1. Factorize by identifying common factors.

(i) $6x + 12$ (ii) $15y^2 + 20y$

(iii) $-12x^2 - 3x$ (iv) $4a^2b + 8ab^2$

(v) $xy - 3x^2 + 2x$ (vi) $3a^2b - 9ab^2 + 15ab$

Solution: **Factorization by identifying common factors:**

(i) $6x + 12$:

Factor out the common factor 6:

$$6x + 12 = 6(x + 2)$$

(ii) **$15y^2 + 20y$:**

Factor out the common factor $5y$:

$$15y^2 + 20y = 5y(3y + 4)$$

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(iii) $-12x^2 - 3x$:

Factor out the common factor $-3x$:

$$-12x^2 - 3x = -3x(4x + 1)$$

(iv) $4a^2b + 8ab^2$:

Factor out the common factor $4ab$:

$$4a^2b + 8ab^2 = 4ab(a + 2b)$$

(v) $xy^2 - 3xy + 2x$

Factor out the common factor x:

$$xy^2 - 3xy + 2x = x(y^2 - 3y + 2)$$

Further factorize if possible:

$$x(y^2 - 3y + 2) = x(y - 1)(y - 2)$$

(vi) $3a^2b - 9ab^2 + 15ab$:

Factor out the common factor $3ab$:

$$3a^2b - 9ab^2 + 15ab = 3ab(a - 3b + 5)$$

2. Factorize and represent pictorially:

(i) $5x + 15$ (ii) $x^2 + 4x + 3$

(iii) $x^2 + 6x + 8$ (iv) $x^2 + 4x + 4$

Solution: **Factorization:**

(i) $5x + 15$:

Factor out the common factor 5:

$$5x + 15 = 5(x + 3)$$

(ii) $x^2 + 4x + 3$:

Factorize by finding two numbers that multiply to 3 and add up to 4, which are 1 and 3:

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

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(iii) $\quad x^2 + 6x + 8$:

Factorize by finding two numbers that multiply to 8 and add up to 6, which are 2 and 4:

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

(iv) $\quad x^2 + 4x + 4$:

Factorize by recognizing this as a perfect square trinomial:

$$x^2 + 4x + 4 = (x + 2)^2$$

3. Factorize:

(i) $x^2 + x - 12$

(ii) $x^2 + 7x + 10$

(iii) $x^2 - 6x + 8$

(iv) $x^2 - x - 56$

(v) $x^2 - 10x - 24$

(vi) $y^2 + 4y - 12$

(vii) $y^2 + 13y + 36$

(viii) $x^2 - x - 2$

Solution: **Factorization:**

(i) $\quad x^2 + x - 12$:

Factorize by finding two numbers that multiply to -12 and add up to 1, which are 4 and -3:

$$x^2 + x - 12 = (x + 4)(x - 3)$$

(ii) $\quad x^2 + 7x + 10$:

Factorize by finding two numbers that multiply to 10 and add to 7, which are 2 and 5:

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

(iii) $\quad x^2 - 6x + 8$:

Factorize by finding two numbers that multiply to 8 and add up to -6, which are -4 and -2:

$$x^2 - 6x + 8 = (x - 4)(x - 2)$$

(iv) $\quad x^2 - x - 56$:

Factorize by finding two numbers that multiply to -56 and add up to -1, which are -8 and 7:

$$x^2 - x - 56 = (x - 8)(x + 7)$$

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(v) $x^2 - 10x - 24$:

Factorize by finding two numbers that multiply to -24 and add up to -10, which are -12 and 2:

$$x^2 - 10x - 24 = (x - 12)(x + 2)$$

(vi) $y^2 + 4y - 12$:

Factorize by finding two numbers that multiply to -12 and add up to 4, which are 6 and -2:

$$y^2 + 4y - 12 = (y + 6)(y - 2)$$

(vii) $y^2 + 13y + 36$:

Factorize by finding two numbers that multiply to 36 and add up to 13, which are 9 and 4:

$$y^2 + 13y + 36 = (y + 9)(y + 4)$$

(viii) $x^2 - x - 2$:

Factorize by finding two numbers that multiply to -2 and add up to -1, which are -2 and 1:

$$x^2 - x - 2 = (x - 2)(x + 1)$$

4. Factorize:

(i) $2x^2 + 7x + 3$ (ii) $2x^2 + 11x + 15$

(iii) $4x^2 + 13x + 3$ (iv) $3x^2 + 5x + 2$

(v) $3y^2 - 11y + 6$ (vi) $2y^2 - 5y + 2$

(vii) $4z^2 - 11z + 6$ (viii) $6 + 7x - 3x^2$

***Solution:* Factorization:**

(i) $2x^2 + 7x + 3$:

Factorize by finding two numbers that multiply to $2 \times 3 = 6$ and add up to 7, which are 6 and 1:

$$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3)$$
$$= (2x + 1)(x + 3)$$

(ii) $2x^2 + 11x + 15$:

