Exercise 5.2

1. **Maximize $f(x, y) = 2x + 5y$; subject to the constraints**
   $2y - x \leq 8; \quad x - y \leq 4; \quad x \geq 0; \quad y \geq 0$

Solution:

Problem:

Maximize $f(x, y) = 2x + 5y$, subject to the constraints:

$$2y - x \leq 8, \quad x - y \leq 4, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $2y - x = 8$ (Equation 1)
- $x - y = 4$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $2y - x = 8$ (EQUATION 1):

Rearranging, we get:

$$x = 2y - 8$$

We can find two points for plotting the line:

- When $y = 0, x = 2(0) - 8 = -8$ (but $x \geq 0$ does not hold, so the point $(-8, 0)$ is not valid.
- When $y = 4, x = 2(4) - 8 = 0$ (so the point $(0,4)$ is valid.)

Thus, the line $2y - x = 8$ passes through $(0,4)$.

GRAPHING $x - y = 4$ (EQUATION 2):

Rearranging, we get:

$$x = y + 4$$

We can find two points for plotting the line: (4, 0), (0, –4)

- When $y = 0, x = 0 + 4 = 4$ (so the point is $(4,0)$).
- When $x = 0, y = -4$ (but $y \geq 0$, so the point $(0, -4)$ is not valid).

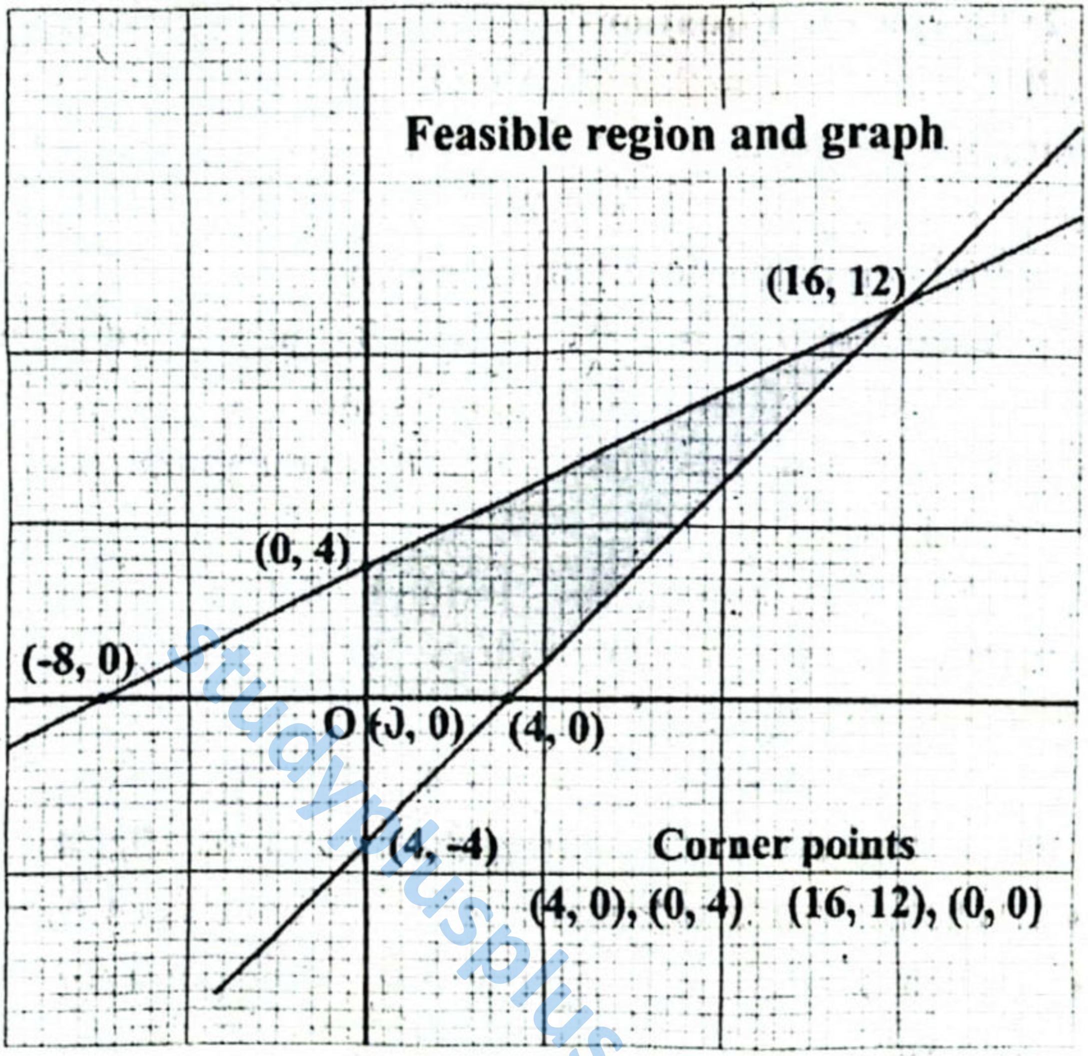
Thus, the line $x - y = 4$ passes through $(4,0)$.

