $\{8,16,24\}$.
- **Intersection $A \cap B$:** Place $\{24\}$ in the overlapping region of the circles.
- **Outside both A and B:** Place the remaining elements of U ($\{2,4,10,14,20,22,26,28\}$) outside the circles.

2. Let, $U = \{x : x \text{ is an integer and } 0 < x < 150\}$,
$G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 \leq m\ 12\}$ and
$H = \{x : x \text{ is a square}\}$

(i) List all elements of sets G and H in tabular form

(ii) Find $G \cup H$ **(iii) Find $G \cap H$**

We are working with the universal set $U = \{x: x \text{ is an integer and } 0 < x < 150\}$, and the following subsets:

- $G = \{x: x = 2m, \text{ where } 0 \leq m \leq 12\}$
- $H = \{x: x$ is a perfect square

(i) List all elements of G and H:

ELEMENTS OF G:

- $G = \{x: x = 2m \text{ for } m \in \{0,1,2,\ldots,12\}\}$
- Calculate:

$x = 2\cdot 0, 2\cdot 1, 2\cdot 2, \ldots, 2\cdot 12 \Rightarrow G = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$

Elements of H:

- $H = \{x: x = n^2 \text{ for integers } n, 0 < n^2 < 150\}$
- Calculate:

$n = 1, 2, 3, \ldots, 12 \Rightarrow H = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

(ii) Find $G \cup H$:

The union $G \cup H$ is the set of all elements that belong to G, H, or both.

- $G = \{2,4,6,8,10,12,14,16,18,20,22,24\}$
- $H = \{1,4,9,16,25,36,49,64,81,100,121,144\}$

Union:

$$G \cup H = \{1,2,4,6,8,9,10,12,14,16,18,20,22,24,25,36,49,64,81,100,121,144\}$$

(iii) Find $G \cap H$:

The intersection $G \cap H$ is the set of elements common to both G and H.

Common elements:

$$G = \{2,4,6,8,10,12,14,16,18,20,22,24\}$$
$$H = \{1,4,9,16,25,36,49,64,81,100,121,144\}$$

Intersection:

$$G \cap H = \{4,16\}$$

3. **Consider the sets $P = \{x : x$ is a prime number and $0 < x \leq 20\}$ and $Q = \{x : x$ is a divisor of 210 and $0 < x < 20\}$**

(i) Find $P \cap Q$ **(ii) Find $P \cup Q$**

Solution: We are working with the sets:

- $P = \{x: x$ is a prime number and $0 < x \leq 20\}$
- $Q = \{x: x$ is a divisor of 210 and $0 < x < 20\}$

Step 1: List the elements of P and Q

ELEMENTS OF P:

Prime numbers less than or equal to 20 are:

$$P = \{2,3,5,7,11,13,17,19\}$$

ELEMENTS OF Q:

Divisors of 210 less than 20 are:

Prime factorization of 210:

$$210 = 2 \cdot 3 \cdot 5 \cdot 7$$

Divisors: $\{1,2,3,5,6,7,10,14,15\}$

$$Q = \{1,2,3,5,6,7,10,14,15\}$$

(i) Find $P \cup Q$:

The union $P \cup Q$ is the set of elements that belong to either P, Q, or both.

$$P = \{2,3,5,7,11,13,17,19\}, \quad Q = \{1,2,3,5,6,7,10,14,15\}$$

Union:

$$P \cup Q = \{1,2,3,5,6,7,10,11,13,14,15,17,19\}$$

(ii) Find $P \cap Q$:

The intersection $P \cap Q$ is the set of elements common to both P and Q.

$$P = \{2,3,5,7,11,13,17,19\}, \quad Q = \{1,2,3,5,6,7,10,14,15\}$$

Intersection:

$$P \cap Q = \{2,3,5,7\}$$

Final Answers:

- **Union ($P \cup Q$):**
  $$P \cup Q = \{1,2,3,5,6,7,10,11,13,14,15,17,19\}$$
- **Intersection ($P \cap Q$):**
  $$P \cap Q = \{2,3,5,7\}$$

4. **Verify the commutative properties of union and intersection for the following pairs of sets:**

(i) $A = \{1, 2, 3, 4, 5\}, B = \{4, 6, 8, 10\}$

(ii) N, Z **(iii)** $A = \{x \| x \in R \wedge x\}, B = R$

Solution: The **commutative properties of union and intersection** state that:

- **Union:** $A \cup B = B \cup A$
- **Intersection:** $A \cap B = B \cap A$

We will verify these properties for the given pairs of sets.

