Exercise 5.1

1. Solve and represent the solution set on a real line.

(i) $12x + 30 = -6$ **(ii)** $\frac{x}{3} + 6 = -12$

(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$ **(iv)** $2 = 7(2x + 4) + 12x$

(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$ **(vi)** $\frac{-5x}{10} = 9 - \frac{10}{5}x$

Solution: Let's solve each equation step by step, and represent their solutions on the real line as requested. I'll follow the pattern you provided in the example.

(i) $\mathbf{12x + 30 = -6}$

STEP 1: SUBTRACT 30 FROM BOTH SIDES.

$$12x + 30 - 30 = -6 - 30$$
$$12x = -36$$

STEP 2: DIVIDE BOTH SIDES BY 12.

$$x = \frac{-36}{12} = -3$$

STEP 3: CHECK THE SOLUTION.

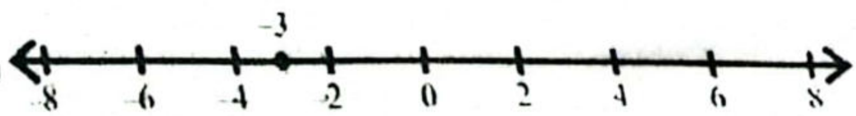
Substitute $x = -3$ into the original equation:

$$12(-3) + 30 = -6$$
$$-36 + 30 = -6$$
$$-6 = -6 \quad \text{(True)}$$

SOLUTION: $x = -3$

Represent the solution on the real line:

Real line: • -3

$x = -3$ 

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(ii) $\frac{x}{3} + 6 = -12$

STEP 1: SUBTRACT 6 FROM BOTH SIDES.

$$\frac{x}{3} + 6 - 6 = -12 - 6$$

$$\frac{x}{3} = -18$$

STEP 2: MULTIPLY BOTH SIDES BY 3.

$$x = -18 \times 3 = -54$$

STEP 3: CHECK THE SOLUTION.

Substitute $x = -54$ into the original equation:

$$\frac{-54}{3} + 6 = -12$$

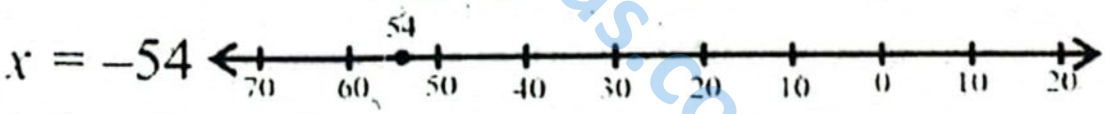
$$-18 + 6 = -12$$

$$-12 = -12 \quad \text{(True)}$$

SOLUTION: $x = -54$

Represent the solution on the real line:

Real line: • -54



(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

STEP 1: FIND A COMMON DENOMINATOR FOR THE FRACTIONS ON THE LEFT SIDE (4).

$$\frac{2x}{4} - \frac{3x}{4} = \frac{1}{12}$$

$$\frac{2x - 3x}{4} = \frac{1}{12}$$

$$\frac{-x}{4} = \frac{1}{12}$$

STEP 2: MULTIPLY BOTH SIDES BY 12 TO ELIMINATE THE DENOMINATOR.

$$12 \times \frac{-x}{4} = 12 \times \frac{1}{12}$$

$$-3x = 1$$

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STEP 3: DIVIDE BOTH SIDES BY -3.

$$x = \frac{1}{-3} = -\frac{1}{3}$$

STEP 4: CHECK THE SOLUTION.

Substitute $x = -\frac{1}{3}$ into the original equation:

$$\frac{-\frac{1}{3}}{2} - \frac{3 \times -\frac{1}{3}}{4} = \frac{1}{12}$$

$$-\frac{1}{6} + \frac{1}{4} = \frac{1}{12}$$

Convert fractions to have a common denominator of 12:

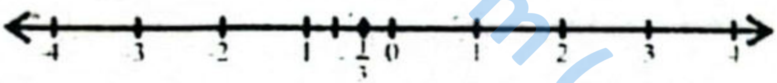
$$-\frac{2}{12} + \frac{3}{12} = \frac{1}{12}$$

$$\frac{1}{12} = \frac{1}{12} \quad \text{(True)}$$

SOLUTION: $x = -\frac{1}{3}$

Represent the solution on the real line:

Real line: $\bullet -\frac{1}{3}$

$x = -\frac{1}{3}$ 

(iv) $\mathbf{2 = 7(2x + 4) + 12x}$

Let's solve the equation step by step:

Given equation:

$$2 = 7(2x + 4) + 12x$$

Step 1: Expand the brackets

Distribute the 7 to both terms inside the parentheses:

$$2 = 7 \times 2x + 7 \times 4 + 12x$$

$$2 = 14x + 28 + 12x$$

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Step 2: Combine like terms

$$2 = 14x + 12x + 28$$

$$2 = 26x + 28$$

Step 3: Isolate the variable

Subtract 28 from both sides: $2 - 28 = 26x$

$$-26 = 26x$$

Step 4: Solve for x

Divide both sides by 26: $x = \frac{-26}{26} = -1$

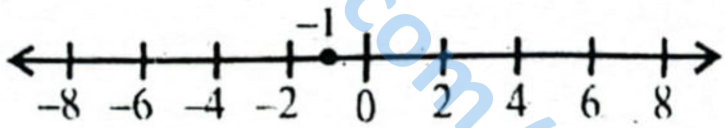
Solution:

The solution to the equation is: $x = -1$

Representing on a Real Line:

To represent the solution on a real line, plot a single point at $x = -1$.

$x = -1$



(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

STEP 1: FIND A COMMON DENOMINATOR FOR THE FRACTIONS (12).

$$\frac{4(2x-1)}{12} - \frac{9x}{12} = \frac{10}{12}$$

$$\frac{8x - 4 - 9x}{12} = \frac{10}{12}$$

$$\frac{-x - 4}{12} = \frac{10}{12}$$

STEP 2: MULTIPLY BOTH SIDES BY 12.

$$-x - 4 = 10$$

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STEP 3: ADD 4 TO BOTH SIDES.

$$-x = 14$$

STEP 4: MULTIPLY BOTH SIDES BY -1.

$$x = -14$$

STEP 5: CHECK THE SOLUTION.

Substitute $x = -14$ into the original equation:

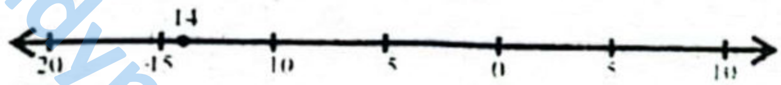
$$\frac{2(-14) - 1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$$

Simplifying this shows that the equation holds true.

SOLUTION: $x = -14$

Represent the solution on the real line:

Real line: • −14

$x = -14$ 

(vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$

Let's solve the given equation step by step:

Given equation:

$$\frac{-5x}{10} = 9 - \frac{10}{5}x$$

Step 1: Simplify both sides

First, simplify the terms in the equation:

- $\frac{-5x}{10} = -\frac{x}{2}$ (since $\frac{5}{10} = \frac{1}{2}$)
- $\frac{10}{5} = 2$

So, the equation becomes:

$$-\frac{x}{2} = 9 - 2x$$

Step 2: Eliminate the fraction

To eliminate the fraction, multiply both sides of the equation by 2:

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$$2 \times \left(-\frac{x}{2}\right) = 2 \times (9 - 2x)$$

This gives:

$$-x = 18 - 4x$$

Step 3: Isolate the variable

Now, add $4x$ to both sides to move all the terms with x to one side:

$$-x + 4x = 18 - 4x + 4x$$

$$3x = 18$$

Step 4: Solve for x

Now, divide both sides by 3:

$$x = \frac{18}{3} = 6$$

Solution:

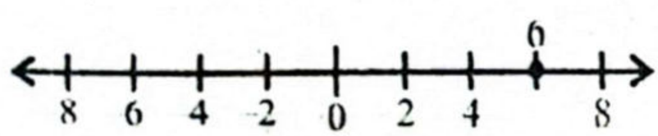
The solution to the equation is:

$$x = 6$$

Representing on a Real Line:

To represent the solution on a real line, plot a single point at $x = 6$.

$x = 6$



2. ***Solve each inequality and represent the solution on a real line.***

(i) $x - 6 \leq -2$ **(ii)** $-9 > -16 + x$

(iii) $3 + 2x \geq 3$ **(iv)** $6(x + 10) \leq 0$

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(v) $\frac{5}{3}x-\frac{3}{4}<\frac{-1}{12}$ (vi) $\frac{1}{4}x-\frac{1}{2}\leq -1+\frac{1}{2}x$

Solution: Let's solve each inequality step-by-step and represent their solutions on a real line.

