Exercise 3.3

1. For $A = \{1, 2, 3, 4\}$, find the following relations in A. State the domain and range of each relation.

(i) $\{(x, y) \mid y = x\}$ (ii) $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$ (iv) $\{(x, y) \mid x + y > 5\}$

Let's go through each of the given relations one by one, finding the domain and range for each.

(i) Relation: $\{(x, y) \mid y = x\}$

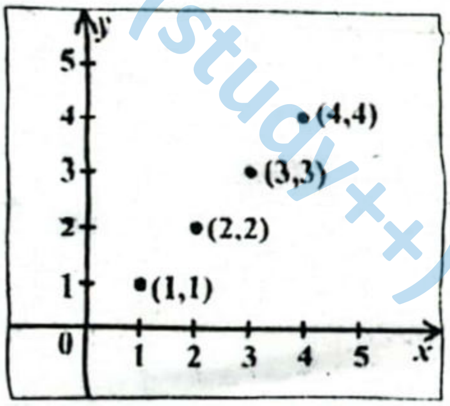
This relation represents all pairs where the second component (y) is equal to the first component (x). Since $A = \{1,2,3,4\}$, the relation will consist of pairs where $x = y$.

Relation:

$$\{(1,1), (2,2), (3,3), (4,4)\}$$

- **Domain:** The set of all possible first elements, which is A, i.e., $\{1,2,3,4\}$.
- **Range:** The set of all second elements, which is also A, i.e., $\{1,2,3,4\}$.

$\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
Domain of (i) = $\{1, 2, 3, 4\}$
Range of (i) = $\{1, 2, 3, 4\}$



(ii) Relation: $\{(x, y) \mid y + x = 5\}$

This relation represents all pairs where the sum of x and y is 5.

We can solve for y in terms of x:

$$y = 5 - x$$

Given $A = \{1,2,3,4\}$, we will check the pairs:

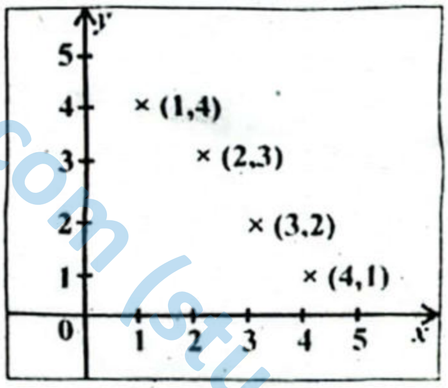
- For $x = 1$, $y = 5 - 1 = 4$, so the pair is $(1,4)$.
- For $x = 2$, $y = 5 - 2 = 3$, so the pair is $(2,3)$.
- For $x = 3$, $y = 5 - 3 = 2$, so the pair is $(3,2)$.
- For $x = 4$, $y = 5 - 4 = 1$, so the pair is $(4,1)$.

Relation:

$$\{(1,4), (2,3), (3,2), (4,1)\}$$

- **Domain**: The set of all possible first elements, which is $\{1,2,3,4\}$.
- **Range**: The set of all second elements, which is also $\{1,2,3,4\}$.

$\{(1, 4), (2, 3), (3, 2), (4, 1)\}$
Domain of (ii) = $\{1, 2, 3, 4\}$
Range of (ii) = $\{1, 2, 3, 4\}$



(iii) Relation: $\{(x, y) \mid x + y < 5\}$

This relation represents all pairs where the sum of x and y is less than 5.

We check each possible pair:

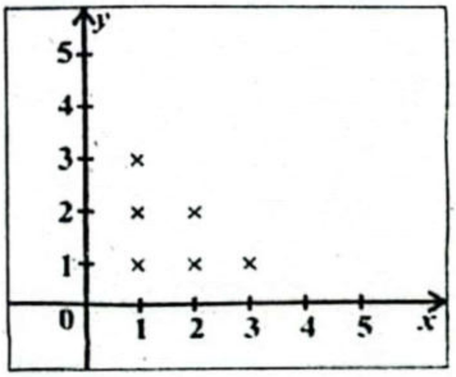
- For $x = 1$, y can be 1,2,3 (since $1 + 4 = 5$, which is not less than 5).
- For $x = 2$, y can be 1,2 (since $2 + 3 = 5$, which is not less than 5).
- For $x = 3$, y can only be 1 (since $3 + 2 = 5$, which is not less than 5).
- For $x = 4$, no values of y work because all pairs give sums greater than or equal to 5.

Relation:

$$\{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$$

- **Domain**: The set of all possible first elements, which is $\{1,2,3\}$.
- **Range**: The set of all second elements, which is $\{1,2,3\}$.