(i) **$A = \{1, 2, 3, 4, 5\}, B = \{4, 6, 8, 10\}$**

VERIFY UNION:

$$A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\} = \{1,2,3,4,5,6,8,10\}$$
$$B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,8,10\}$$

$\therefore A \cup B = B \cup A.$

VERIFY INTERSECTION:

$$A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\} = \{4\}$$
$$B \cap A = \{4,6,8,10\} \cap \{1,2,3,4,5\} = \{4\}$$

$\therefore A \cap B = B \cap A.$

(ii) $A = \mathbb{N}$ (natural numbers), $B = \mathbb{Z}$ (integers)

VERIFY UNION:

$$A \cup B = \mathbb{N} \cup \mathbb{Z} = \mathbb{Z}$$
$$B \cup A = \mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$$

$\therefore A \cup B = B \cup A.$

VERIFY INTERSECTION:

$$A \cap B = \mathbb{N} \cap \mathbb{Z} = \mathbb{N}$$
$$B \cap A = \mathbb{Z} \cap \mathbb{N} = \mathbb{N}$$

$\therefore A \cap B = B \cap A.$

(iii) $A = \{x \mid x \in \mathbb{R}$ and $x \geq 0\}, B = \mathbb{R}$

NOTE ON A:

The set $A = \{x \mid x \in \mathbb{R} \text{ and } x \geq 0\}$

then, if $A = \mathbb{R}$:

- Union: $A \cup B = \mathbb{R} \cup \mathbb{R} = \mathbb{R}$
- Intersection: $A \cap B = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$.

  $\therefore\ A \cup B = B \cup A$ *and* $A \cap B = B \cap A$

5. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$,

Verify De Morgan's Laws for these sets. Draw Venn diagram.

Solution: **De Morgan's Laws** are two important rules about set operations that relate unions and intersections of sets to their complements. They are:

- **First Law (Union of Complements):**

$$(A \cup B)^c = A^c \cap B^c$$

- **Second Law (Intersection of Complements):**

$$(A \cap B)^c = A^c \cup B^c$$

Let us verify these laws for a universal set $U = \{a, b, c, d, e, f, g, h, i, j\}$ and arbitrary sets A and B.

Step 1: Choose Example Sets A and B

Let:

$$A = \{a, c, e, g\}, \quad B = \{b, d, f, h\}$$

The universal set is:

$$U = \{a, b, c, d, e, f, g, h, i, j\}$$

Step 2: Find Complements of A and B

The complements of A and B are all the elements in U that are not in A or B, respectively.

- $A^c = U - A = \{b, d, f, h, i, j\}$
- $B^c = U - B = \{a, c, e, g, i, j\}$

Step 3: Verify De Morgan's First Law

We need to check if:

$$(A \cup B)^c = A^c \cap B^c$$

STEP 3.1: FIND $(A \cup B)$

The union $A \cup B$ is the set of all elements that are in either A or B or both.

$$A \cup B = \{a, c, e, g, b, d, f, h\}$$

STEP 3.2: FIND $(A \cup B)^c$

The complement of $A \cup B$ is the set of elements in U that are **not** in $A \cup B$:

$$(A \cup B)^c = U - (A \cup B) = \{i, j\}$$

STEP 3.3: FIND $A^c \cap B^c$

Now, find the intersection of A^c and B^c:

$$A^c \cap B^c = \{b, d, f, h, i, j\} \cap \{a, c, e, g, i, j\} = \{i, j\}$$

STEP 3.4: CONCLUSION FOR FIRST LAW

We find that:

$$(A \cup B)^c = \{i, j\} = A^c \cap B^c$$

Thus, **De Morgan's First Law** is verified:

$$(A \cup B)^c = A^c \cap B^c$$

Step 4: Verify De Morgan's Second Law

We need to check if:

$$(A \cap B)^c = A^c \cup B^c$$

STEP 4.1: FIND $(A \cap B)$

The intersection $A \cap B$ is the set of elements that are in both A and B. Since A and B have no common elements:

$$A \cap B = \emptyset$$

STEP 4.2: FIND $(A \cap B)^c$

The complement of the empty set is the entire set U:

$$(A \cap B)^c = U - \emptyset = \{a, b, c, d, e, f, g, h, i, j\}$$

STEP 4.3: FIND $A^c \cup B^c$

Now, find the union of A^c and B^c:

$$A^c \cup B^c = \{b, d, f, h, i, j\} \cup \{a, c, e, g, i, j\}$$
$$= \{a, b, c, d, e, f, g, h, i, j\}$$

STEP 4.4: CONCLUSION FOR SECOND LAW

We find that:

$$(A \cap B)^c = \{a, b, c, d, e, f, g, h, i, j\} = A^c \cup B^c$$

Thus, **De Morgan's Second Law** is verified:

$$(A \cap B)^c = A^c \cup B^c$$

6. **If U = {1, 2, 3, ..., 20} and A = {1, 3, 5, ..., 19}, verify the following:**

(i) $A \cup A' = U$ (ii) $A \cap U = A$

(iii) $A \cap A' = \phi$

Let's verify the following set properties for $U = \{1,2,3,\ldots,20\}$ and $A = \{1,3,5,7,\ldots,19\}$:

(i) $A \cup A' = U$

STEP 1: DEFINE THE COMPLEMENT OF A (i.e., A')

The complement of A, denoted A', consists of all elements in U that are **not** in A. Since A consists of all odd numbers from 1 to 19:

$$A = \{1,3,5,7,9,11,13,15,17,19\}$$

Thus, the complement of A (in U) is:

$$A' = \{2,4,6,8,10,12,14,16,18,20\}$$

STEP 2: FIND $A \cup A'$

The union of A and A' is the set of all elements that are in A or in A', or in both:

$$A \cup A' = \{1,3,5,7,9,11,13,15,17,19\} \cup \{2,4,6,8,10,12,14,16,18,20\}$$

Since A and A' together cover all the elements in U, we have:

$$A \cup A' = U$$

$$= \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$$

CONCLUSION FOR (i):

We verify that:

$$A \cup A' = U$$

Thus, the first property is verified.

(ii) $A \cap U = A$

STEP 1: FIND $A \cap U$

The intersection $A \cap U$ is the set of all elements that are both in A and in U. Since A is already a subset of U, we have:

$$A \cap U = A = \{1,3,5,7,9,11,13,15,17,19\}$$

CONCLUSION FOR (ii):

We verify that:

$$A \cap U = A$$

Thus, the second property is verified.

(iii) $A \cap A' = \phi$

STEP 1: FIND $A \cap A'$

The intersection $A \cap A'$ is the set of all elements that are both in A and in A'. Since A consists of odd numbers and A' consists of even numbers, there are no elements that are both in A and in A'. Therefore, the intersection is the empty set:

$$A \cap A' = \varnothing$$

CONCLUSION FOR (iii):

We verify that:

$$A \cap A' = \varnothing$$

Thus, the third property is verified.

7. **In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each students likes to play at least one of the two games. How many students like to play both games?**

Solution: We can solve this problem using the principle of inclusion-exclusion. Let's define:

- C as the set of students who like to play cricket.
- H as the set of students who like to play hockey.

From the problem, we know:

- Total number of students in the class = 55
- Number of students who like cricket $|C| = 34$
- Number of students who like hockey $|H| = 30$
- Every student likes at least one of the two games, so $|C \cup H| = 55$.

The formula for the union of two sets is:

$$|C \cup H| = |C| + |H| - |C \cap H|$$

Where $|C \cap H|$ represents the number of students who like both cricket and hockey.

