



Exercise 6.4



1. Find the value of the following trigonometric ratios without using the calculator.

(i) $\sin 30^\circ$

(iii) $\tan 30^\circ$

(v) $\sec 60^\circ$

(vii) $\cot 60^\circ$

(ix) $\sec 30^\circ$

(xi) $\sin 45^\circ$

(ii) $\cos 30^\circ$

(iv) $\tan 60^\circ$

(vi) $\cos 60^\circ$

(viii) $\sin 60^\circ$

(x) $\operatorname{cosec} 30^\circ$

(xii) $\cos 45^\circ$

Solution: To find the values of these trigonometric ratios without using a calculator, we rely on standard trigonometric values



derived from the special triangles ($30^\circ-60^\circ-90^\circ$ and $45^\circ-45^\circ-90^\circ$ triangles).

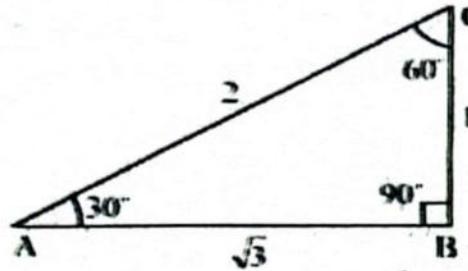
Special Triangles Recap

- **$30^\circ-60^\circ-90^\circ$ Triangle:**
 - Shortest side (30°) = 1,
 - Hypotenuse = 2,
 - Other side (60°) = $\sqrt{3}$.
- **$45^\circ-45^\circ-90^\circ$ Triangle:**
 - Legs = 1,
 - Hypotenuse = $\sqrt{2}$.

Values for Each Ratio

(i) $\sin 30^\circ$:

$$\sin 30^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{2}$$

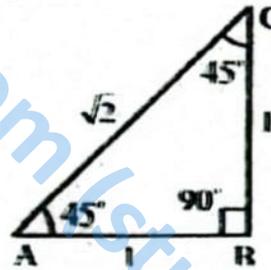


Answer: $\frac{1}{2}$

(ii) $\cos 30^\circ$:

$$\cos 30^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$

Answer: $\frac{\sqrt{3}}{2}$



(iii) $\tan 30^\circ$:

$$\tan 30^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Answer: $\frac{\sqrt{3}}{3}$

(iv) $\tan 60^\circ$:

$$\tan 60^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \sqrt{3}$$

Answer: $\sqrt{3}$

(v) $\sec 60^\circ$:

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2.$$

Answer: 2

(vi) $\cos 60^\circ$:

$$\cos 60^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{2}.$$

Answer: $\frac{1}{2}$

(vii) $\cot 60^\circ$:

$$\cot 60^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

Answer: $\frac{\sqrt{3}}{3}$

(viii) $\sin 60^\circ$:

$$\sin 60^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}.$$

Answer: $\frac{\sqrt{3}}{2}$

(ix) $\sec 30^\circ$:

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

Answer: $\frac{2\sqrt{3}}{3}$

(x) $\csc 30^\circ$:

$$\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2.$$

Answer: 2

(xi) $\sin 45^\circ$:

$$\sin 45^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

Answer: $\frac{\sqrt{2}}{2}$



(xii) $\cos 45^\circ$:

$$\cos 45^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Answer: $\frac{\sqrt{2}}{2}$

2. Find the values of each part:

(i) $2 \sin 60^\circ \cos 60^\circ$

Solution: $2 \sin 60^\circ \cos 60^\circ$

Using the known values for sine and cosine at 60° :

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

Now, substitute these values into the expression:

$$2 \sin 60^\circ \cos 60^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

Thus:

$$2 \sin 60^\circ \cos 60^\circ = \frac{\sqrt{3}}{2}$$

(ii) $2 \cos 60^\circ \sin 60^\circ$

Solution: $2 \cos 60^\circ \sin 60^\circ$

Using the same values for sine and cosine:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

Now, substitute these values into the expression:

$$2 \cos 60^\circ \sin 60^\circ = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Thus:

$$2 \cos 60^\circ \sin 60^\circ = \frac{\sqrt{3}}{2}$$

(iii) $2 \sin 45^\circ + 2 \cos 45^\circ$

Solution: $2 \sin 45^\circ + 2 \cos 45^\circ$

Using the known values for sine and cosine at 45° :

$$\sin 45^\circ = \frac{\sqrt{2}}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}$$



