Exercise 4.3

1. **Find HCF by factorization method.**

(i) $21x^2y, 35xy^2$ (ii) $4x^2 - 9y^2, 2x^2 - 3xy$

(iii) $x^3 - 1, x^2 + x + 1$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

(v) $t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

(vi) $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Solution:

(i) $21x^2y, 35xy^2$

- **Factorizing** $21x^2y$:

$$21x^2y = 3 \times 7 \times x \times x \times y$$

- **Factorizing** $35xy^2$:

$$35xy^2 = 5 \times 7 \times x \times y \times y$$

- **Finding the common factors**: The common factor is $7xy$.

  HCF = $7xy$

(ii) $\mathbf{4x^2 - 9y^2, 2x^2 - 3xy}$

1. **Factorizing** $4x^2 - 9y^2$:

$$4x^2 - 9y^2 = (2x - 3y)(2x + 3y)$$

2. **Factorizing** $2x^2 - 3xy$:

$$2x^2 - 3xy = x(2x - 3y)$$

3. **Finding the common factors**: The common factor is $(2x - 3y)$.

   HCF = $(2x - 3y)$

(iii) $\mathbf{x^3 - 1, x^2 + x + 1}$

1. **Factorizing** $x^3 - 1$:

$$x^3 - 1 = x^3 - 1^3 = (x - 1)(x^2 + x + 1)$$

2. **Factorizing** $x^2 + x + 1$: This expression cannot be simplified further.

3. **Finding the common factors**: The common factor is $(x^2 + x + 1)$.

   HCF = $(x^2 + x + 1)$

(iv) **Given Expressions:**

$$a^3 + 2a^2 - 3a, \quad 2a^3 + 5a^2 - 3a$$

Step 1: Factorize each expression

FACTORIZING $a^3 + 2a^2 - 3a$:

$$a^3 + 2a^2 - 3a = a \cdot (a^2 + 2a - 3)$$

Now factorize $a^2 + 2a - 3$:

$$a^2 + 2a - 3 = (a + 3)(a - 1)$$

Thus:

$$a^3 + 2a^2 - 3a = a \cdot (a + 3) \cdot (a - 1)$$

FACTORIZING $2a^3 + 5a^2 - 3a$:

$$2a^3 + 5a^2 - 3a = a \cdot (2a^2 + 5a - 3)$$

Now factorize $2a^2 + 5a - 3$:

$$2a^2 + 5a - 3 = (a + 3)(2a - 1)$$

Thus:

$$2a^3 + 5a^2 - 3a = a \cdot (a + 3) \cdot (2a - 1)$$

Step 2: Identify common factors

The factorizations are:

$$a^3 + 2a^2 - 3a = a \cdot (a + 3) \cdot (a - 1)$$

$$2a^3 + 5a^2 - 3a = a \cdot (a + 3) \cdot (2a - 1)$$

The common factors are:

$$a \quad \text{and} \quad (a + 3)$$

Step 3: Write the HCF

$$\text{HCF} = a(a + 3)$$

(v) $\quad t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

1. **Factors of** $t^2 - 3t - 4$:

   are $(t - y)\,(t + 1)$

2. **Factors of** $t^2 + 5t + 4$:

   are $(t + 1)\,(t + 4)$

3. **Factors of** $t^2 - 1$:

   are $(t + 1)\,(t - 1)$

   Common factors is

   $t + 1$

   $\therefore \qquad \text{H.C.F} = t + 1$

(vi) $\quad x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

1. **Factorizing** $x^2 + 15x + 56$:

$$x^2 + 15x + 56 = (x + 7)(x + 8)$$

2. **Factorizing** $x^2 + 5x - 24$:

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

3. **Factorizing** $x^2 + 8x$:

$$x^2 + 8x = x(x + 8)$$

4. **Finding the common factors**: The common factor is $(x + 8)$.

HCF $= (x + 8)$

2. **Find HCF of the following expressions by using division method:**

(i) $27x^3 + 9x^2 - 3x - 9,\ 3x - 2$

(ii) $x^3 - 9x^2 + 21x - 15,\ x^2 - 4x + 3$

(iii) $2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12$

(iv) $2x^3 - 4x^2 + 6x,\ x^3 - 2x,\ 3x^2 - 6x$

Solution: To find the Highest Common Factor (HCF) of the given expressions using the **division method**, we will follow the same approach that was used in the previous example, performing polynomial division step by step.