Substitute the known values into the formula:

$$55 = 34 + 30 - |C \cap H|$$

Simplifying:

$$55 = 64 - |C \cap H|$$

Solving for $|C \cap H|$:

$$|C \cap H| = 64 - 55 = 9$$

So, 9 students like to play both cricket and hockey.

8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi and 10 can speak both Urdu and Punjabi. How many can speak all three languages?

Solution: To solve this problem, we can use the principle of inclusion-exclusion. Let's define the following:

- U as the set of employees who can speak Urdu,
- E as the set of employees who can speak English,
- P as the set of employees who can speak Punjabi.

We are given the following information:

- $|U| = 250$ (employees who can speak Urdu),
- $|E| = 150$ (employees who can speak English),
- $|P| = 50$ (employees who can speak Punjabi),
- $|U \cap E| = 40$ (employees who can speak both Urdu and English),
- $|E \cap P| = 30$ (employees who can speak both English and Punjabi),
- $|U \cap P| = 10$ (employees who can speak both Urdu and Punjabi).

We need to find $|U \cap E \cap P|$, the number of employees who can speak all three languages. We can use the inclusion-exclusion formula for three sets:

$$|U \cup E \cup P| = |U| + |E| + |P| - |U \cap E| - |E \cap P| - |U \cap P| + |U \cap E \cap P|$$

Since there are 500 employees in total, we know that:

$$|U \cup E \cup P| = 500$$

Now substitute the known values into the formula:

$$500 = 250 + 150 + 50 - 40 - 30 - 10 + |U \cap E \cap P|$$

Simplifying:

$$500 = 450 - 80 + |U \cap E \cap P|$$

$$500 = 370 + |U \cap E \cap P|$$

Solving for $|U \cap E \cap P|$:

$$|U \cap E \cap P| = 500 - 370 = 130$$

Thus, **130 employees** can speak all three languages.

9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 27. How many people are wearing caps?

Solution: Let's break down the problem using set notation and the principle of inclusion-exclusion.

Given:

- Let B represent the set of people wearing blue shirts.
- Let G represent the set of people wearing green shirts.
- Let C represent the set of people wearing caps.

From the problem, we are given:

- $|B| = 19$ (people wearing blue shirts)
- $|G| = 15$ (people wearing green shirts)
- $|B \cap G| = 3$ (people wearing both blue and green shirts)
- $|B \cap C| = 4$ (people wearing both a cap and a blue shirt)
- $|G \cap C| = 2$ (people wearing both a cap and a green shirt)
- $|B \cup G \cup C| = 27$ (total number of people with a blue or green shirt or a cap)

We need to determine $|C|$, the number of people wearing caps.

Step 1: Use the inclusion-exclusion principle.

The inclusion-exclusion formula for three sets is:

$$|B \cup G \cup C| = |B| + |G| + |C| - |B \cap G| - |B \cap C| - |G \cap C| + |B \cap G \cap C|$$

Substitute the known values into the formula:

$$25 = 19 + 15 + |C| - 3 - 4 - 2 + |B \cap G \cap C|$$

Simplifying:

$$25 = 25 + |C| + |B \cap G \cap C|$$

So, we have:

$$25 = 25 + |C| + |B \cap G \cap C|$$

$$|C| + |B \cap G \cap C| = 0$$

Thus, we can express this as:

$$|C| = |B \cap G \cap C| \, 0$$

Step 2: Find the value of $|B \cap G \cap C|$.

Since we know the total number of people wearing either a blue or green shirt or a cap is 25, and we already have the other sets' information, we find that the value of (|C| = 6.

10. In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Let's define the sets:

- L represents the set of participants with laptops.
- T represents the set of participants with tablets.
- B represents the set of participants with books.

Given information:

- $|L| = 17$ (participants with laptops)
- $|T| = 11$ (participants with tablets)
- $|L \cap T| = 9$ (participants with both laptops and tablets)
- $|L \cap B| = 6$ (participants with both laptops and books)
- $|T \cap B| = 4$ (participants with both tablets and books)
- $|L \cap T \cap B| = 8$ (participants with laptops, tablets, and books)
- $|L \cup T \cup B| = 35$ (total number of participants with laptops, tablets, or books)

We need to find $|B|$, the number of participants with books.