Step 3: Plot the Constraints

Now, we plot the inequalities:

- For $2y - x \leq 8$, the region below the line $2y - x = 8$ (i.e., the half-plane below the line).
- For $x - y \leq 4$, the region below the line $x - y = 4$ (i.e., the half-plane below the line).
- The region for $x \geq 0$, the right half-plane.
- The region for $y \geq 0$, the upper half-plane.

These constraints, when plotted on a graph, will create a bounded *region called the* **feasible region**.



Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points.**

INTERSECTION OF $2y - x = 8$ AND $x - y = 4$:

Solve the system of equations:

- $2y - x = 8$
- $x - y = 4$

From the second equation, solve for x:

$$x = y + 4$$

Substitute $x = y + 4$ into the first equation:

$$2y-(y+4)=8 \Rightarrow 2y-y-4=8 \Rightarrow y=12$$

Substitute $y = 12$ into $x = y + 4$:

$$x = 12 + 4 = 16$$

Thus, the intersection point is (16,12).

INTERSECTION OF $2y - x = 8$ WITH THE Y-AXIS ($x = 0$):

When $x = 0$, substitute into $2y - x = 8$:

$$2y = 8 \Rightarrow y = 4$$

Thus, the point is $(0,4)$.

INTERSECTION OF $x - y = 4$ WITH THE X-AXIS ($y = 0$):

When $y = 0$, substitute into $x - y = 4$:

$$x = 4$$

Thus, the point is $(4,0)$.

Step 5: Maximize $f(x, y) = 2x + 5y$ at the Corner Points

Evaluate $f(x, y) = 2x + 5y$ at each of the corner points:

- At $(0,4)$:
$$f(0,4) = 2(0) + 5(4) = 0 + 20 = 20$$
- At $(4,0)$:
$$f(4,0) = 2(4) + 5(0) = 8 + 0 = 8$$
- At $(16,12)$:
$$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$$

Step 6: Conclusion

The maximum value of $f(x, y) = 2x + 5y$ is $\boxed{92}$, and it occurs at the point $(16,12)$.

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $2y - x = 8$, $x - y = 4$, the x-axis, and the y-axis.
- The corner points of the feasible region are $(0,4)$, $(4,0)$, and $(16,12)$.

2. **Maximize $f(x, y) = x + 3y$; subject to the constraints**
$2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Solution: Let's solve the problem step by step following the format you've requested.

Problem:

Maximize $f(x, y) = x + 3y$, subject to the constraints:

$$2x + 5y \leq 30, \quad 5x + 4y \leq 20, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $2x + 5y = 30$ (Equation 1)
- $5x + 4y = 20$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $2x + 5y = 30$ (EQUATION 1):

Rearranging, we get:

$$5y = 30 - 2x \quad \text{or} \quad y = \frac{30 - 2x}{5}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{30-2(0)}{5} = 6$ (so the point is (0,6)).
- When $y = 0$, $2x = 30 \Rightarrow x = 15$
  (so the point is (15,0)).

Thus, the line $2x + 5y = 30$ passes through the points (0,6) and (15,0).

GRAPHING $5x + 4y = 20$ (EQUATION 2):

Rearranging, we get:

$$4y = 20 - 5x \quad \text{or} \quad y = \frac{20 - 5x}{4}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{20-5(0)}{4} = 5$ (so the point is (0,5)).
- When $y = 0$, $5x = 20 \Rightarrow x = 4$
  (so the point is (4,0)).

Thus, the line $5x + 4y = 20$ passes through the points (0,5) and (4,0).

Step 3: Plot the Constraints

Now, we plot the inequalities:

- For $2x + 5y \le 30$, the region below the line $2x + 5y = 30$.
- For $5x + 4y \le 20$, the region below the line $5x + 4y = 20$.
- The region for $x \ge 0$, the right half-plane.
- The region for $y \ge 0$, the upper half-plane.

These constraints, when plotted on a graph, will create a bounded region called the **feasible region**.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $2x + 5y = 30$ AND $5x + 4y = 20$:

Solve the system of equations:

- $2x + 5y = 30$
- $5x + 4y = 20$

We can solve this system using the substitution or elimination method. Let's use the elimination method.

First, multiply the first equation by 5 and the second equation by 2 to align the coefficients of x:

$$5(2x + 5y) = 5(30) \quad \text{and} \quad 2(5x + 4y) = 2(20)$$

$$10x + 25y = 150 \quad \text{and} \quad 10x + 8y = 40$$

Now subtract the second equation from the first:

$$(10x + 25y) - (10x + 8y) = 150 - 40$$

$$17y = 110 \Rightarrow y = \frac{110}{17} \approx 6.47$$

Substitute $y = 6.47$ into one of the original equations, say $2x + 5y = 30$:

$$2x + 5(6.47) = 30 \Rightarrow 2x + 32.35 = 30 \Rightarrow 2x = -2.35 \Rightarrow x \approx -1.18$$

Since $x \geq 0$, this solution does not satisfy the constraint, so no valid intersection occurs here.