(i) $27x^3 + 9x^2 - 3x - 9, 3x - 2$

Step 1: Perform polynomial division of $27x^3 + 9x^2 - 3x - 9$ **by** $3x - 2$.

1. Divide the first term of the dividend by the first term of the divisor:

$$\frac{27x^3}{3x} = 9x^2$$

2. Multiply the divisor $3x - 2$ by $9x^2$:

$$9x^2(3x - 2) = 27x^3 - 18x^2$$

3. Subtract the result from the dividend:

$$(27x^3 + 9x^2 - 3x - 9) - (27x^3 - 18x^2)$$
$$= 27x^3 + 9x^2 - 3x - 9 - 27x^3 + 18x^2$$
$$= 27x^2 - 3x - 9$$

4. Divide the first term of the new polynomial by the first term of the divisor:

$$\frac{27x^2}{3x} = 9x$$

5. Multiply the divisor by $9x$:

$$9x(3x - 2) = 27x^2 - 18x$$

6. Subtract the result from the new polynomial:

$$(27x^2 - 3x - 9) - (27x^2 - 18x)$$
$$= 27x^2 - 3x - 9 - 27x^2 + 18x = 15x - 9$$

7. Divide the first term of the new polynomial by the first term of the divisor:

$$\frac{15x}{3x} = 5$$

8. Multiply the divisor by 5:

$$5(3x - 2) = 15x - 10$$

9. Subtract the result from the new polynomial:

$$(15x - 9) - (15x - 10) = 15x - 9 - 15x + 10 = 1$$

So, the remainder is 1. Since the remainder is 1, this means there are no further common factors between the two polynomials.

HCF = 1

(ii) $x^3 - 9x^2 + 21x - 15, x^2 - 4x + 3$

Step 1: Perform polynomial division of $x^3 - 9x^2 + 21x - 15$ **by** $x^2 - 4x + 3$.

1. Divide the first term of the dividend by the first term of the divisor:

$$\frac{x^3}{x^2} = x$$

2. Multiply the divisor $x^2 - 4x + 3$ by x:

$$x(x^2 - 4x + 3) = x^3 - 4x^2 + 3x$$

3. Subtract the result from the dividend:

$$(x^3 - 9x^2 + 21x - 15) - (x^3 - 4x^2 + 3x)$$
$$= x^3 - 9x^2 + 21x - 15 - x^3 + 4x^2 - 3x$$
$$= -5x^2 + 18x - 15$$

4. Divide the first term of the new polynomial by the first term of the divisor:

$$\frac{-5x^2}{x^2} = -5$$

5. Multiply the divisor by -5:

$$-5(x^2 - 4x + 3) = -5x^2 + 20x - 15$$

6. Subtract the result from the new polynomial:

$$(-5x^2 + 18x - 15) - (-5x^2 + 20x - 15)$$
$$= -5x^2 + 18x - 15 + 5x^2 - 20x + 15 = -2x$$

Now, the remainder is $-2x$, and since it is a non-zero term and cannot be further divided by the divisor $x^2 - 4x + 3$, the **HCF** is 1.

HCF = 1

(iii) $\mathbf{2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12}$

Step 1: Perform polynomial division of $2x^3 + 2x^2 + 2x + 2$ **by** $6x^3 + 12x^2 + 6x + 12$.

1. Divide the first term of the dividend by the first term of the divisor:

$$\frac{2x^3}{6x^3} = \frac{1}{3}$$

2. Multiply the divisor $6x^3 + 12x^2 + 6x + 12$ by $\frac{1}{3}$:

$$\frac{1}{3}(6x^3 + 12x^2 + 6x + 12) = 2x^3 + 4x^2 + 2x + 4$$

3. Subtract the resu'lt from the dividend:

$$(2x^3 + 2x^2 + 2x + 2) - (2x^3 + 4x^2 + 2x + 4)$$
$$= 2x^3 + 2x^2 + 2x + 2 - 2x^3 - 4x^2 - 2x - 4$$
$$= -2x^2 - 2$$

4. Divide the first term of the new polynomial by the first term of the divisor:

$$\frac{-2x^2}{6x^3} = \text{not possible to divide further.}$$

The remainder is $-2x^2 - 2$, so the **HCF** here is 1.