Step 1: Use the inclusion-exclusion principle.

The inclusion-exclusion formula for three sets is:

$$|L \cup T \cup B| = |L| + |T| + |B| - |L \cap T| - |L \cap B| - |T \cap B| + |L \cap T \cap B|$$

Substitute the known values:

$$35 = 17 + 11 + |B| - 9 - 6 - 4 + 8$$

Simplifying: $35 = 36 - 19 + |B|$

$$35 = 17 + |B|$$

$$|B| = 35 - 17$$

$$|B| = 18$$

Conclusion:

The number of participants who have books is $\boxed{18}$.

11. A shopping mall has 150 employees labelled 1 to 150, representing the Universal set *U*. The employees fall into the following categories:

- **Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.**
- **Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.**
- **Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.**

(a) Find $(A \cup B') \cap C$ (b) Find $n\{A \cap (B' \cap C')\}$

Part (a): Find $(A \cup B') \cap C$

Step 1: Understand the given sets:

- *A*: Employees with a salary range of 30k-45k, labelled from 50 to 89.
- *B*: Employees with a salary range of 50k-80k, labelled from 101 to 150.
- *C*: Employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.
- B': Complement of set *B*, i.e., employees who are not in *B*, labelled from 1 to 100 (since *B* is from 101 to 150).

Step 2: Find the union $A \cup B'$:

- *A* consists of employees 50 to 89.
- B′ consists of employees 1 to 100.

- Therefore, $A \cup B'$ will contain employees from 1 to 100, i.e., employees 1 to 100.

Step 3: Find the intersection $(A \cup B') \cap C$:

- C consists of employees 1 to 49 and 90 to 100.
- $A \cup B'$ consists of employees 1 to 100.
- Therefore, $(A \cup B') \cap C$ will consist of employees who are in both $A \cup B'$ and C, i.e., employees 1 to 49 and 90 to 100.
- Therefore, $(A \cup B') \cap C = \{1,2, \dots ,49,90,91, \dots ,100\}$.

Conclusion for part (a):

$$(A \cup B') \cap C = \{1,2, \dots ,49,90,91, \dots ,100\}$$

Part (b): Find $n(A \cap (B' \cap C))$

Step 1: Understand $A \cap (B' \cap C)$:

- $B' \cap C$ will be the employees who are in both B' (from 1 to 100) and C (employees 1 to 49 and 90 to 100).
- So, $B' \cap C$ consists of employees from 1 to 49 and 90 to 100.

Step 2: Find the intersection $A \cap (B' \cap C)$:

- A consists of employees 50 to 89.
- $B' \cap C$ consists of employees 1 to 49 and 90 to 100.
- The intersection of these two sets is the set of employees who are both in A (50 to 89) and in $B' \cap C$ (1 to 49 and 90 to 100).
- The only employees who belong to both sets are those in the range 90 to 89, which is **empty**.

Conclusion for part (b):

$$n(A \cap (B' \cap C)) = 0$$

Final answers:

(a) $(A \cup B') \cap C = \{1,2, \dots ,49,90,91, \dots ,100\}$

(b) $n(A \cap (B' \cap C)) = 0$

12. **In a secondary school with 125 students participate in at least one of the following sports: Cricket, football or hockey.**

- **60 students play cricket.**
- **70 students play football.**
- **40 students play hockey.**
- **25 students play both cricket and football.**
- **15 students play both football and hockey.**
- **10 students play both cricket and hockey.**

(a) How many students play all three sports?

(b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Solution: Let's solve the problem step-by-step.

Given Data:

- Total students: 125
- Students who play cricket (C): 60
- Students who play football (F): 70
- Students who play hockey (H): 40
- Students who play both cricket and football ($C \cap F$): 25
- Students who play both football and hockey ($F \cap H$): 15
- Students who play both cricket and hockey ($C \cap H$): 10

Part (a): How many students play all three sports?

We need to find the number of students who play all three sports, which is the intersection of all three sets: $C \cap F \cap H$.

We will use the principle of inclusion and exclusion to calculate the total number of students who play at least one sport.

