Exercise 4.4

1. Find the square root of the following polynomials by factorization method:

(i)	$x^2 - 8x + 16$	**(ii)**	$9x^2 + 12x + 4$
(iii)	$36a^2 + 84a + 49$	**(iv)**	$64y^2 - 32y + 4$
(v)	$200t^2 - 120t + 18$	**(vi)**	$40x^2 + 120x + 90$

Solution: Let's find the square roots of the given polynomials using the factorization method, following the example's solution pattern.

(i) $\boldsymbol{x^2 - 8x + 16}$

$$x^2 - 8x + 16 = (x^2 - 8x + 16) = (x - 4)^2$$

Now, take the square root of both sides:

$$\sqrt{x^2 - 8x + 16} = \sqrt{(x-4)^2}$$
$$= \pm(x - 4)$$

So, the square root is:

$$\boxed{x - 4}$$

(ii) $\mathbf{9x^2 + 12x + 4}$

$$9x^2 + 12x + 4 = 9\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) = 9\left(x + \frac{2}{3}\right)^2$$

Taking the square root of both sides:

$$\sqrt{9x^2 + 12x + 4} = \sqrt{9\left(x + \frac{2}{3}\right)^2}$$

$$= 3\left(x + \frac{2}{3}\right)$$

So, the square root is:

$$\boxed{3\left(x + \frac{2}{3}\right)}$$

(iii) $\mathbf{36a^2 + 84a + 49}$

$$36a^2 + 84a + 49 = (6a + 7)^2$$

Now, take the square root of both sides:

$$\sqrt{36a^2 + 84a + 49} = \sqrt{(6a + 7)^2}$$

$$= \pm(6a + 7)$$

So, the square root is:

$$\boxed{6a + 7}$$

(iv) $\mathbf{64y^2 - 32y + 4}$

$$64y^2 - 32y + 4 = 4(16y^2 - 8y + 1) = 4(4y - 1)^2$$

Now, take the square root of both sides:

$$\sqrt{64y^2 - 32y + 4} = \sqrt{4(4y - 1)^2}$$

$$= 2(4y - 1)$$

So, the square root is:

$$\boxed{2(4y - 1)}$$

(v) $\mathbf{200t^2 - 120t + 18}$

$$200t^2 - 120t + 18 = 2(100t^2 - 60t + 9) = 2(10t - 3)^2$$

Now, take the square root of both sides:

$$\sqrt{200t^2 - 120t + 18} = \sqrt{2(10t-3)^2}$$
$$= \sqrt{2} \cdot (10t - 3)$$

So, the square root is:

$$\boxed{\sqrt{2} \cdot (10t - 3)}$$

(vi) $\mathbf{40x^2 + 120x + 90}$

$$40x^2 + 120x + 90 = 10(4x^2 + 12x + 9) = 10(2x+3)^2$$

Now, take the square root of both sides:

$$\sqrt{40x^2 + 120x + 90} = \sqrt{10(2x+3)^2}$$
$$= \sqrt{10} \cdot (2x + 3)$$

So, the square root is:

$$\boxed{\sqrt{10} \cdot (2x + 3)}$$

2. Find the square root of the following polynomials by division method:

(i) $4x^4 - 28x^3 + 37x^2 + 42x + 9$

(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

(iii) $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

(iv) $4x^4 - 12x^3 + 37x^2 - 42x + 49$

Solution:

(i) $\mathbf{4x^4 - 28x^3 + 37x^2 + 42x + 9}$

<u>STEP-BY-STEP LONG DIVISION</u>

1. **First term:** Divide $4x^4$ by x^2:

Result: $2x^2$.

Multiply $2x^2$ by $x^2 - 7x + 3$:

$$2x^2 \cdot (x^2 - 7x + 3) = 4x^4 - 14x^3 + 6x^2.$$

Subtract from the original polynomial:

$$(4x^4 - 28x^3 + 37x^2 + 42x + 9) - (4x^4 - 14x^3 + 6x^2).$$

Result:

$$-14x^3 + 31x^2 + 42x + 9.$$

2. **Next term:** Divide $-14x^3$ by x^2:

Result: $-7x$.