HCF = 1

(iv) $\mathbf{2x^3 - 4x^2 + 6x, x^3 - 2x, 3x^2 - 6x}$

Step 1: Perform polynomial division of $2x^3 - 4x^2 + 6x$ **by** $x^3 - 2x$.

1. Divide the first term of the dividend by the first term of the divisor:

$$\frac{2x^3}{x^3} = 2$$

2. Multiply the divisor $x^3 - 2x$ by 2:

$$2(x^3 - 2x) = 2x^3 - 4x$$

3. Subtract the result from the dividend:

$$(2x^3 - 4x^2 + 6x) - (2x^3 - 4x) = 2x^3 - 4x^2 + 6x - 2x^3 + 4x$$
$$= -4x^2 + 10x$$

4. Divide the first term of the new polynomial by the first term of the divisor:

$$\frac{-4x^2}{x^3} = \text{not possible to divide further.}$$

Thus, the **HCF** for this case is x.

HCF = 1

3. Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b, 4ab^2, 6ab$ **(ii)** $x^2 + x,\ x^3 + x^2$

(iii) $a^2 - 4a + 4, a^2 - 2a$ **(iv)** $x^4 - 16,\ x^3 - 4x$

(v) $16 - 4x^2,\ x^2 + x - 6,\ 4 - x^2$

Solution: To find the LCM using the **prime factorization** method, we need to factorize each expression (or term) into its prime factors (including variables) and then select the highest powers of all primes and variables involved. Here's how we'll proceed:

(i) $2a^2b, 4ab^2, 6ab$

Step 1: Prime factorization of each term

- $2a^2b = 2 \cdot a^2 \cdot b$
- $4ab^2 = 2^2 \cdot a \cdot b^2$
- $6ab = 2 \cdot 3 \cdot a \cdot b$

Step 2: LCM is obtained by taking the highest powers of all the primes and variables:

- Highest power of $2 = 2^2$ (from $4ab^2$)
- Highest power of $3 = 3$ (from $6ab$)
- Highest power of $a = a^2$ (from $2a^2b$)
- Highest power of $b = b^2$ (from $4ab^2$)

Thus, the **LCM** is:

$$LCM = 2^2 \cdot 3 \cdot a^2 \cdot b^2 = 12a^2b^2$$

(ii) $\boldsymbol{x^2 + x, x^3 + x^2}$

Step 1: Factor each expression:

- $x^2 + x = x(x+1)$
- $x^3 + x^2 = x^2(x+1)$

Step 2: LCM is obtained by taking the highest powers of all factors:

- Highest power of $x = x^2$ (from $x^2(x+1)$)
- Highest power of $(x+1) = (x+1)$ (common factor in both terms)

Thus, the **LCM** is:

$$LCM = x^2 \cdot (x+1) = x^2(x+1)$$

(iii) $\boldsymbol{a^2 - 4a + 4, a^2 - 2a}$

Step 1: Factor each expression:

- $a^2 - 4a + 4 = (a-2)^2$ (Perfect square trinomial)
- $a^2 - 2a = a(a-2)$

Step 2: LCM is obtained by taking the highest powers of all factors:

- Highest power of $a = a$ (from $a(a-2)$)
- Highest power of $(a-2) = (a-2)^2$ (from $(a-2)^2$)

Thus, the **LCM** is:

$$LCM = a \cdot (a-2)^2$$

Given Expressions:

- $x^4 - 16$
- $x^3 - 4x$

Step 1: Factor each expression

Factorizing $x^4 - 16$:

$$x^4 - 16 = (x^2)^2 - 4^2$$

This is a difference of squares:

$$x^4 - 16 = (x^2 + 4)(x^2 - 4)$$

Now factorize $x^2 - 4$:

$$x^2 - 4 = (x + 2)(x - 2)$$

Thus:

$$x^4 - 16 = (x^2 + 4)(x + 2)(x - 2)$$

Factorizing $x^3 - 4x$:

$$x^3 - 4x = x(x^2 - 4)$$

Now factorize $x^2 - 4$:

$$x^2 - 4 = (x + 2)(x - 2)$$

Thus:

$$x^3 - 4x = x(x + 2)(x - 2)$$

Step 2: LCM is obtained by taking the highest powers of all factors

- Highest power of $x = x$ (from $x(x + 2)(x - 2)$)
- Highest power of $(x^2 + 4) = (x^2 + 4)$ (from $(x^2 + 4)(x + 2)(x - 2)$)
- Highest power of $(x + 2) = (x + 2)$ (common factor in both terms)
- Highest power of $(x - 2) = (x - 2)$ (common factor in both terms)