Now, substitute these values into the expression:

$$2\sin 45^\circ + 2\cos 45^\circ = 2 \times \frac{\sqrt{2}}{2} + 2 \times \frac{\sqrt{2}}{2} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}.$$

Thus:

$$2\sin 45^\circ + 2\cos 45^\circ = 2\sqrt{2}.$$

(iv) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Solution: $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Using the known values for sine and cosine:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}, \quad \sin 30^\circ = \frac{1}{2}.$$

Now, substitute these values into the expression:

$$\begin{aligned} \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} \\ &= 1. \end{aligned}$$

Thus:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = 1.$$

(v) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

Solution: $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

This is a standard identity for $\cos(A + B)$, so:

$$\begin{aligned} \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ &= \cos(60^\circ + 30^\circ) = \cos 90^\circ \\ &= 0. \end{aligned}$$

Thus:

$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0.$$

(vi) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Solution: $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

This is a standard identity for $\sin(A - B)$, so:

$$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin(60^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

Thus:

$$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \frac{1}{2}.$$

(vii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Solution: $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

This is a standard identity for $\cos(A - B)$, so:



$$\begin{aligned}\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ &= \cos(60^\circ - 30^\circ) = \cos 30^\circ \\ &= \frac{\sqrt{3}}{2}.\end{aligned}$$

Thus:

$$\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \frac{\sqrt{3}}{2}.$$

(viii) $\tan 30^\circ \cot 30^\circ + 1$

Solution: $\tan 30^\circ \cot 30^\circ + 1$

Using the known values for tangent and cotangent at 30° :

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \cot 30^\circ = \sqrt{3}.$$

Now, substitute these values into the expression:

$$\tan 30^\circ \cot 30^\circ + 1 = \frac{1}{\sqrt{3}} \times \sqrt{3} + 1 = 1 + 1 = 2.$$

Thus:

$$\tan 30^\circ \cot 30^\circ + 1 = 2.$$

3. If $\sin 45^\circ$ and $\cos 45^\circ$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) $2 \sin 45^\circ + 2 \cos 45^\circ$

(i) $2 \sin 45^\circ + 2 \cos 45^\circ$

Substitute the known value for $\sin 45^\circ$ and $\cos 45^\circ$:

$$2 \sin 45^\circ + 2 \cos 45^\circ = 2 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Now, simplify $\frac{4}{\sqrt{2}}$:

$$\frac{4}{\sqrt{2}} = \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}.$$

Thus:

$$2 \sin 45^\circ + 2 \cos 45^\circ = 2\sqrt{2}.$$

(ii) $3 \cos 45^\circ + 4 \sin 30^\circ$

Substitute the known values for $\cos 45^\circ$ and $\sin 30^\circ$:



$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}$$

Now, substitute these values into the expression:

$$3\cos 45^\circ + 4\sin 30^\circ = 3 \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{2} = \frac{3}{\sqrt{2}} + 2.$$

Now, simplify $\frac{3}{\sqrt{2}}$:

$$\frac{3}{\sqrt{2}} = \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{2}.$$

Thus, the expression becomes:

$$\frac{3\sqrt{2}}{2} + 2.$$

To combine the terms, write 2 as $\frac{4}{2}$:

$$\frac{3\sqrt{2}}{2} + \frac{4}{2} = \frac{3\sqrt{2} + 4}{2}.$$

Thus:

$$3\cos 45^\circ + 4\sin 30^\circ = \frac{3\sqrt{2} + 4}{2}.$$

(iii) $5\cos 45^\circ - 3\sin 45^\circ$

Substitute the known values for $\cos 45^\circ$ and $\sin 45^\circ$:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

Now, substitute these values into the expression:

$$\begin{aligned} 5\cos 45^\circ - 3\sin 45^\circ &= 5 \times \frac{1}{\sqrt{2}} - 3 \times \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{5-3}{\sqrt{2}} \\ &= \frac{2}{\sqrt{2}} \end{aligned}$$

Now, simplify $\frac{2}{\sqrt{2}}$:

$$\frac{2}{\sqrt{2}} = \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

Thus:

$$5\cos 45^\circ - 3\sin 45^\circ = \sqrt{2}.$$

