Exercise 6.1

1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:

(i) 65° (ii) 135° (iii) −40°

(iv) 210° (v) −150°

Solution:

(i) **65°**

- 65° lies between 0° and 90°.

  Quadrant: I

- Co-terminal angle: Add 360°:

$$65^\circ + 360^\circ = 425^\circ$$

Co-terminal angle: 425°

(ii) **135°**

- 135° lies between 90° and 180°.

  Quadrant: II

- Co-terminal angle: Add 360°:

$$135^\circ + 360^\circ = 495^\circ$$

Co-terminal angle: 495°

(iii) **−40°**

- −40° is a negative angle, measured clockwise. Adding 360° to bring it within 0° to 360°:

$$-40^\circ + 360^\circ = 320^\circ$$

- 320° lies between 270° and 360°.

  Quadrant: IV

- Co-terminal angle: 320°

(iv) **210°**

- 210° lies between 180° and 270°.

  Quadrant: III

- Co-terminal angle: Add 360°:

$$210^\circ + 360^\circ = 570^\circ$$

Co-terminal angle: 570°

(v) **−150°**

- −150° is a negative angle. Adding 360°:

$$-150^\circ + 360^\circ = 210^\circ$$

- *210° lies between 180° and 270°.*

  Quadrant: III

- Co-terminal angle: 210°

2. **Convert the following into degrees, minutes and seconds:**

(i) **123.456°** **(ii)** **58.7891°**

(iii) **90.5678°**

Solutions:

(i) 123.456°

- **Degrees (D):**

$$123.456^\circ \rightarrow 123^\circ$$

- **Minutes (M):**

Multiply the decimal part (0.456) by 60:

$$0.456 \times 60 = 27.36$$

27 minutes (27′), decimal part 0.36 remains.

- **Seconds (S):**

Multiply the decimal part (0.36) by 60:

$$0.36 \times 60 = 21.6$$

21.6 ≈ 22 seconds (22″).

Final Answer:

$$123.456^\circ = 123^\circ 27' 22''$$

(ii) 58.7891°

- **Degrees (D):**

$$58.7891^\circ \rightarrow 58^\circ$$

- **Minutes (M):**

Multiply the decimal part (0.7891) by 60:

$$0.7891 \times 60 = 47.346$$

47 minutes (47′), decimal part 0.346 remains.

- **Seconds (S):**

Multiply the decimal part (0.346) by 60:

$$0.346 \times 60 = 20.76$$

20.76 ≈ 21 seconds (21″).

Final Answer:

$$58.7891^\circ = 58^\circ 47' 21''$$

(iii) **90.5678°**

- **Degrees (D):**

$$90.5678° \rightarrow 90°$$

- **Minutes (M):**
  Multiply the decimal part (0.5678) by 60:

$$0.5678 \times 60 = 34.068$$

  34 minutes ($34'$), decimal part 0.068 remains.

- **Seconds (S):**
  Multiply the decimal part (0.068) by 60:

$$0.068 \times 60 = 4.08$$

  $4.08 \approx 4$ seconds ($4''$).

Final Answer:

$$90.5678° = 90°34'4''$$

3. Convert the following into decimal degrees.

(i) 65° 32' 15" **(ii) 42° 18' 45"**

(iii) 78° 45' 36"

Solution: **Convert 65° 32' 15" to decimal degrees:**

1. **Degrees**: 65° remains as is.
2. **Minutes to decimal**:

$$\frac{32}{60} = 0.5333\ldots$$

3. **Seconds to decimal**:

$$\frac{15}{3600} = 0.004166\ldots$$

4. **Add them together**:

$$65 + 0.5333 + 0.004166 = 65.5375$$

Final Result: 65.5375°.

Convert 42° 18' 45" to decimal degrees:

1. **Degrees**: 42° remains as 42.
2. **Minutes to decimal degrees**:

$$\frac{18}{60} = 0.3$$

3. **Seconds to decimal degrees:**

$$\frac{45}{3600} = 0.0125$$

4. **Add them together:**

$$42 + 0.3 + 0.0125 = 42.3125$$

Final Result: 42.3125°.

Convert 78° 45' 36" to decimal degrees:

1. **Degrees:** 78° remains as 78°.
2. **Minutes to decimal degrees:**

$$\frac{45}{60} = 0.75$$

3. **Seconds to decimal degrees:**

$$\frac{36}{3600} = 0.01$$

4. **Add them together:**

$$78 + 0.75 + 0.01 = 78.76$$

Final Result: 78.76°.

4. Convert the following into radian.

(i) **36°** (ii) **22.5°** (iii) **67.5°**

Solution: **Convert 36° to radians:**

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$36° \times \frac{\pi}{180} = \frac{\pi}{5} \text{ rad} \quad \text{or approximately } 0.628 \text{ rad.}$$

Final Result: $\frac{\pi}{5}$ rad or 0.629 rad.