Thus:

$$\text{LCM} = x(x^2 + 4)(x + 2)(x - 2)$$

(v) $\mathbf{16 - 4x^2, x^2 + x - 6, 4 - x^2}$

Step 1: Factor each expression:

- $16 - 4x^2 = 4(4 - x^2) = 4(2 - x)(2 + x)$
- $x^2 + x - 6 = (x - 2)(x + 3)$
- $4 - x^2 = (2 - x)(2 + x)$

Step 2: LCM is obtained by taking the highest powers of all factors:

- Highest power of $4 = 4$ (from $4(2 - x)(2 + x)$)
- Highest power of $(2 - x) = (2 - x)$ (common factor in two terms)
- Highest power of $(2 + x) = (2 + x)$ (common factor in two terms)

- Highest power of $(x-2) = (x-2)$ (from $(x-2)(x+3)$)
- Highest power of $(x+3) = (x+3)$ (from $(x-2)(x+3)$)

Thus, the **LCM** is:

$$LCM = 4 \cdot (2-x) \cdot (2+x) \cdot (x-2) \cdot (x+3)$$

4. The HCF of two polynomials is $y - 7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.

Solution: We are given the following:

- The **HCF** of two polynomials is $y - 7$.
- The **LCM** of the two polynomials is $y^3 - 10y^2 + 11y + 70$.
- One of the polynomials is $y^2 - 5y - 14$.

We are asked to find the other polynomial.

Step 1: Recall the relationship between HCF, LCM, and the product of two polynomials.

The relationship between the **HCF** (HCF(A, B)), **LCM** (LCM(A, B)), and the product of the two polynomials A and B is given by:

$$\text{HCF}(A, B) \times \text{LCM}(A, B) = A \times B$$

Let the unknown polynomial be P. Therefore, we have:

$$(y-7) \times (y^3 - 10y^2 + 11y + 70) = (y^2 - 5y - 14) \times P$$

Step 2: Simplify the equation.

We need to solve for P, the unknown polynomial.

First, let's factor $y^2 - 5y - 14$ to make it easier to work with. We factor the quadratic:

$$y^2 - 5y - 14 = (y-7)(y+2)$$

Thus, the equation becomes:

$$(y-7) \times (y^3 - 10y^2 + 11y + 70) = (y-7)(y+2) \times P$$

Step 3: Cancel the common factor $y - 7$.

Since $y - 7$ is common on both sides of the equation, we can cancel it out (assuming $y \neq 7$):

$$y^3 - 10y^2 + 11y + 70 = (y+2) \times P$$

Step 4: Divide by $y+2$ to solve for P.

Now, we divide both sides of the equation by $y+2$ to isolate P:

$$P = \frac{y^3 - 10y^2 + 11y + 70}{y+2}$$

Step 5: Perform polynomial division.

Let's divide $y^3 - 10y^2 + 11y + 70$ by $y+2$ using polynomial division.

- **Divide** y^3 by y, which gives y^2.
- Multiply y^2 by $y+2$, which gives $y^3 + 2y^2$.
- Subtract $(y^3 + 2y^2)$ from $y^3 - 10y^2 + 11y + 70$ to get $-12y^2 + 11y + 70$.
- **Divide** $-12y^2$ by y, which gives $-12y$.
- Multiply $-12y$ by $y+2$, which gives $-12y^2 - 24y$.
- Subtract $(-12y^2 - 24y)$ from $-12y^2 + 11y + 70$ to get $35y + 70$.
- **Divide** $35y$ by y, which gives 35.
- Multiply 35 by $y+2$, which gives $35y + 70$.
- Subtract $(35y + 70)$ from $35y + 70$ to get 0.