Step 5: Evaluate the Objective Function at the Corner Points

The feasible region's corner points are:

- (0,6)
- (15,0)
- (4,0)

Evaluate the objective function $f(x, y) = x + 3y$ at these points:

- At (0,6):

$$f(0,6) = 0 + 3(6) = 18$$

- At (15,0):

$$f(15,0) = 15 + 3(0) = 15$$

- At (4,0):

$$f(4,0) = 4 + 3(0) = 4$$

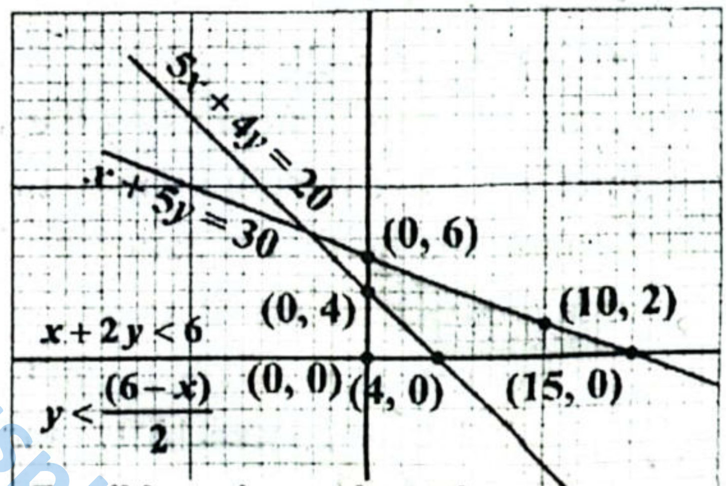
Step 6: Conclusion

The maximum value of $f(x, y) = x + 3y$ is $\boxed{18}$, and it occurs at the point (0,6).

Feasible Region and Graph:

The feasible region is the area bounded by the lines $2x + 5y = 30$, $5x + 4y = 20$, the x-axis, and the y-axis.

The corner points of the feasible region are (0,6), (15,0), and (4,0).



Feasible region and graph

3. **Maximize z = 2x + 3y; subject to the constraints:**

$$2x + y \leq 4; \quad 4x - y \leq 4; \quad x \geq 0; \quad y \geq 0$$

Solution: Let's solve the given linear programming problem step by step following the pattern as requested.

Problem:

Maximize $z = 2x + 3y$, subject to the constraints:

$$2x + y \leq 4, \quad 4x - y \leq 4, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $2x + y = 4$ (Equation 1)
- $4x - y = 4$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $2x + y = 4$ (EQUATION 1):

Rearranging, we get:

$$y = 4 - 2x$$

We can find two points for plotting the line:

- When $x = 0, y = 4 - 2(0) = 4$ (so the point is (0,4)).
- When $y = 0,\ 2x = 4 \quad \Rightarrow \quad x = 2$ (so the point is (2,0)).

Thus, the line $2x + y = 4$ passes through the points (0,4) and (2,0).

GRAPHING $4x - y = 4$ (EQUATION 2):

Rearranging, we get:

$$y = 4x - 4$$

We can find two points for plotting the line:

- When $x = 0, y = 4(0) - 4 = -4$ (so the point is $(0, -4)$, but this is below the x-axis and not within the feasible region since $y \geq 0$).
- When $y = 0,\ 4x = 4 \quad \Rightarrow \quad x = 1$ (so the point is (1,0)).

Thus, the line $4x - y = 4$ passes through the points (1,0) and, for $x = 0$, the point $(0, -4)$, but we ignore the negative value of y.

Step 3: Plot the Constraints

Now, we plot the inequalities on the graph:

- For $2x + y \leq 4$, the region below the line $2x + y = 4$.
- For $4x - y \leq 4$, the region below the line $4x - y = 4$.
- The region for $x \geq 0$, the right half-plane.
- The region for $y \geq 0$, the upper half-plane.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $2x + y = 4$ AND $4x - y = 4$:

Solve the system of equations:

- $2x + y = 4$
- $4x - y = 4$

Add the two equations:

$$(2x + y) + (4x - y) = 4 + 4$$

$$6x = 8 \quad \Rightarrow \quad x = \frac{8}{6} = \frac{4}{3}$$

Substitute $x = \frac{4}{3}$ into the first equation $2x + y = 4$:

$$2\left(\frac{4}{3}\right) + y = 4 \quad \Rightarrow \quad \frac{8}{3} + y = 4 \quad \Rightarrow \quad y = 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$$

So, the intersection point is $\left(\frac{4}{3}, \frac{4}{3}\right)$.

INTERSECTION OF $2x + y = 4$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $2x + y = 4$:

$$2x + 0 = 4 \quad \Rightarrow \quad x = 2$$

So, the intersection point is (2,0).

INTERSECTION OF $4x - y = 4$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $4x - y = 4$:

$$4x - 0 = 4 \quad \Rightarrow \quad x = 1$$

So, the intersection point is (1,0).