$\{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$
Domain of (iii) = $\{1,2,3\}$
Range of (iii) = $\{1,2,3\}$



(iv) **Relation:** $\{(x,y) \mid x+y>5\}$

This relation represents all pairs where the sum of x and y is greater than 5.

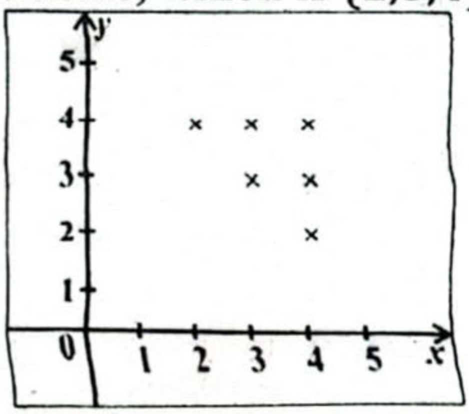
We check each possible pair:

- For $x=1$, no values of y work because all sums are less than or equal to 5.
- For $x=2$, no values of y work because all sums are less than or equal to 5.
- For $x=3$, y can only be 3,4 (since $3+3=6$ and $3+4=7$).
- For $x=4$, y can be 2,3,4 (since $4+2=6$, $4+3=7$, and $4+4=8$).

Relation: $\{(3,3),(3,4),(4,2),(4,3),(4,4)\}$

- **Domain**: The set of all possible first elements, which is $\{3,4\}$.
- **Range**: The set of all second elements, which is $\{2,3,4\}$.

$\{(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)\}$
Domain of (iv) = $\{2,3,4\}$
Range of (iv) = $\{2,3,4\}$



2. **Which of the following diagrams represents functions and of which type?**

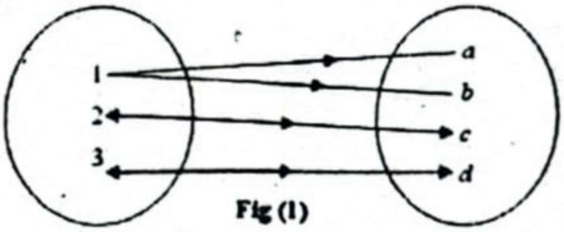


Fig (1)

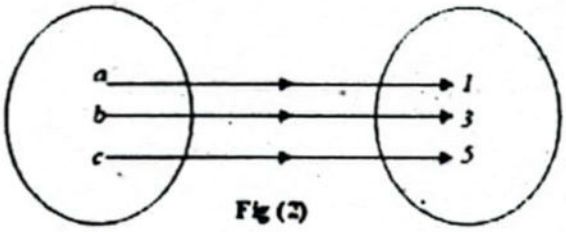


Fig (2)

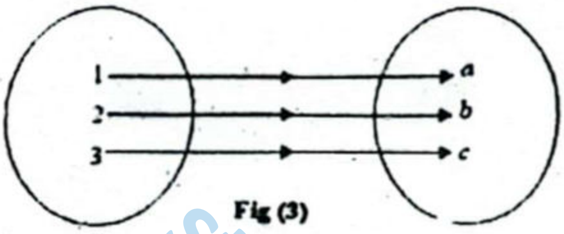


Fig (3)

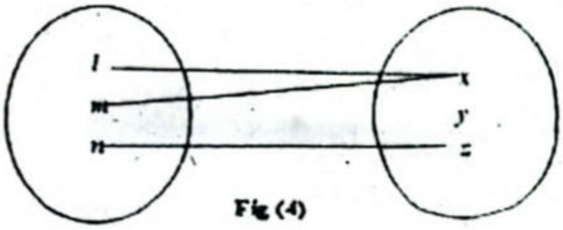


Fig (4)

3. **If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:**

(i) $g(0)$ (ii) $g(-3)$

(iii) $g\left(\frac{2}{3}\right)$ (iv) $h(1)$

(v) $h(-4)$ (vi) $h\left(-\frac{1}{2}\right)$

Let's compute the values for each of the given expressions:

Given:

- $g(x) = 3x + 2$
- $h(x) = x^2 + 1$

(i) $g(0)$

Substitute $x = 0$ into $g(x)$:

$$g(0) = 3(0) + 2 = 0 + 2 = 2$$

(ii) $g(-3)$

Substitute $x = -3$ into $g(x)$:

$$g(-3) = 3(-3) + 2 = -9 + 2 = -7$$

(iii) $g\left(\frac{2}{3}\right)$

Substitute $x = \frac{2}{3}$ into $g(x)$:

$$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$$

(iv) $h(1)$

Substitute $x = 1$ into $h(x)$:

$$h(1) = 1^2 + 1 = 1 + 1 = 2$$

(v) $h(-4)$

Substitute $x = -4$ into $h(x)$:

$$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$$

(vi) $h\left(-\frac{1}{2}\right)$

Substitute $x = -\frac{1}{2}$ into $h(x)$:

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1 = \frac{1}{4} + 1 = \frac{1}{4} + \frac{4}{4} = \frac{5}{4}$$

4. Given that $f(x) = ax + b + 1$, where c and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

We are given that $f(x) = ax + b + 1$, and we are also given the following values:

- $f(3) = 8$
- $f(6) = 14$

We can use these values to form two equations and solve for a and b.

Step 1: Use $f(3) = 8$

Substitute $x = 3$ into the function $f(x) = ax + b + 1$:

$$f(3) = a(3) + b + 1 = 8$$
$$3a + b + 1 = 8$$
$$3a + b = 7 \quad \text{(Equation 1)}$$

Step 2: Use $f(6) = 14$

Substitute $x = 6$ into the function $f(x) = ax + b + 1$:

$$f(6) = a(6) + b + 1 = 14$$
$$6a + b + 1 = 14$$
$$6a + b = 13 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $3a + b = 7$
- $6a + b = 13$

To eliminate b, subtract Equation 1 from Equation 2:

$$(6a + b) - (3a + b) = 13 - 7$$
$$6a - 3a = 6$$

$$3a = 6$$
$$a = 2$$

S... **Find b**

...itute $a = 2$ into Equation 1:

$$3(2) + b = 7$$
$$6 + b = 7$$
$$b = 1$$

5. **Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.**

We are given that $g(x) = ax + b + 5$, and we are also given the following values:

- $g(-1) = 0$
- $g(2) = 10$

We can use these values to form two equations and solve for a and b.

Step 1: Use $g(-1) = 0$

Substitute $x = -1$ into the function $g(x) = ax + b + 5$:

$$g(-1) = a(-1) + b + 5 = 0$$
$$-a + b + 5 = 0$$
$$-b + a = 5 \quad \text{(Equation 1)}$$

Step 2: Use $g(2) = 10$

Substitute $x = 2$ into the function $g(x) = ax + b + 5$:

$$g(2) = a(2) + b + 5 = 10$$
$$2a + b + 5 = 10$$
$$2a + b = 5 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $-a + b = 5$
- $2a + b = 5$

To eliminate b, subtract Equation 1 from Equation 2:

$$(2a + b) - (-a + b) = 5 - 5$$
$$2a + b + a - b = 0$$
$$3a = 0$$
$$a = 0$$

Step 4: Find b

Now substitute $a = 0$ into Equation 2:

$$2(0) + b = 5$$
$$b = 5$$

6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Solution: We are given the function $f(x) = 5x + 1$ and the equation $f(x) = 32$. We need to find the value of x that satisfies this equation.

Step 1: Set up the equation

$$f(x) = 32$$

Substitute $f(x) = 5x + 1$ into this equation:

$$5x + 1 = 32$$

Step 2: Solve for x

Subtract 1 from both sides:

$$5x = 32 - 1$$
$$5x = 31$$

Now, divide both sides by 5:

$$x = \frac{31}{5}$$

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

We are given the function $f(x) = cx^2 + d$, where c and d are constant numbers. We are also given:

- $f(1) = 6$
- $f(-2) = 10$

We can use these values to form two equations and solve for c and d.

Step 1: Use $f(1) = 6$

Substitute $x = 1$ into the function $f(x) = cx^2 + d$:

$$f(1) = c(1)^2 + d = 6$$
$$c + d = 6 \quad \text{(Equation 1)}$$

Step 2: Use $f(-2) = 10$

Substitute $x = -2$ into the function $f(x) = cx^2 + d$:

$$f(-2) = c(-2)^2 + d = 10$$

$$4c + d = 10 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $c + d = 6$
- $4c + d = 10$

To eliminate d, subtract Equation 1 from Equation 2:

$$(4c + d) - (c + d) = 10 - 6$$

$$4c - c = 4$$

$$3c = 4$$

$$c = \frac{4}{3}$$

Step 4: Find d

Now substitute $c = \frac{4}{3}$ into Equation 1:

$$\frac{4}{3} + d = 6$$

$$d = 6 - \frac{4}{3}$$

To subtract the fractions, express 6 as $\frac{18}{3}$:

$$d = \frac{18}{3} - \frac{4}{3} = \frac{14}{3}$$