FORMULA FOR INCLUSION-EXCLUSION:

$$n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$$

We are given the following values:

- $n(C \cup F \cup H) = 125$ (total number of students)

- $n(C) = 60$
- $n(F) = 70$
- $n(H) = 40$
- $n(C \cap F) = 25$
- $n(F \cap H) = 15$
- $n(C \cap H) = 10$

We need to find $n(C \cap F \cap H)$, the number of students who play all three sports.

SUBSTITUTING INTO THE FORMULA:

$$125 = 60 + 70 + 40 - 25 - 15 - 10 + n(C \cap F \cap H)$$

Simplifying:

$$125 = 170 - 50 + n(C \cap F \cap H)$$
$$125 = 120 + n(C \cap F \cap H)$$
$$n(C \cap F \cap H) = 125 - 120$$
$$n(C \cap F \cap H) = 5$$

So, **5 students** play all three sports.

Part (b): Draw a Venn diagram

Now, let's draw the Venn diagram based on the given information and the calculated value.

We have:

- 5 students in $C \cap F \cap H$ (all three sports).
- The number of students who play both cricket and football but not hockey: $n(C \cap F) - n(C \cap F \cap H) = 25 - 5 = 20$.
- The number of students who play both football and hockey but not cricket: $n(F \cap H) - n(C \cap F \cap H) = 15 - 5 = 10$.
- The number of students who play both cricket and hockey but not football: $n(C \cap H) - n(C \cap F \cap H) = 10 - 5 = 5$.

Now, the number of students who play only cricket (and not the other two sports):

$$n(C) - \left(n(C \cap F) + n(C \cap H) - n(C \cap F \cap H)\right)$$
$$= 60 - (25 + 10 - 5) = 60 - 30 = 30$$

Similarly, the number of students who play only football:

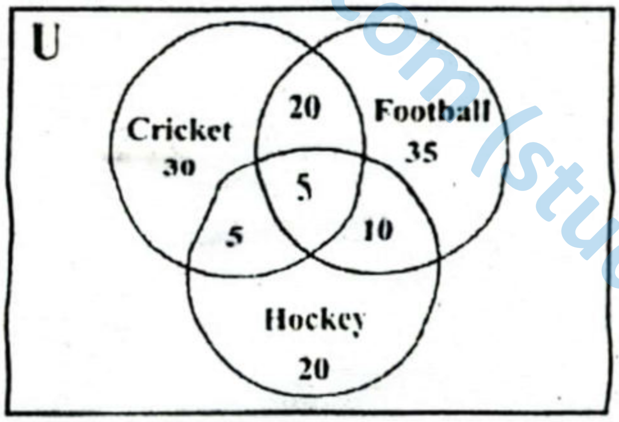
$$n(F) - \left(n(C \cap F) + n(F \cap H) - n(C \cap F \cap H)\right)$$
$$= 70 - (25 + 15 - 5) = 70 - 35 = 35$$

And the number of students who play only hockey:

$$n(H) - \left(n(C \cap F) + n(F \cap H) - n(C \cap F \cap H)\right)$$
$$= 40 - (10 + 15 - 5) = 40 - 20 = 20$$

VENN DIAGRAM STRUCTURE:

- 30 students play only cricket.
- 35 students play only football.
- 20 students play only hockey.
- 20 students play both cricket and football, but not hockey.
- 10 students play both football and hockey, but not cricket.
- 5 students play all three sports.



13. A *survey* was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- **40 people said they liked nihari.**
- **65 people said they liked biryani.**
- **50 people said they liked korma.**
- **20 people said they liked nihari and biryani.**
- **35 people said they liked biryani and korma.**
- **27 people said they liked nihari and korma.**

$$n(C) - \big(n(C \cap F) + n(C \cap H) - n(C \cap F \cap H)\big)$$
$$= 60 - (25 + 10 - 5) = 60 - 30 = 30$$

Similarly, the number of students who play only football:

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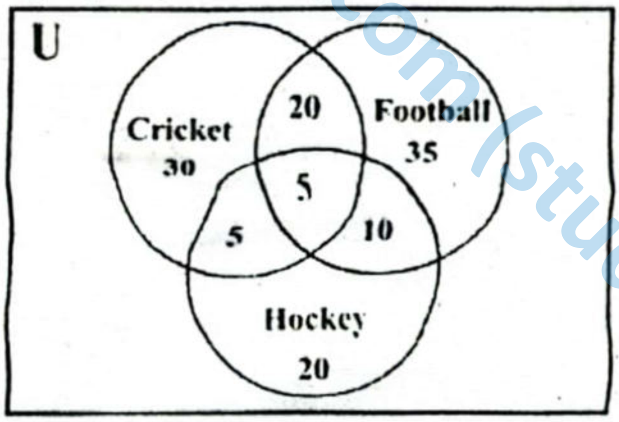

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- 12 people said they liked all three foods nihari, biryani and korma.

(a) **At least how many people like nihari, biryani or korma?**

(b) **How many people did not like nihari or korma?**

(c) **How many people like only one the following foods: nihari, biryani or korma?**

(d) **Draw a Venn diagram.**

Let's solve the questions step by step using the principle of inclusion-exclusion.

Given:

- Total number of people: 130
- People who like:
  - Nihari (N): 40
  - Biryani (B): 65
  - Korma (K): 50
  - Nihari and Biryani ($N \cap B$): 20
  - Biryani and Korma ($B \cap K$): 35
  - Nihari and Korma ($N \cap K$): 27
  - All three foods ($N \cap B \cap K$): 12

(A) AT LEAST HOW MANY PEOPLE LIKE NIHARI, BIRYANI, OR KORMA?

To find the number of people who like at least one of the three foods, we use the inclusion-exclusion principle:

$$|N \cup B \cup K| = |N| + |B| + |K| - |N \cap B| - |B \cap K| - |N \cap K| + |N \cap B \cap K|$$

Substitute the given values:

$$|N \cup B \cup K| = 40 + 65 + 50 - 20 - 35 - 27 + 12$$

Now, calculate:

$$|N \cup B \cup K| = 40 + 65 + 50 - 20 - 35 - 27 + 12 = 85$$

So, **at least 85 people** like Nihari, Biryani, or Korma.

(B) HOW MANY PEOPLE DID NOT LIKE NIHARI OR KORMA?

The number of people who did not like Nihari or Korma can be found by subtracting the number of people who like either Nihari or Korma (i.e., $|N \cup K|$) from the total number of people.

First, we calculate $|N \cup K|$ using the inclusion-exclusion principle:

$$|N \cup K| = |N| + |K| - |N \cap K|$$

Substitute the given values:

$$|N \cup K| = 40 + 50 - 27 = 63$$

Now, subtract from the total number of people:

$$\text{People who did not like Nihari or Korma} = 130 - 63 = 67$$

So, **67 people did not like Nihari or Korma.**

(C) HOW MANY PEOPLE LIKE ONLY ONE OF THE FOLLOWING FOODS: NIHARI, BIRYANI, OR KORMA?

To find the number of people who like only one of the foods, we need to subtract the people who like two or more foods from the total number of people who like each food.

- **People who like only Nihari**:

$$|N| - |N \cap B| - |N \cap K| + |N \cap B \cap K|$$
$$40 - 20 - 27 + 12 = 5$$

- **People who like only Biryani**:

$$|B| - |N \cap B| - |B \cap K| + |N \cap B \cap K|$$
$$65 - 20 - 35 + 12 = 22$$

- **People who like only Korma**:

$$|K| - |N \cap K| - |B \cap K| + |N \cap B \cap K|$$
$$50 - 27 - 35 + 12 = 0$$

Now, add the people who like only one of the foods:

$$5 \text{ (only Nihari)} + 22 \text{ (only Biryani)} + 0 \text{ (only Korma)} = 27$$

So, **27 people like only one of the foods: Nihari, Biryani, or Korma.**

Final Answers:

- (a) At least 85 people like Nihari, Biryani, or Korma.
- (b) 67 people did not like Nihari or Korma.
- (c) 27 people like only one of Nihari, Biryani, or Korma.

Draw a Venn diagram