Step 5: Evaluate the Objective Function at the Corner Points

The feasible region's corner points are:

- $\left(\frac{4}{3}, \frac{4}{3}\right)$
- (2,0)
- (1,0)

Evaluate the objective function $z = 2x + 3y$ at these points:

- At $\left(\frac{4}{3}, \frac{4}{3}\right)$:

$$z = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right) = \frac{8}{3} + \frac{12}{3} = \frac{20}{3} \approx 6.67$$

- At (2,0):

$$z = 2(2) + 3(0) = 4$$

- At (1,0):

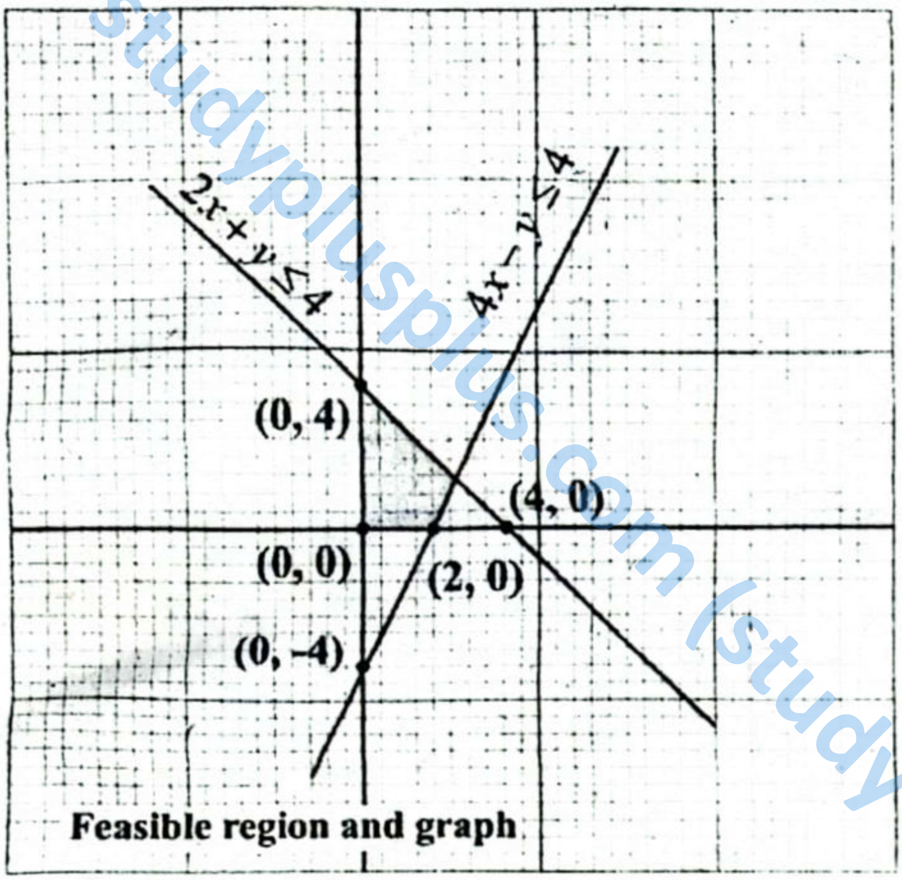
$$z = 2(1) + 3(0) = 2$$

Step 6: Conclusion

The maximum value of $z = 2x + 3y$ is $\boxed{\frac{20}{3}} \approx 6.67$, and it occurs at the point $\left(\frac{4}{3}, \frac{4}{3}\right)$.

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $2x + y = 4$, $4x - y = 4$, the x-axis, and the y-axis.
- The corner points of the feasible region are $\left(\frac{4}{3}, \frac{4}{3}\right)$, (2,0), and (1,0).



Feasible region and graph

4. **Minimize $z = 2x + y$; subject to the constraints:**

$$x + y \geq 3; \quad 7x + 5y \leq 35; \quad x \geq 0; \quad y \geq 0$$

Solution: Let's solve the given linear programming problem step by step.

Problem:

Minimize $z = 2x + y$, subject to the constraints:

$$x + y \geq 3, \quad 7x + 5y \leq 35, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $x + y = 3$ (Equation 1)
- $7x + 5y = 35$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $x + y = 3$ (EQUATION 1):

Rearranging, we get:

$$y = 3 - x$$

We can find two points for plotting the line:

- When $x = 0, y = 3 - 0 = 3$ (so the point is $(0,3)$).
- When $y = 0, x = 3 - 0 = 3$ (so the point is $(3,0)$).

Thus, the line $x + y = 3$ passes through the points $(0,3)$ and $(3,0)$.

GRAPHING $7x + 5y = 35$ (EQUATION 2):

Rearranging, we get:

$$y = \frac{35 - 7x}{5}$$

We can find two points for plotting the line:

- When $x = 0, y = \frac{35-7(0)}{5} = \frac{35}{5} = 7$ (so the point is $(0,7)$).
- When $y = 0,\ 7x = 35 \Rightarrow x = 5$ (so the point is $(5,0)$).

Thus, the line $7x + 5y = 35$ passes through the points $(0,7)$ and $(5,0)$.

Step 3: Plot the Constraints

Now, we plot the inequalities on the graph:

- For $x + y \geq 3$, the region above the line $x + y = 3$.
- For $7x + 5y \leq 35$, the region below the line
- $7x + 5y = 35$.
- *The region for $x \geq 0$, the right half-plane.*
- The region for $y \geq 0$, the upper half-plane.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $x + y = 3$ AND $7x + 5y = 35$:

Solve the system of equations:

- $x + y = 3$
- $7x + 5y = 35$

From the first equation, solve for y:

$$y = 3 - x$$

Substitute $y = 3 - x$ into the second equation:

$$7x + 5(3 - x) = 35$$

$$7x + 15 - 5x = 35$$

$$2x = 20 \quad \Rightarrow \quad x = 10$$

Substitute $x = 10$ into the first equation $x + y = 3$:

$$10 + y = 3 \quad \Rightarrow \quad y = -7$$

Since $y = -7$ is not a valid point in the feasible region (as $y \geq 0$), this intersection point is not valid.