Factorize by finding two numbers that multiply to $2 \times 15 = 30$ and add up to 11, which are 6 and 5:

$$2x^2 + 11x + 15 = 2x^2 + 6x + 5x + 15$$
$$= 2x(x + 3) + 5(x + 3) = (2x + 5)(x + 3)$$

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(iii) $\mathbf{4x^2 + 13x + 3}$**:**

Factorize by finding two numbers that multiply to $4 \times 3 = 12$ and add up to 13, which are 12 and 1:

$$4x^2 + 13x + 3 = 4x^2 + 12x + x + 3 = 4x(x+3) + 1(x+3)$$
$$= (4x+1)(x+3)$$

(iv) $\mathbf{3x^2 + 5x + 2}$**:**

Factorize by finding two numbers that multiply to $3 \times 2 = 6$ and add up to 5, which are 3 and 2:

$$3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2 = 3x(x+1) + 2(x+1)$$
$$= (3x+2)(x+1)$$

(v) $\mathbf{3y^2 - 11y + 6}$**:**

Factorize by finding two numbers that multiply to $3 \times 6 = 18$ and add up to -11, which are -9 and -2:

$$3y^2 - 11y + 6 = 3y^2 - 9y - 2y + 6 = 3y(y-3) - 2(y-3)$$
$$= (3y-2)(y-3)$$

(vi) $\mathbf{2y^2 - 5y + 2}$**:**

Factorize by finding two numbers that multiply to $2 \times 2 = 4$ and add up to -5, which are -4 and -1:

$$2y^2 - 5y + 2 = 2y^2 - 4y - y + 2 = 2y(y-2) - 1(y-2)$$
$$= (2y-1)(y-2)$$

(vii) $\mathbf{4z^2 - 11z + 6}$**:**

Factorize by finding two numbers that multiply to $4 \times 6 = 24$ and add up to -11, which are -8 and -3:

$$4z^2 - 11z + 6 = 4z^2 - 8z - 3z + 6 = 4z(z-2) - 3(z-2)$$
$$= (4z-3)(z-2)$$

(viii) $\mathbf{6 + 7x - 3x^2}$**:**

Rearrange the terms:

$$6 + 7x - 3x^2 = -3x^2 + 7x + 6$$

Factorize by finding two numbers that multiply to $-3 \times 6 = -18$ and add up to 7, which are 9 and -2:

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$$-3x^2 + 7x + 6 = -3x^2 + 9x - 2x + 6$$
$$= -3x(x-3) - 2(x-3) = -(3x-2)(x-3)$$

Type – II: Factorization of the expression of the types

$a^4 + a^2b^2 + b^4$ or $a^4 + b^4$

Let's factorize $a^4 + a^2b^2 + b^4$

$$a^4 + a^2b^2 + b^4$$
$$= a^4 + b^4 + a^2b^2$$
$$= (a^2)^2 + (b^2)^2 + a^2b^2$$
$$= (a^2)^2 + (b^2)^2 + a^2b^2$$
$$= (a^2)^2 + (b^2)^2 + 2a^2b^2 - ab^2$$

(Adding and subtracting $2a^2b^2$)

$$= (a^2 + b^2)^2 - a^2b^2$$
$$= (a^2 + b^2)^2 - (ab)^2$$
$$= (a^2 + b^2 - ab)(a^2 + b^2 + ab)$$
$$= (a^2 - ab + b^2)(a^2 + ab + b^2)$$

Remember!

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Activity

- Prepare cards by writing several expressions.
- Divide students in small groups.
- Each group will draw a card and factorize the expression.
- The group which completes the most correct factorizations in a set time will win.

Example 9: Factorize: $x^4 + x^2 + 25$

Solution: $x^4 + x^2 + 25$

$$= x^4 + 25 + x^2$$
$$= (x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) + x^2$$

(Adding and subtracting $2(x^2)(5)$)

$$= (x^2 + 5)^2 - 10x^2 + x^2$$
$$= (x^2 + 5)^2 - 9x^2$$
$$= (x^2 + 5)^2 - (3x)^2$$
$$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$$
$$= (x^2 - 3x + 5)(x^2 + 3x + 5)$$

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Example 10: Factorize: $x^4 + y^4$

Solution: $x^4 + y^4$

$= (x^2)^2 + (y^2)^2$

$= (x^2)^2 + (y^2)^2 + 2(x^2)(y^2) - 2(x^2)(y^2)$

(Adding and subtracting $2x^2y^2$)

$= (x^2 + y^2)^2 - (\sqrt{2}xy)^2$

$= (x^2 + y^2 - \sqrt{2}\,xy)(x^2 + y^2 + \sqrt{2}\,xy)$

$= (x^2 - \sqrt{2}\,xy + y^2)(x^2 + \sqrt{2}\,xy + y^2)$

> **Try Yourself**
> Factorize:
> (i) $16x^4 + y^4$
> (ii) $81x^4 + \frac{1}{81x^4} - 5$