(i) $x-6\leq -2$

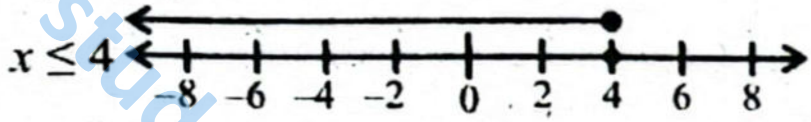
Solution:

$$x-6\leq -2$$

Add 6 to both sides:

$$x\leq 4$$

Solution set: $(-\infty, 4]$



(ii) $-9>-16+x$

Solution:

$$-9>-16+x$$

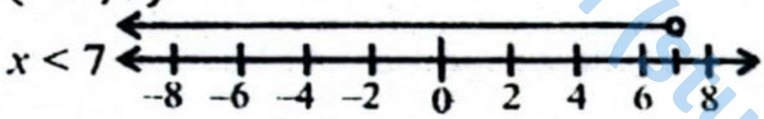
Add 16 to both sides:

$$7>x$$

or

$$x<7$$

Solution set: $(-\infty, 7)$



(iii) $3+2x\geq 3$

Solution:

$$3+2x\geq 3$$

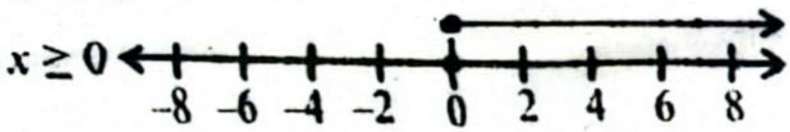
Subtract 3 from both sides:

$$2x\geq 0$$

Divide both sides by 2:

$$x\geq 0$$

Solution set: $[0,\infty)$



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(iv) $6(x+10) \le 0$

Solution:

$$6(x+10) \le 0$$

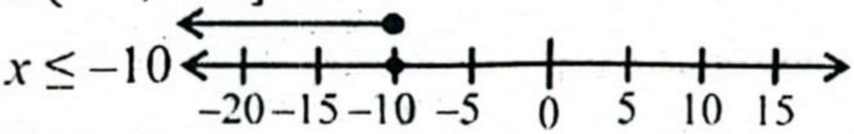
Divide both sides by 6:

$$x + 10 \le 0$$

Subtract 10 from both sides:

$$x \le -10$$

Solution set: $(-\infty, -10]$



(v) $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$

Solution:

$$\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$$

Add $\frac{3}{4}$ to both sides:

$$\frac{5}{3}x < \frac{-1}{12} + \frac{3}{4}$$

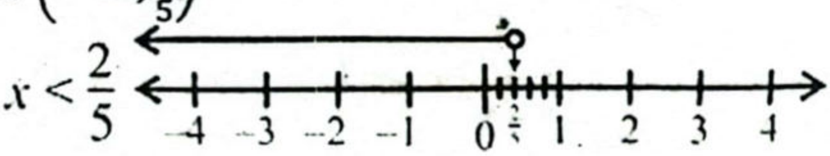
Convert $\frac{3}{4}$ to $\frac{9}{12}$:

$$\frac{5}{3}x < \frac{-1}{12} + \frac{9}{12} = \frac{8}{12} = \frac{2}{3}$$

Multiply both sides by $\frac{3}{5}$ to solve for x:

$$x < \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

Solution set: $\left(-\infty, \frac{2}{5}\right)$



(vi) $\frac{1}{4}x - \frac{1}{2} \le -1 + \frac{1}{2}x$

Solution:

$$\frac{1}{4}x - \frac{1}{2} \le -1 + \frac{1}{2}x$$

Add $\frac{1}{2}$ to both sides:

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$$\frac{1}{4}x \leq -\frac{1}{2} + \frac{1}{2}x$$