INTERSECTION OF $x + y = 3$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $x + y = 3$:

$$x + 0 = 3 \quad \Rightarrow \quad x = 3$$

So, the intersection point is (3,0).

INTERSECTION OF $7x + 5y = 35$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $7x + 5y = 35$:

$$7x + 0 = 35 \quad \Rightarrow \quad x = 5$$

So, the intersection point is (5,0).

INTERSECTION OF $x + y = 3$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $x + y = 3$:

$$0 + y = 3 \quad \Rightarrow \quad y = 3$$

So, the intersection point is (0,3).

INTERSECTION OF $7x + 5y = 35$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $7x + 5y = 35$:

$$0 + 5y = 35 \quad \Rightarrow \quad y = 7$$

So, the intersection point is (0,7).

Step 5: Evaluate the Objective Function at the Corner Points

The feasible region's corner points are:

- (3,0) • (5,0)
- (0,3) • (0,7)

Evaluate the objective function $z = 2x + y$ at these points:

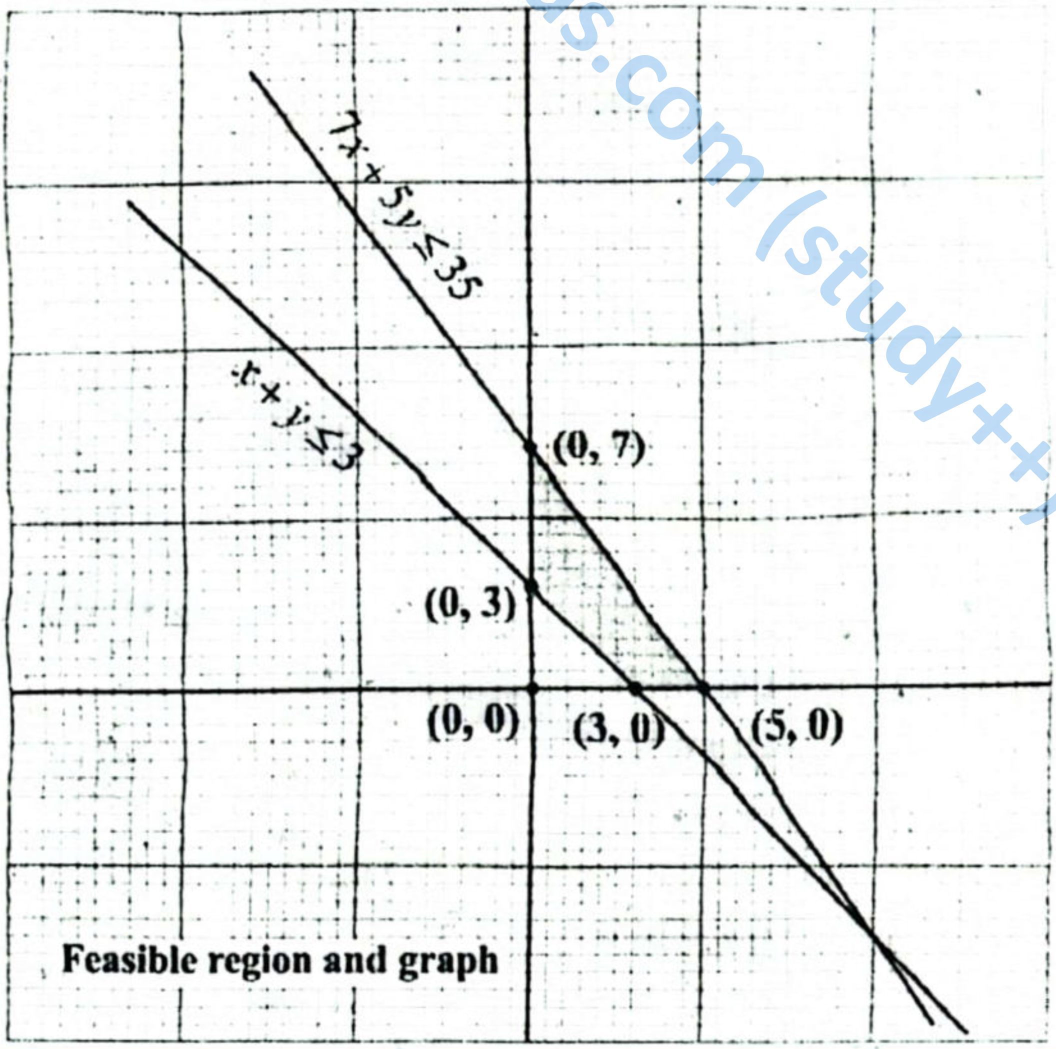
- At (3,0): $z = 2(3) + 0 = 6$
- At (5,0): $z = 2(5) + 0 = 10$
- At (0,3): $z = 2(0) + 3 = 3$
- At (0,7): $z = 2(0) + 7 = 7$

Step 6: Conclusion

The **minimum value** of $z = 2x + y$ is $\boxed{3}$, and it occurs at the point (0,3).