Multiply $-7x$ by $x^2 - 7x + 3$:

$$-7x \cdot (x^2 - 7x + 3) = -7x^3 + 49x^2 - 21x.$$

Subtract:

$$(-14x^3 + 31x^2 + 42x + 9) - (-7x^3 + 49x^2 - 21x).$$

Result:

$$-18x^2 + 63x + 9.$$

3. **Next term:** Divide $-18x^2$ by x^2:

Result: -3.

Multiply -3 by $x^2 - 7x + 3$:

$$-3 \cdot (x^2 - 7x + 3) = -3x^2 + 21x - 9.$$

Subtract:

$$(-18x^2 + 63x + 9) - (-3x^2 + 21x - 9).$$

Result:

$$0.$$

Final Answer:

The square root of $4x^4 - 28x^3 + 37x^2 + 42x + 9$ is:

$$2x^2 - 7x + 3.$$

(ii) $\mathbf{121x^4 - 198x^3 - 183x^2 + 216x + 144}$

STEP-BY-STEP LONG DIVISION

1. **First term:** Divide $121x^4$ by x^2:

Result: $11x^2$.

Multiply $11x^2$ by $11x^2 - 9x + 12$:

$$11x^2 \cdot (11x^2 - 9x + 12) = 121x^4 - 99x^3 + 132x^2.$$

Subtract:

$$(121x^4 - 198x^3 - 183x^2 + 216x + 144) - (121x^4 - 99x^3 + 132x^2).$$

Result:

$$-99x^3 - 315x^2 + 216x + 144.$$

2. **Next term:** Divide $-99x^3$ by $11x^2$:

Result: $-9x$.

Multiply $-9x$ by $11x^2 - 9x + 12$:

$$-9x \cdot (11x^2 - 9x + 12) = -99x^3 + 81x^2 - 108x.$$

Subtract:

$$(-99x^3 - 315x^2 + 216x + 144) - (-99x^3 + 81x^2 - 108x).$$

Result:

$$-396x^2 + 324x + 144.$$

3. **Next term:** Divide $-396x^2$ by $11x^2$:

Result: -12.

Multiply -12 by $11x^2 - 9x + 12$:

$$-12 \cdot (11x^2 - 9x + 12) = -132x^2 + 108x - 144.$$

Subtract:

$$(-396x^2 + 324x + 144) - (-132x^2 + 108x - 144).$$

Result: 0.

Final Answer:

The square root of $121x^4 - 198x^3 - 183x^2 + 216x + 144$ is:

$$11x^2 - 9x + 12.$$

(iii) $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

Step 1: Write the polynomial

Given polynomial:

$$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$$

Step 2: First term of the square root

Take the square root of the first term x^4:

$$\sqrt{x^4} = x^2$$

Step 3: Subtract $(x^2)^2$ from the original polynomial

$$(x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4) - x^4$$
$$= -10x^3y + 27x^2y^2 - 10xy^3 + y^4$$

Step 4: Second term of the square root

Divide $-10x^3y$ by $2x^2$:

$$\frac{-10x^3y}{2x^2} = -5xy$$

Step 5: Subtract $(-5xy)^2$

$$(-5xy)^2 = 25x^2y^2$$

$$(-10x^3y + 27x^2y^2 - 10xy^3 + y^4) - (-10x^3y + 25x^2y^2)$$
$$= 2x^2y^2 - 10xy^3 + y^4$$

Step 6: Third term of the square root

Divide $2x^2y^2$ by $2x^2$:

$$\frac{2x^2y^2}{2x^2} = y^2$$

Step 7: Final answer

The square root is:

$$x^2 - 5xy + y^2$$

Verification

$$(x^2 - 5xy + y^2)^2 = x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$$

Thus, the square root is:

$$x^2 - 5xy + y^2$$

(iv) $\mathbf{4x^4 - 12x^3 + 37x^2 - 42x + 49}$

STEP-BY-STEP LONG DIVISION

1. **First term:** Divide $4x^4$ by x^2:

   Result: $2x^2$.

   Multiply $2x^2$ by $2x^2 - 3x + 7$:

   $$2x^2 \cdot (2x^2 - 3x + 7) = 4x^4 - 6x^3 + 14x^2.$$

   Subtract:

   $$(4x^4 - 12x^3 + 37x^2 - 42x + 49) - (4x^4 - 6x^3 + 14x^2).$$

   Result: $-6x^3 + 23x^2 - 42x + 49$.