Convert 22. 5° to radians:

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$22.5° \times \frac{\pi}{180} = \frac{\pi}{8} \text{ rad} \quad \text{or approximately } 0.3929 \text{ rad.}$$

Final Result:

$$\frac{\pi}{8} \text{ rad} \quad \text{or } 0.3929 \text{ rad.}$$

Convert 67.5° to radians:

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$67.5^\circ \times \frac{\pi}{180} = \frac{3\pi}{8} \text{ rad} \quad \text{or approximately } 1.179 \text{ rad.}$$

Final Result:

$$\frac{3\pi}{8} \text{ rad} \quad \text{or } 1.179 \text{ rad.}$$

5. Convert the following into degrees.

(i) $\frac{\pi}{16}$ **rad** (ii) $\frac{11\pi}{5}$ **rad**

(iii) $\frac{7\pi}{6}$ **rad**

***Solution*: Convert $\frac{\pi}{16}$ rad to degrees:**

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{\pi}{16} \times \frac{180}{\pi} = \frac{180}{16} = 11.25^\circ$$

Final Result:

$$11.25^\circ$$

Convert $\frac{11\pi}{5}$ rad to degrees:

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{11\pi}{5} \times \frac{180}{\pi} = \frac{11 \times 180}{5} = 396^\circ$$

Final Result:

$$396°$$

Convert $\frac{7\pi}{6}$ rad to degrees:

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{7\pi}{6} \times \frac{180}{\pi} = \frac{7 \times 180}{6} = 210°$$

Final Result:

$$210°$$

6. **Find the arc length and area of a sector with:**

(i) $r = 6$ **cm and central angle** $\theta = \frac{\pi}{3}$ **radians.**

(ii) $r = \frac{4.8}{\pi}$ **cm and central angle** $\theta = \frac{5\pi}{6}$ **radians.**

Solution:

Problem 1. Given:

- Radius $(r) = 6\,\text{cm}$
- Central angle $(\theta) = \frac{\pi}{3}$ radians
- **Arc Length (ℓ):**

  Formula:

$$\ell = r\theta$$

  Substituting the values:

$$\ell = 6 \times \frac{\pi}{3} = 2\pi\ \text{cm}$$

- **Area of the Sector (A):**

  Formula:

$$A = \frac{1}{2}r^2\theta$$

  Substituting the values:

$$A = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = \frac{1}{2} \times 36 \times \frac{\pi}{3} = 6\pi\ \text{cm}^2$$

Result:

- Arc Length: 2π cm
- Area: $6\pi\ \text{cm}^2$

Problem 2:

Given:

- Radius $(r) = 8$ cm
- Central angle $(\theta) = \frac{5\pi}{6}$ radians
- **Arc Length** (ℓ):

  Formula:

  $$\ell = r\theta$$

  Substituting the values:

  $$\ell = 8 \times \frac{5\pi}{6} = \frac{40\pi}{6} = \frac{20\pi}{3}\ \text{cm}$$

- **Area of the Sector** (A):

  Formula:

  $$A = \frac{1}{2}r^2\theta$$

  Substituting the values:

$$A = \frac{1}{2} \times 8^2 \times \frac{5\pi}{6} = \frac{1}{2} \times 64 \times \frac{5\pi}{6} = \frac{160\pi}{6} = \frac{80\pi}{3}\ \text{cm}^2$$

Result:

- Arc Length: $\frac{20\pi}{3}$ cm
- Area: $\frac{80\pi}{3}\ \text{cm}^2$

7. **If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.**

Solution: **Step 1: Formula for the area of a sector**

The area of a sector is given by:

$$\text{Area of sector} = \frac{\text{Central angle}}{360^\circ} \times \pi r^2$$

Step 2: Calculate the area of the sector

Substitute the given values:

- Central angle = 60°

- Radius $(r) = 12$ cm

$$\text{Area of sector} = \frac{60}{360} \times \pi(12)^2$$

Simplify the fraction:

$$\frac{60}{360} = \frac{1}{6}$$

Now calculate:

$$\text{Area of sector} = \frac{1}{6} \times \pi \times 144 = \frac{144\pi}{6} = 24\pi \text{ cm}^2$$

Approximating $\pi \approx 3.1416$:

$$\text{Area of sector} \approx 24 \times 3.1416 = 75.3984 \text{ cm}^2$$

Step 3: Find the total area of the circle

The area of the full circle is given by:

$$\text{Total area of the circle} = \pi r^2$$

Substitute $r = 12$:

$$\text{Total area of the circle} = \pi(12)^2 = 144\pi \text{ cm}^2$$

Approximating $\pi \approx 3.1416$:

$$\text{Total area of the circle} \approx 144 \times 3.1416 = 452.3904 \text{ cm}^2$$

Step 4: Calculate the percentage of the circle represented by the sector

The percentage is given by:

$$\text{Percentage} = \frac{\text{Area of sector}}{\text{Total area of the circle}} \times 100$$

Substitute the values:

$$\text{Percentage} = \frac{24\pi}{144\pi} \times 100 = \frac{24}{144} \times 100 = 16.67\%$$

8. **Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.**

Solution: Let's solve the problem step by step in detail to calculate the percentage of the area of a sector subtending an angle $\frac{\pi}{8}$ radians.