Subtract $\frac{1}{4}x$ from both sides: $+\frac{1}{2} \leq -\frac{1}{4}x$

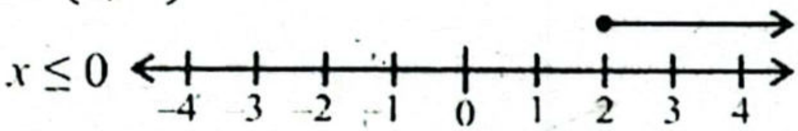
Multiply both sides by -4 (which reverses the inequality):

$$2 \geq -x$$

or

$$x \geq 2$$

Solution set: $x\ (2, \infty)$



3. Shade the solution region for the following linear inequalities in *xy*-plane:

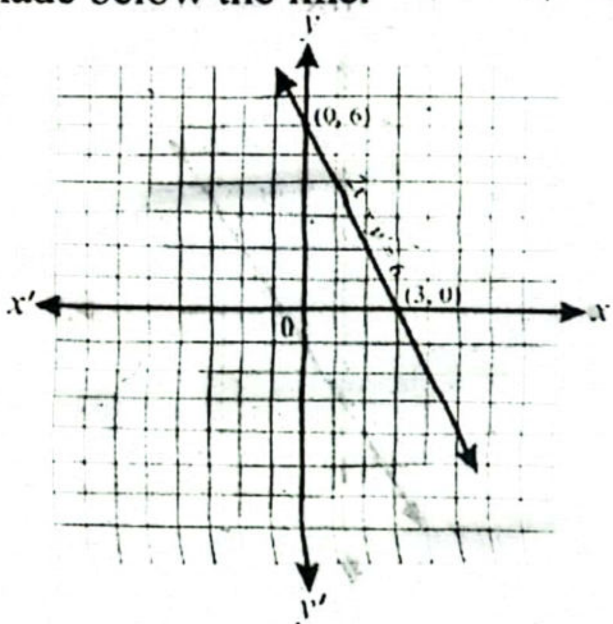
(i)	$2x + y < 6$	**(ii)**	$3x + 7y \geq 21$
(iii)	$3x - 2y \geq 6$	**(iv)**	$5x - 4y \leq 20$
(v)	$2x + 1 \geq 0$	**(vi)**	$3y - 4 \leq 0$

Solution:

(i) $\mathbf{2x + y < 6}$

- **Boundary Line**: $2x + y = 6$.
  - Points: When $x = 0, y = 6$; when $y = 0, x = 3$.
  - Line: Solid line connecting $(0,6)$ and $(3,0)$.
- **Test Point**: Substitute $(0,0)$:

  $2(0) + 0 < 6 \quad \Rightarrow \quad 0 < 6 \quad$ (true).
  - Shade below the line.



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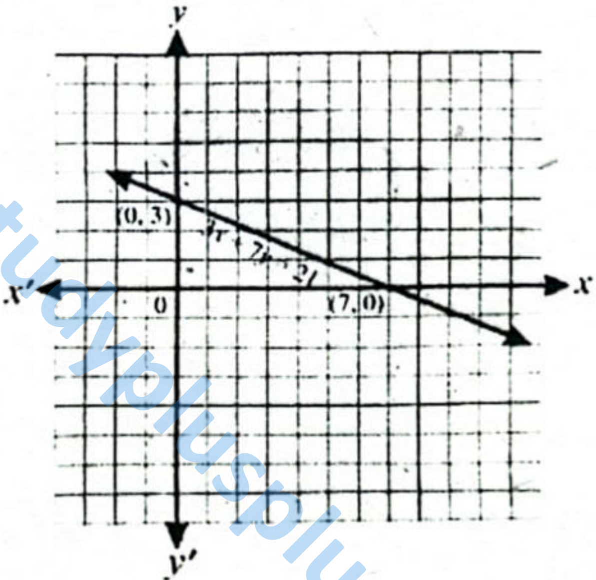
(ii) $3x + 7y \geq 21$

- **Boundary Line**: $3x + 7y = 21$.
  - Points: When $x = 0, y = 3$; when $y = 0, x = 7$.
  - Line: Solid line connecting (0,3) and (7,0).
- **Test Point**: Substitute (0,0):

  $$3(0) + 7(0) \geq 21 \quad \Rightarrow \quad 0 \geq 21 \quad \text{(false)}.$$

  - Shade above the line.