Example 11: Factorize: $a^4 + 64$

Solution: $a^4 + 64$

$= (a^2)^2 + (8)^2$

$= (a^2)^2 + (8)^2 + 2(a^2)(8) - 2(a^2)(8)$

(Adding and subtracting $2(a^2)(8)$)

$= (a^2 + 8)^2 - 16a^2$

$= (a^2 + 8)^2 - (4a)^2$

$= (a^2 + 8 - 4a)(a^2 + 8 + 4a)$

$= (a^2 - 4a + 8)(a^2 + 4a + 8)$

Type – III: Factorization of the expression of the types

- $(ax^2 + bx + c)(ax^2 + bx + d) + k$
- $(x + a)(x + b)(x + c)(x + d) + k$
- $(x + a)(x + b)(x + c)(x + d) + kx^2$

For explanation consider the following examples:

Example 12: **Factorize: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$**

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

$= (y + 4)(y + 6) - 3$ (Let $y = x^2 + 5x$)

$= y^2 + 6y + 4y + 24 - 3$

$= y^2 + 10y + 21$

$= y^2 + 7y + 3y + 21$

$= y(y + 7) + 3(y + 7)$

$= (y + 7)(y + 3)$

$= (x^2 + 5x + 7)(x^2 + 5x + 3)$ $(\because y = x^2 + 5x)$

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Example 13: Factorize: $(x+2)(x+3)(x+4)(x+5)-15$

Solution: $(x+2)(x+3)(x+4)(x+5)-15$

Re-arrange the given expression because $2+5=3+4$

$$[(x+2)(x+5)][(x+3)(x+4)]-15$$
$$=(x^2+5x+2x+10)(x^2+4x+3x+12)-15$$
$$=(x^2+7x+10)(x^2+7x+12)-15$$
$$=(y+10)(y+12)-15 \qquad \text{Let } y=x^2+7x$$
$$=y^2+12y+10y+120-15$$
$$=y^2+22y+105$$
$$=y^2+15y+7y+105$$
$$=y(y+15)+7(y+15)$$
$$=(y+15)(y+7)$$
$$=(x^2+7x+15)(x^2+7x+7) \qquad (\because y=x^2+7x)$$

Example 14: Factorize: $(x-2)(x+2)(x+1)(x-4)+2x^2$

Solution: $(x-2)(x+2)(x+1)(x-4)+2x^2$

$$=[(x-2)(x+2)][(x+1)(x-4)]+2x^2$$
$$[\because (-2)\times 2 = 1\times(-4)]$$
$$=(x^2-2^2)(x^2-4x+x-4)+2x^2$$
$$=(x^2-4)(x^2-3x-4)+2x^2$$
$$=y(y-3x)+2x^2 \qquad \text{Let } y=x^2-4$$
$$=y^2-3xy+2x^2$$
$$=y^2-2xy-xy+2x^2$$
$$=y(y-2x)-x(y-2x)$$
$$=(y-2x)(y-x)$$
$$=(x^2-4-2x)(x^2-4-x) \qquad (\because y=x^2-4)$$
$$=(x^2-2x-4)(x^2-x-4)$$

Type – IV: Factorization of the expression of the forms

- $a^3+3a^2b+3ab^2+b^3$
- $a^3-3a^2b+3ab^2-b^3$

Factorization of such types of expressions is explained in the following examples:

Remember!
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

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Example 15: **Factorize:** $8x^3 + 60x^2 + 150x + 125$

Solution: $8x^3 + 60x^2 + 150x + 125$

$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$

$= (2x + 5)^3$

$= (2x + 5)(2x + 5)(2x + 5)$

Example 16: **Factorize:** $x^3 - 18x^2 + 108x - 216$

Solution: $x^3 - 18x^2 + 108x - 216$

$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3$

$= (x-6)^3$

$= (x-6)(x-6)(x-6)$

Type – V: Factorization of the expression of the form $a^3 \pm b^3$

The expression $a^3 + b^3$ is a sum of cubes and it can be factorized using the following identity:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

The expression $a^3 - b^3$ is a difference of cubes and it can be factorized using the following identity:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 17: **Factorize:** $8x^3 + 27$

Solution: $8x^3 + 27$

$= (2x)^3 + (3)^3$

$= (2x + 3)[(2x)^2 - (2x)(3) + (3)^2]$

$= (2x + 3)(4x^2 - 6x + 9)$

Example 18: **Factorize:** $x^3 - 27y^3$

Solution: $x^3 - 27y^3$

$= (x)^3 - (3y)^3$

$= (x - 3y)[(x)^2 + (x)(3y) + (3y)^2]$

$= (x - 3y)(x^2 + 3xy + 9y^2)$

Do you know?

$(a + b)^2 \neq a^2 + b^2$

$(a - b)^2 \neq a^2 - b^2$

$(a + b)^3 \neq a^3 + b^3$

$(a - b)^3 \neq a^3 - b^3$

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