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $x + y = 3$, $7x + 5y = 35$, the x-axis, and the y-axis.
- The corner points of the feasible region are (3,0), (5,0), (0,3), and (0,7).



Feasible region and graph

5. **Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints:**

$$2x + y \leq 8; \quad x + 2y \leq 14; \quad x \geq 0; \quad y \geq 0$$

Let's solve the given linear programming problem step by step.

Problem:

Maximize $f(x, y) = 2x + 3y$, subject to the constraints:

$$2x + y \leq 8, \quad x + 2y \leq 14, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $2x + y = 8$ (Equation 1)
- $x + 2y = 14$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $2x + y = 8$ (EQUATION 1):

Rearranging the equation, we get:

$$y = 8 - 2x$$

We can find two points for plotting the line:

- When $x = 0$, $y = 8 - 2(0) = 8$ (so the point is (0,8)).
- When $y = 0$, $2x = 8 \Rightarrow x = 4$ (so the point is (4,0)).

Thus, the line $2x + y = 8$ passes through the points (0,8) and (4,0).

GRAPHING $x + 2y = 14$ (EQUATION 2):

Rearranging the equation, we get:

$$y = \frac{14 - x}{2}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{14-0}{2} = 7$ (so the point is (0,7)).
- When $y = 0$, $x = 14$ (so the point is (14,0)).

Thus, the line $x + 2y = 14$ passes through the points (0,7) and (14,0).

Step 3: Plot the Constraints

Now, we plot the inequalities on the graph:

- For $2x + y \leq 8$, the region **below** the line $2x + y = 8$.

- For $x + 2y \leq 14$, the region **below** the line $x + 2y = 14$.
- The region for $x \geq 0$, the **right** half-plane.
- The region for $y \geq 0$, the **upper** half-plane.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $2x + y = 8$ AND $x + 2y = 14$:

Solve the system of equations:

- $2x + y = 8$
- $x + 2y = 14$

From the first equation, solve for y:

$$y = 8 - 2x$$

Substitute $y = 8 - 2x$ into the second equation:

$$x + 2(8 - 2x) = 14$$

$$x + 16 - 4x = 14$$

$$-3x = -2 \quad \Rightarrow \quad x = \frac{2}{3}$$

Substitute $x = \frac{2}{3}$ into the first equation $2x + y = 8$:

$$2\left(\frac{2}{3}\right) + y = 8 \quad \Rightarrow \quad \frac{4}{3} + y = 8 \quad \Rightarrow \quad y = 8 - \frac{4}{3} = \frac{24}{3} - \frac{4}{3} = \frac{20}{3}$$

So, the intersection point is $\left(\frac{2}{3}, \frac{20}{3}\right)$.

INTERSECTION OF $2x + y = 8$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $2x + y = 8$:

$$2x + 0 = 8 \quad \Rightarrow \quad x = 4$$

So, the intersection point is (4,0).

INTERSECTION OF $x + 2y = 14$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $x + 2y = 14$:

$$x + 0 = 14 \quad \Rightarrow \quad x = 14$$

So, the intersection point is (14,0).

INTERSECTION OF $2x + y = 8$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $2x + y = 8$:

$$0 + y = 8 \quad \Rightarrow \quad y = 8$$

So, the intersection point is (0,8).

INTERSECTION OF $x + 2y = 14$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $x + 2y = 14$:

$$0 + 2y = 14 \quad \Rightarrow \quad y = 7$$

So, the intersection point is (0,7).

Step 5: Evaluate the Objective Function at the Corner Points

The feasible region's corner points are:

- (4,0)
- (14,0)
- (0,8)
- (0,7)
- $\left(\frac{2}{3}, \frac{20}{3}\right)$

Evaluate the objective function $f(x, y) = 2x + 3y$ at these points:

- At (4,0): $f(4,0) = 2(4) + 3(0) = 8$
- At (14,0): $f(14,0) = 2(14) + 3(0) = 28$
- At (0,8): $f(0,8) = 2(0) + 3(8) = 24$
- At (0,7): $f(0,7) = 2(0) + 3(7) = 21$
- At $\left(\frac{2}{3}, \frac{20}{3}\right)$:

$$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{4}{3} + \frac{60}{3} = \frac{64}{3} \approx 21.33$$