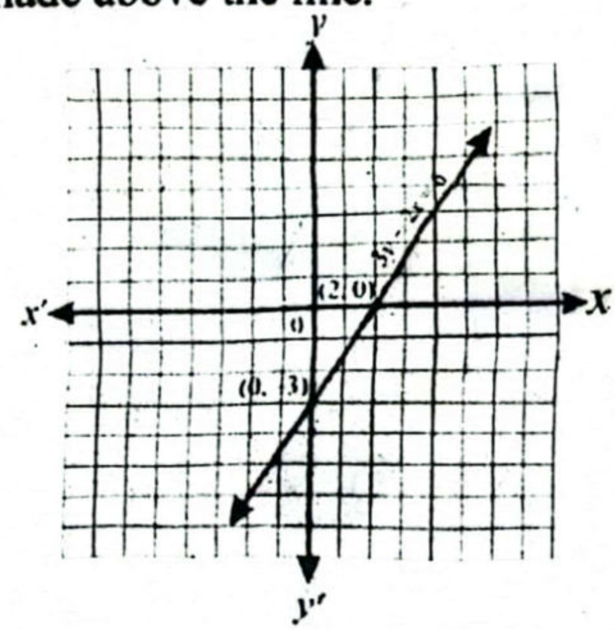


(iii) $3x - 2y \geq 6$

- **Boundary Line**: $3x - 2y = 6$.
  - Points: When $x = 0, y = -3$; when $y = 0, x = 2$.
  - Line: Solid line connecting (0,−3) and (2,0).
- **Test Point**: Substitute (0,0):

  $$3(0) - 2(0) \geq 6 \quad \Rightarrow \quad 0 \geq 6 \quad \text{(false)}.$$

  - Shade above the line.



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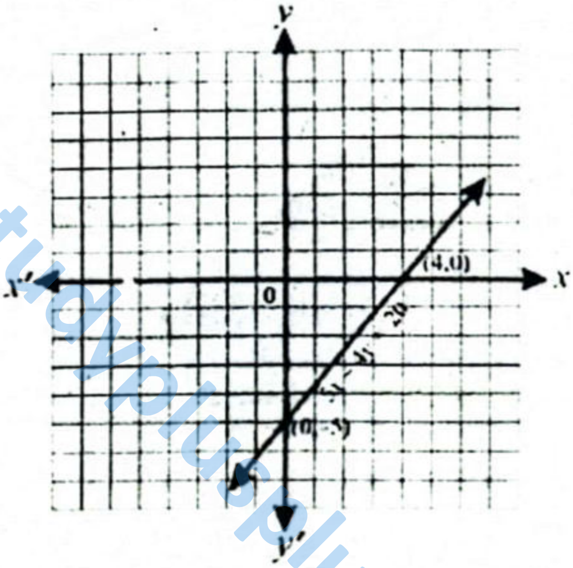
(iv) $5x - 4y \leq 20$

- **Boundary Line**: $5x - 4y = 20$.
  - Points: When $x = 0, y = -5$; when $y = 0, x = 4$.
  - Line: Solid line connecting $(0, -5)$ and $(4,0)$.
- **Test Point**: Substitute $(0,0)$:

$$5(0) - 4(0) \leq 20 \quad \Rightarrow \quad 0 \leq 20 \quad \text{(true)}.$$

  - Shade below the line.

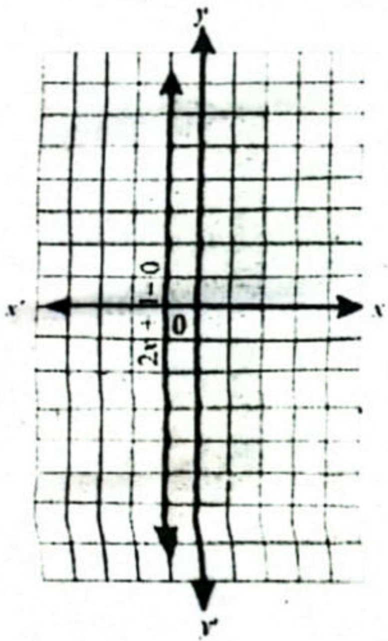


(v). $2x + 1 > 0$

- **Boundary Line**: $2x + 1 = 0$.
  - Point: $x = -\frac{1}{2}$; vertical line passing through $x = -\frac{1}{2}$.
- **Test Point**: Substitute $(0,0)$:

$$2(0) + 1 > 0 \quad \Rightarrow \quad 1 > 0 \quad \text{(true)}.$$

  - Shade to the right of the line.