2. **Next term:** Divide $-6x^3$ by $2x^2$:

   Result: $-3x$.

   Multiply $-3x$ by $2x^2 - 3x + 7$:

   $$-3x \cdot (2x^2 - 3x + 7) = -6x^3 + 9x^2 - 21x.$$

   Subtract:

   $$(-6x^3 + 23x^2 - 42x + 49) - (-6x^3 + 9x^2 - 21x).$$

   Result: $14x^2 - 21x + 49$.

3. **Next term:** Divide $14x^2$ by $2x^2$:

   Result: 7.

   Multiply 7 by $2x^2 - 3x + 7$:

   $$7 \cdot (2x^2 - 3x + 7) = 14x^2 - 21x + 49.$$

   Subtract: $(14x^2 - 21x + 49) - (14x^2 - 21x + 49) = 0$.

Final Answer:

The square root of $4x^4 - 12x^3 + 37x^2 - 42x + 49$ is:

$$2x^2 - 3x + 7.$$

3. An investor's return $R(x)$ in rupees after investing x thousand rupees is given by quadratic expression:

$$R(x) = -x^2 + 6x - 8$$

Factorize the expression and find the investment levels that result in zero return.

Solution: We are given the quadratic expression for the return:

$$R(x) = -x^2 + 6x - 8.$$

We need to **factorize** the expression and determine the investment levels (x) that result in zero return ($R(x) = 0$).

Step 1: Factorize the quadratic expression

1. Rewrite $R(x)$ in standard form:

$$R(x) = -x^2 + 6x - 8.$$

To simplify factorization, **take out a negative sign** from the entire expression:

$$R(x) = -(x^2 - 6x + 8).$$

2. Factorize $x^2 - 6x + 8$:

We need two numbers that **multiply to 8 and add to** -6.

These numbers are -4 and -2.

So, we can write:

$$x^2 - 6x + 8 = (x - 4)(x - 2).$$

3. Include the negative sign back:

$$R(x) = -(x - 4)(x - 2).$$

Step 2: Find investment levels for zero return

To find the values of x where $R(x) = 0$, set the expression equal to zero:

$$-(x - 4)(x - 2) = 0.$$

Since the negative sign does not affect the solutions, we solve:

$$(x-4)(x-2)=0.$$

This gives two solutions:

$$x-4=0 \Rightarrow x=4$$
$$x-2=0 \Rightarrow x=2.$$

Final Answer:

The factorized form of the quadratic expression is:

$$R(x)=-(x-4)(x-2).$$

The investment levels that result in **zero return** are:

$$x=4 \quad \text{and} \quad x=2.$$

These values represent investments of **2 thousand rupees** and **4 thousand rupees.**

4. A company's profit $P(x)$ in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x)=x^3-15x^2+75x-125$$

Find the break-even point(s), where the profit is zero.

Solution: To find the break-even points where the profit $P(x)=0$, we solve the given cubic expression for x:

$$P(x)=x^3-15x^2+75x-125.$$

Step 1: Solve for $P(x)=0$

Set $P(x)=0$:

$$x^3-15x^2+75x-125=0.$$

We solve this equation by **factoring** the cubic polynomial.

Step 2: Find a factor using the Rational Root Theorem

The possible rational roots of the cubic equation are factors of the constant term -125 divided by factors of the leading coefficient 1.

Factors of -125 are $\pm1, \pm5, \pm25, \pm125$.

We test these possible roots by substituting into $P(x)$:

1. Start with $x=5$:

$$P(5) = (5)^3 - 15(5)^2 + 75(5) - 125.$$

Simplify: $P(5) = 125 - 375 + 375 - 125 = 0.$

Since $P(5) = 0$, $x = 5$ is a root of the equation.

Step 3: Factorize the cubic expression

Since $x = 5$ is a root, $(x - 5)$ is a factor of $P(x)$.

We perform polynomial division to divide $P(x)$ by $(x - 5)$.

DIVIDE $x^3 - 15x^2 + 75x - 125$ BY $x - 5$:

2. **First term:** Divide x^3 by x:

   Result: x^2.