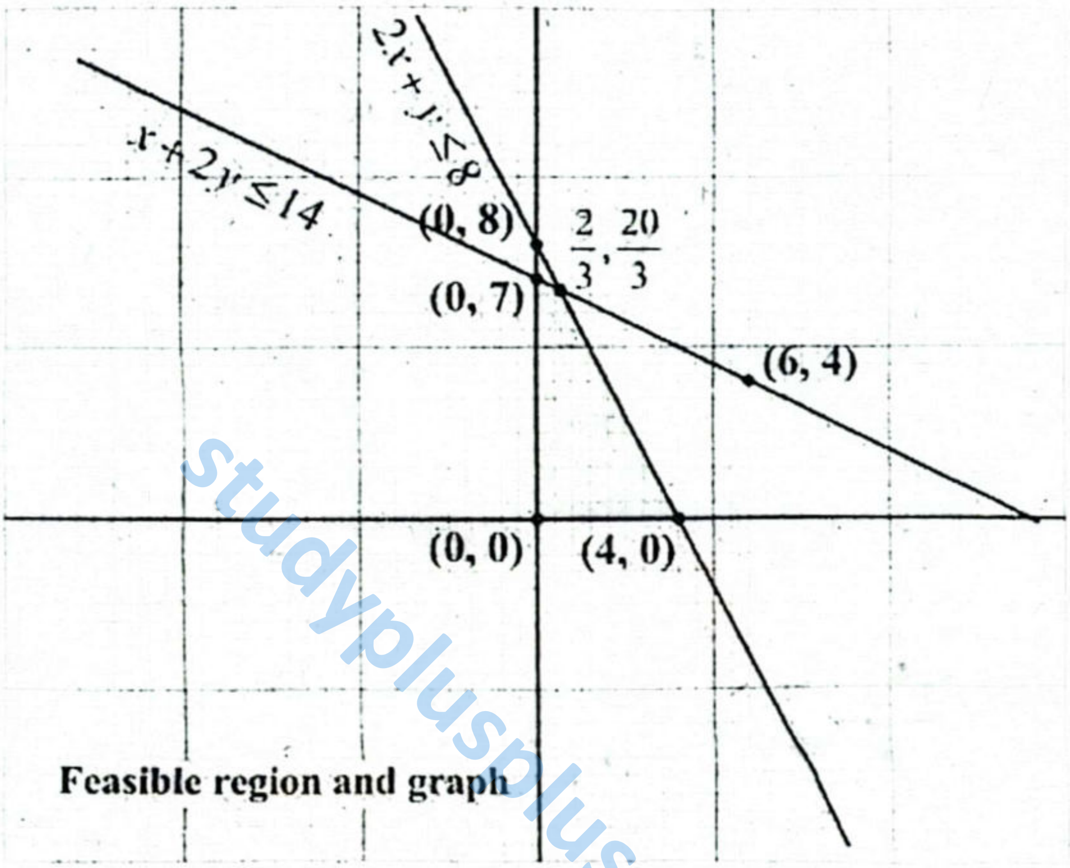
Step 6: Conclusion

The **maximum value** of $f(x, y) = 2x + 3y$ is $\boxed{28}$, and it occurs at the point (14,0).

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $2x + y = 8$, $x + 2y = 14$, the x-axis, and the y-axis.

- The corner points of the feasible region are (4,0), (14,0), (0,8), (0,7), and $\left(\frac{2}{3}, \frac{20}{3}\right)$.



Feasible region and graph

6. **Find minimum and maximum values of $z = 3x + y$; subject to the constraints:**

$$3x + 5y \geq 15; \quad x + 6y \geq 9; \quad x \geq 0; \quad y \geq 0$$

Solution: Let's solve the linear programming problem step by step to find the minimum and maximum values of $z = 3x + y$, subject to the constraints:

Problem:

Maximize and minimize $z = 3x + y$, subject to the constraints:

$$3x + 5y \geq 15$$
$$x + 6y \geq 9$$
$$x \geq 0, \quad y \geq 0$$

Step 1: Convert Inequalities to Equalities

The associated equations for the inequalities are:

- $3x + 5y = 15$ (Equation 1)
- $x + 6y = 9$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $3x + 5y = 15$ (EQUATION 1):

Rearranging the equation to solve for y:

$$y = \frac{15 - 3x}{5}$$

We can find two points for plotting the line:

- When $x = 0, y = \frac{15-3(0)}{5} = 3$ (so the point is $(0,3)$).
- When $y=0, \quad 3x=15 \quad \Rightarrow \quad x=5$ (so the point is $(5,0)$).

Thus, the line $3x + 5y = 15$ passes through the points $(0,3)$ and $(5,0)$.

GRAPHING $x + 6y = 9$ (EQUATION 2):

Rearranging the equation to solve for y:

$$y = \frac{9 - x}{6}$$

We can find two points for plotting the line:

- When $x = 0, y = \frac{9-0}{6} = 1.5$ (so the point is $(0,1.5)$).
- When $y = 0, x = 9$ (so the point is $(9,0)$).

Thus, the line $x + 6y = 9$ passes through the points $(0,1.5)$ and $(9,0)$.

Step 3: Plot the Constraints and Identify the Feasible Region

We need to plot the inequalities on the graph:

- For $3x + 5y \geq 15$, the feasible region is **above** the line $3x + 5y = 15$.
- For $x + 6y \geq 9$, the feasible region is **above** the line $x + 6y = 9$.
- For $x \geq 0$, the feasible region is in the **right half-plane**.
- For $y \geq 0$, the feasible region is in the **upper half-plane**.