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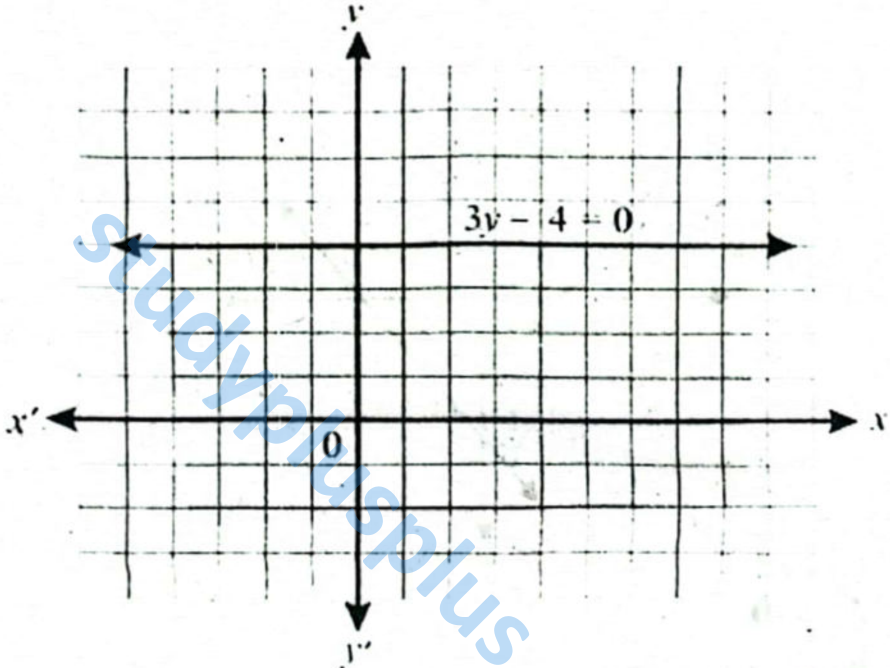
(vi) $3y - 4 \le 0$

- **Boundary Line**: $3y - 4 = 0$.
    - Point: $y = \frac{4}{3}$; horizontal line passing through $y = \frac{4}{3}$.
- **Test Point**: Substitute (0,0):

$$3(0) - 4 \le 0 \quad \Rightarrow \quad -4 \le 0 \text{ (true)},$$

    - Shade below the line.



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4. **Indicate the solution region of the following linear inequalities by shading:**

(i) $2x - 3y \le 6$, $2x + 3y \le 12$ **(ii)** $x + y \ge 5$, $-y + x \le 1$

(iii) $3x + 7y \ge 21$, $x - y \le 2$ **(iv)** $4x - 3y \le 12$, $x \ge -\frac{3}{2}$

(v) $3x - 7y \ge 21$, $y \le 4$ **(vi)** $5x + 7y \le 35$, $x - 2y \le 2$

Solution:

(i) $2x - 3y \le 6$ **and** $2x + 3y \le 12$

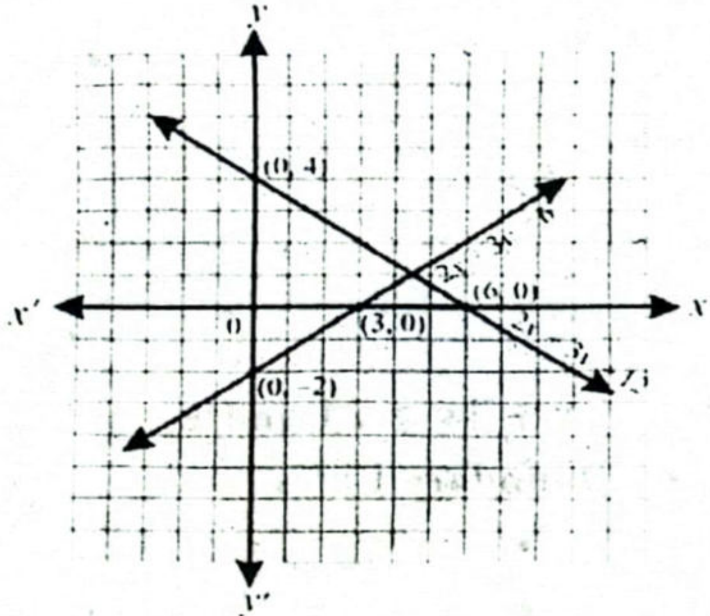
- **Boundary Lines**:
    - $2x - 3y = 6$: Points (3,0) and (0,−2).
    - $2x + 3y = 12$: Points (6,0) and (0,4).

- **Shading:**

a. For $2x - 3y \leq 6$, test (0,0): $2(0) - 3(0) \leq 6$ (true). Shade below the line.

b. For $2x + 3y \leq 12$, test (0,0): $2(0) + 3(0) \leq 12$ (true). Shade below the line.

Solution: The overlapping shaded region is below both lines.