Step 1: Understand the relationship between the angle and the sector area

The area of a sector is proportional to the angle it subtends at the center of a circle. The proportion is given by:

$$\text{Area of sector} = \left(\frac{\text{Angle subtended by the sector}}{\text{Angle for the full circle}}\right) \times \text{Total area of the circle}$$

The percentage of the circle's area covered by the sector is therefore:

$$\text{Percentage of area} = \left(\frac{\text{Angle subtended by the sector}}{\text{Angle for the full circle}}\right) \times 100$$

Step 2: Define the full circle angle and substitute values

The angle for a full circle is 2π radians. The angle subtended by the sector is $\frac{\pi}{8}$ radians. Substituting these values into the formula:

$$\text{Percentage of area} = \left(\frac{\frac{\pi}{8}}{2\pi}\right) \times 100$$

Step 3: Simplify the fraction

Simplify $\frac{\frac{\pi}{8}}{2\pi}$:

$$\frac{\frac{\pi}{8}}{2\pi} = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

Step 4: Multiply by 100 to get the percentage

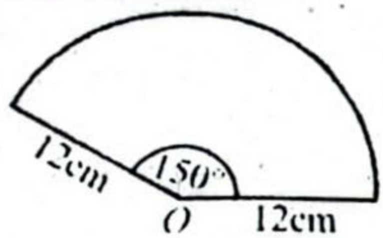
Now calculate the percentage:

$$\text{Percentage of area} = \frac{1}{16} \times 100 = 6.25\%$$

Final Answer:

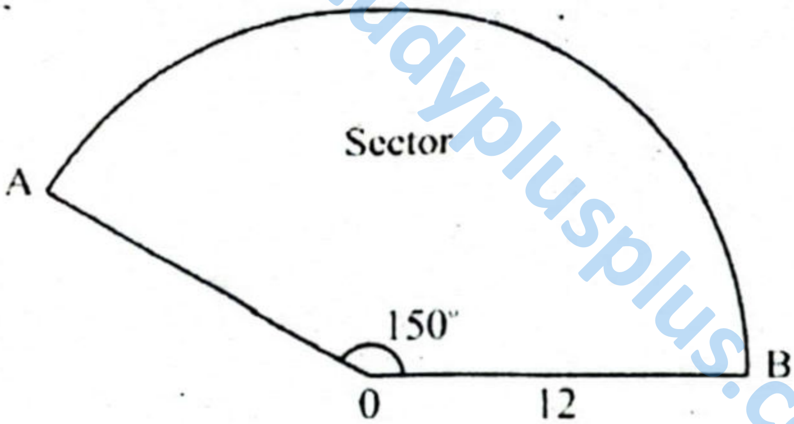
The sector subtending an angle of $\frac{2\pi}{8}$ radians represents **12.5%** of the total area of the circle.

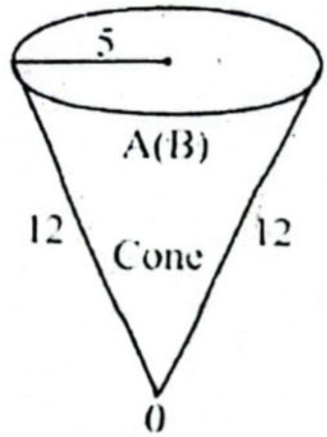
9. . **A circular sector of radius r =12 has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?**



Hint: Arc length of sector = circumference of cone.

Solution: When a circular sector is bent to form a cone, its arc becomes the **circumference** of the cone's base, and the radius of the sector becomes the **slant height** of the cone. Let's solve this step by step.





Step 1: Given values

- Radius of the sector: $r = 12$ (this will become the slant height of the cone).
- Angle of the sector: $\theta = 150°$.

Step 2: Calculate the length of the arc of the sector

The formula for the arc length of a sector is:

$$\text{Arc length} = \theta \times r$$

where θ is in radians. First, convert 150° to radians:

$$\theta \text{ (in radians)} = \frac{150° \times \pi}{180°} = \frac{5\pi}{6} \text{ radians.}$$

Now calculate the arc length:

$$\text{Arc length} = r \times \theta = 12 \times \frac{5\pi}{6} = 10\pi.$$

Step 3: Relate the arc length to the cone's base circumference

When the sector is bent into a cone, the arc length becomes the circumference of the base of the cone. Let the radius of the cone's base be R. The circumference of the base of the cone is:

$$\text{Circumference of cone's base} = 2\pi R.$$

Equating this to the arc length:

$$2\pi R = 10\pi.$$

Solve for R:

$$R = \frac{10\pi}{2\pi} = 5.$$

Step 4: Determine the slant height of the cone

The radius of the sector ($r = 12$) becomes the slant height (L) of the cone:

$$L = 12.$$

Final Answer:

- **Slant height (L)**: 12
- **Radius of the base (R)**: 5