3. Multiply x^2 by $x - 5$:

   $$x^2 \cdot (x - 5) = x^3 - 5x^2.$$

4. Subtract:

   $$(x^3 - 15x^2 + 75x - 125) - (x^3 - 5x^2)$$
   $$= -10x^2 + 75x - 125.$$

5. **Next term:** Divide $-10x^2$ by x:

   Result: $-10x$.

6. Multiply $-10x$ by $x - 5$:

   $$-10x \cdot (x - 5) = -10x^2 + 50x.$$

7. Subtract:

   $$(-10x^2 + 75x - 125) - (-10x^2 + 50x) = 25x - 125.$$

8. **Next term:** Divide $25x$ by x:

   Result: 25.

9. Multiply 25 by $x - 5$:

   $$25 \cdot (x - 5) = 25x - 125.$$

10. Subtract:

    $$(25x - 125) - (25x - 125) = 0.$$

Thus, the division gives:

$$P(x) = (x - 5)(x^2 - 10x + 25).$$

Step 4: Factorize the quadratic expression

The quadratic factor $x^2 - 10x + 25$ is a perfect square trinomial:

$$x^2 - 10x + 25 = (x-5)^2.$$

So, the fully factorized form of $P(x)$ is:

$$P(x) = (x-5)(x-5)^2 = (x-5)^3.$$

Step 5: Solve for $P(x) = 0$

To find the break-even points, set $P(x) = 0$:

$$(x-5)^3 = 0.$$

The solution is: $x = 5$.

Final Answer:

The break-even point occurs at: $x = 5$ units.

At this level of sales, the company's profit is zero.

5. The potential energy $V(x)$ in an electric field varies as a cubic function of distance x, given by:

$$V(x) = 2x^3 - 6x^2 + 4x$$

Determine where the potential energy is zero.

Solution: To determine where the potential energy $V(x)$ is zero, we solve the given cubic equation:

$$V(x) = 2x^3 - 6x^2 + 4x.$$

Step 1: Set $V(x) = 0$

Set the expression for $V(x)$ equal to zero:

$$2x^3 - 6x^2 + 4x = 0.$$

Step 2: Factorize the equation

We observe that all terms in $2x^3 - 6x^2 + 4x$ have a common factor of $2x$. Factor it out:

$$2x(x^2 - 3x + 2) = 0.$$

Now, we have two factors:

1. $2x = 0$, and
2. $x^2 - 3x + 2 = 0$.

Step 3: Solve each factor

1. SOLVE $2x = 0$:

$$x = 0.$$

2. SOLVE $x^2 - 3x + 2 = 0$:

The quadratic equation $x^2 - 3x + 2 = 0$ can be factored. We need two numbers that multiply to 2 and add to -3.

These numbers are -1 and -2.

So, we factorize:

$$x^2 - 3x + 2 = (x-1)(x-2).$$

Set each factor equal to zero:

3: $x - 1 = 0 \Rightarrow x = 1$

4: $x - 2 = 0 \Rightarrow x = 2.$

Step 4: Combine all solutions

The solutions to $V(x) = 0$ are:

$$x = 0,\ x = 1, \text{ and } x = 2.$$

Final Answer:

The potential energy $V(x)$ is zero at:

$$x = 0,\ x = 1, \text{ and } x = 2.$$

6. **In structural engineering, the deflection $Y(x)$ of a beam is given by:**

$$Y(x) = 2x^2 - 8x + 6$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Solution: To determine the points of **zero deflection** where $Y(x) = 0$, we solve the given quadratic equation:

$$Y(x) = 2x^2 - 8x + 6.$$

Step 1: Set $Y(x) = 0$

Set the equation equal to zero:

$$2x^2 - 8x + 6 = 0.$$

Step 2: Solve the quadratic equation

Divide through by 2 to simplify the coefficients:

$$x^2 - 4x + 3 = 0.$$

We now factorize the quadratic equation.

Step 3: Factorize the quadratic expression

We need two numbers that multiply to 3 (constant term) and add to -4 (coefficient of x):

These numbers are -3 and -1.

So, we write:

$$x^2 - 4x + 3 = (x - 3)(x - 1).$$

Step 4: Solve for x

Set each factor equal to zero:

1. $x - 3 = 0 \Rightarrow x = 3$
2. $x - 1 = 0 \Rightarrow x = 1$

Final Answer:

The points of **zero deflection** are:

$$x = 1 \quad \text{and} \quad x = 3.$$