(ii) $x + y \geq 5$ **and** $-y + x \leq 1$

- **Boundary Lines:**
- $x + y = 5$: Points (5,0) and (0,5).
- $-y + x = 1$: Points (1,0) and (0, −1).
- **Shading:**
- For $x + y \geq 5$, test (0,0): $0 + 0 \geq 5$ (false). Shade above the line.
- For $-y + x \leq 1$, test (0,0): $0 - 0 \leq 1$ (true). Shade below the line.

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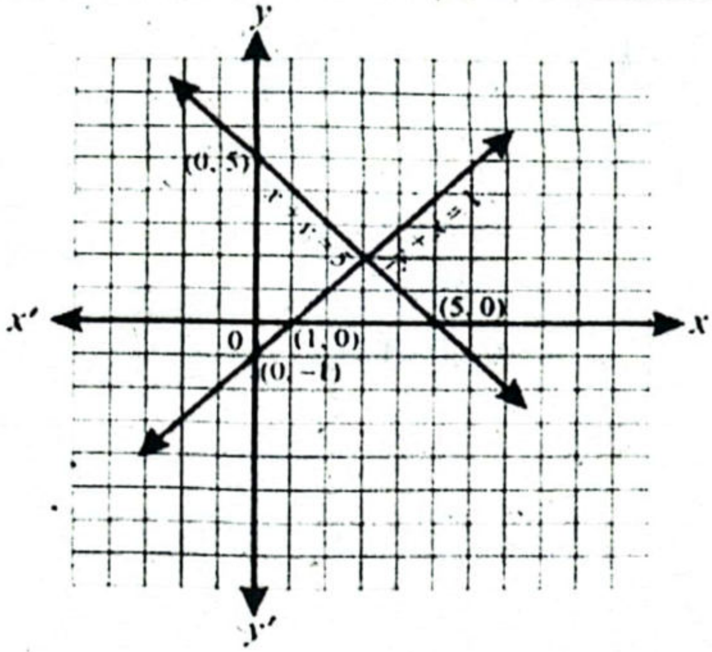
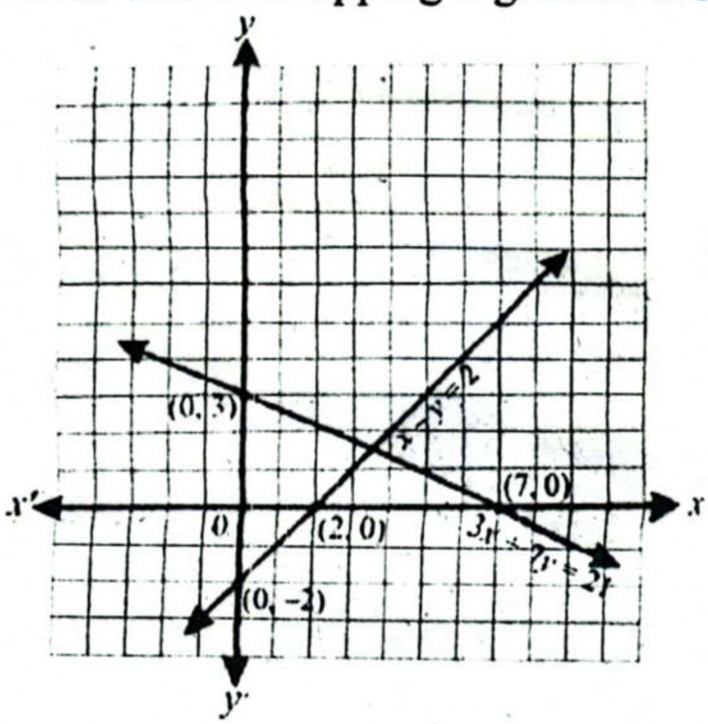
Solution: The region where $x + y \geq 5$ and $-y + x \leq 1$ overlap.



(iii) **$3x + 7y \geq 21$ and $x - y \leq 2$**

- **Boundary Lines**:
  - $3x + 7y = 21$: Points (7,0) and (0,3).
  - $x - y = 2$: Points (2,0) and (0, −2).
- **Shading**:
  - For $3x + 7y \geq 21$, test (0,0): $3(0) + 7(0) \geq 21$ (false). Shade above the line.
  - For $x - y \leq 2$, test (0,0): $0 - 0 \leq 2$ (true). Shade below the line.
- **Solution**: The overlapping region of the shaded areas.



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