The feasible region is the intersection of these regions.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $3x + 5y = 15$ AND $x + 6y = 9$:

Solve the system of equations:

- $3x + 5y = 15$
- $x + 6y = 9$

From the second equation, solve for x:

$$x = 9 - 6y$$

Substitute $x = 9 - 6y$ into the first equation:-

$$3(9 - 6y) + 5y = 15$$
$$27 - 18y + 5y = 15$$
$$27 - 13y = 15$$
$$-13y = -12 \quad \Rightarrow \quad y = \frac{12}{13}$$

Substitute $y = \frac{12}{13}$ into $x = 9 - 6y$:

$$x = 9 - 6\left(\frac{12}{13}\right) = 9 - \frac{72}{13} = \frac{117}{13} - \frac{72}{13} = \frac{45}{13}$$

So, the intersection point is $\left(\frac{45}{13}, \frac{12}{13}\right)$.

INTERSECTION OF $3x + 5y = 15$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into $3x + 5y = 15$:

$$3x = 15 \quad \Rightarrow \quad x = 5$$

So, the intersection point is (5,0).

INTERSECTION OF $x + 6y = 9$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into $x + 6y = 9$:

$$x = 9$$

So, the intersection point is (9,0).

INTERSECTION OF $3x + 5y = 15$ AND THE Y-AXIS (i.e., $x = 0$):

Substitute $x = 0$ into $3x + 5y = 15$:

$$5y = 15 \quad \Rightarrow \quad y = 3$$

So, the intersection point is (0,3).

INTERSECTION OF $x + 6y = 9$ AND THE Y-AXIS (i.e., $x = 0$):

Substitute $x = 0$ into $x + 6y = 9$:

$$6y = 9 \quad \Rightarrow \quad y = 1.5$$

So, the intersection point is (0,1.5).

Step 5: Evaluate the Objective Function at the Corner Points

The corner points of the feasible region are:

- (5,0) • (9,0) • (0,3)
- (0,1.5) • $\left(\frac{45}{13},\frac{12}{13}\right)$

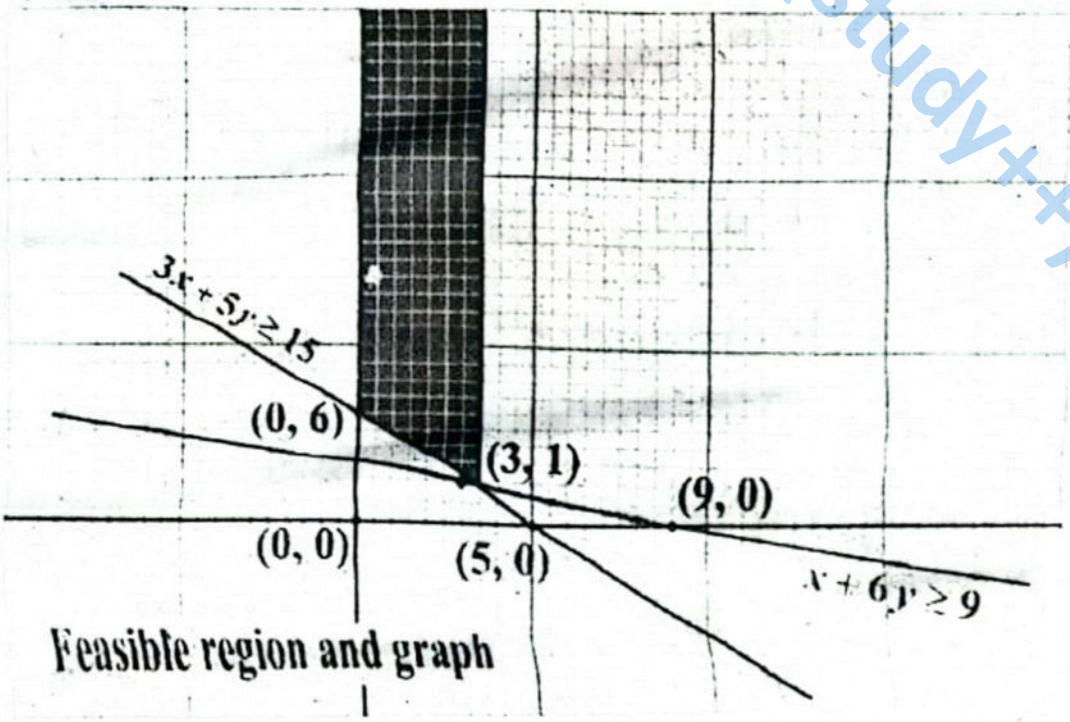
Evaluate the objective function $z = 3x + y$ at these points:

- At (5,0):
$$z = 3(5) + 0 = 15$$
- At (9,0):
$$z = 3(9) + 0 = 27$$
- At (0,3):
$$z = 3(0) + 3 = 3$$
- At (0,1.5):
$$z = 3(0) + 1.5 = 1.5$$
- At $\left(\frac{45}{13},\frac{12}{13}\right)$:
$$z = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{135}{13} + \frac{12}{13} = \frac{147}{13} \approx 11.31$$

Step 6: Conclusion

The **maximum value** of $z = 3x + y$ is $\boxed{27}$, and it occurs at the point (9,0).

The **minimum value** of $z = 3x + y$ is $\boxed{1.5}$, and it occurs at the point (0,1.5).



Feasible region and graph