Thus, the quotient is $y^2 - 12y + 35$, and the remainder is 0.

Final Answer:

The other polynomial P is:

$$P = y^2 - 12y + 35$$

5. The LCM and HCF of two polynomial $p(x)$ and $q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$ respectively. If $p(x) = 4x^2(x^2-a^2)$, find $q(x)$.

Solution: We are given the following information:

- The **LCM** of two polynomials $p(x)$ and $q(x)$ is $36x^3(x+a)(x^3-a^3)$.
- The **HCF** of the two polynomials $p(x)$ and $q(x)$ is $x^2(x-a)$.
- The polynomial $p(x)$ is $4x^2(x^2-a^2)$.

We are asked to find the polynomial $q(x)$.

Step 1: Recall the relationship between LCM, HCF, and the product of two polynomials

The relationship between the **HCF** ($\text{HCF}(p(x), q(x))$), **LCM** ($\text{LCM}(p(x), q(x))$), and the product of the two polynomials $p(x)$ and $q(x)$ is given by:

$$\text{HCF}(p(x), q(x)) \times \text{LCM}(p(x), q(x)) = p(x) \times q(x)$$

Substituting the given values:

$$x^2(x-a) \times 36x^3(x+a)(x^3-a^3) = p(x) \times q(x)$$

We know $p(x) = 4x^2(x^2-a^2)$, so the equation becomes:

$$x^2(x-a) \times 36x^3(x+a)(x^3-a^3) = 4x^2(x^2-a^2) \times q(x)$$

Step 2: Simplify the equation

First, factor $x^2 - a^2$ as $(x-a)(x+a)$:

$$4x^2(x^2-a^2) = 4x^2(x-a)(x+a)$$

Thus, the equation becomes:

$$x^2(x-a) \times 36x^3(x+a)(x^3-a^3)$$
$$= 4x^2(x-a)(x+a) \times q(x)$$

Step 3: Cancel common factors

Cancel x^2 and $(x-a)(x+a)$ from both sides (assuming $x \neq a$):

$$36x^3(x+a)(x^3-a^3) = 4q(x)$$

Step 4: Solve for $q(x)$

Now divide both sides of the equation by 4:

$$q(x) = \frac{36x^3(x+a)(x^3-a^3)}{4}$$

Simplify the right-hand side:

$$q(x) = 9x^3(x+a)(x^3-a^3)$$

Step 5: Simplify the expression further

Recall that $x^3 - a^3$ can be factored as $(x-a)(x^2+ax+a^2)$.

Thus, we have:

$$q(x) = 9x^3(x+a)(x-a)(x^2+ax+a^2)$$

This is the required polynomial $q(x)$.

Final Answer:

The polynomial $q(x)$ is:

$$q(x) = 9x^3(x + a)(x - a)(x^2 + ax + a^2)$$

6. **The HCF and LCM of two polynomials is $(x + a)$ and $12x^2(x + a)(x^2 - a^2)$ respectively. Find the product of the two polynomials.**

Solution: We are given the following information:

- The **HCF** of two polynomials is $(x + a)$.
- The **LCM** of the two polynomials is $12x^2(x + a)(x^2 - a^2)$.

We are asked to find the **product** of the two polynomials.

Step 1: Recall the relationship between LCM, HCF, and the product of two polynomials

The relationship between the **HCF** ($\text{HCF}(p(x), q(x))$), **LCM** ($\text{LCM}(p(x), q(x))$), and the product of the two polynomials $p(x)$ and $q(x)$ is given by:

$$\text{HCF}(p(x), q(x)) \times \text{LCM}(p(x), q(x)) = p(x) \times q(x)$$

Substituting the given values:

$$(x + a) \times 12x^2(x + a)(x^2 - a^2) = p(x) \times q(x)$$

Step 2: Simplify the equation

Now, simplify the expression on the left-hand side:

$$(x + a) \times 12x^2(x + a)(x^2 - a^2) = 12x^2(x + a)^2(x^2 - a^2)$$

Step 3: Final answer

Thus, the product of the two polynomials is:

$$p(x) \times q(x) = 12x^2(x + a)^2(x^2 - a^2)$$

This is the required product of the two polynomials.