# Exercise 7.2

1. **Find the slope and inclination of the line joining the points:**
   **(i)** **(–2, 4) ; (5, 11)** **(ii)** **(3, –2) ; (2, 7)**
   **(iii)** **(4, 6) ; (4, 8)**

***Solution:*** To find the slope and inclination of the lines joining the given points, we'll follow these steps:

**Step 1: Find the Slope**

The slope $m$ of the line joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Step 2: Find the Inclination**

The inclination $\theta$ of the line is the angle that the line makes with the positive x-axis. The formula for inclination is:

$$\tan(\theta) = m$$

Thus, the inclination $\theta$ can be calculated using the inverse tangent (arctan) of the slope:

$$\theta = \tan^{-1}(m)$$

**(i)** **Points $(-2, 4)$ and $(5, 11)$**

FIND THE SLOPE:

$$m = \frac{11 - 4}{5 - (-2)} = \frac{7}{7} = 1$$

FIND THE INCLINATION:

$$\theta = \tan^{-1}(1) = 45^\circ$$

**(ii) Points $(3, -2)$ and $(2, 7)$**

FIND THE SLOPE:

$$m = \frac{7-(-2)}{2-3} = \frac{9}{-1} = -9$$

FIND THE INCLINATION:

$$\theta = \tan^{-1}(-9)$$

Calculating the angle using a calculator:

$$\theta \approx -83.6^\circ$$

Since inclination is measured from the positive x-axis, we add 180° (to convert to a positive angle):

$$\theta \approx 96.4^\circ$$

**(iii) Points $(4, 6)$ and $(4, 8)$**

FIND THE SLOPE:

Since the x-coordinates are the same ($x_1 = x_2 = 4$), the line is vertical. The slope of a vertical line is undefined.

FIND THE INCLINATION:

The inclination of a vertical line is always 90°.

**Summary of Results:**

- **For points** $(-2,4)$ **and** $(5,11)$:
  - Slope: 1
  - Inclination: 45°
- **For points** $(3,-2)$ **and** $(2,7)$:
  - Slope: $-9$
  - Inclination: 96.4°
- **For points** $(4,6)$ **and** $(4,8)$:
  - Slope: Undefined
  - Inclination: 90°

**Sketch of the Lines:**

- ***For the line joining*** $(-2,4)$ ***and*** $(5,11)$:
  - This is a line with a slope of 1, which means it rises 1 unit for every 1 unit it moves to the right.

This line will have an inclination of 45°, making it a diagonal line at a 45° angle to the x-axis.

- **For the line joining $(3,-2)$ and $(2,7)$:**
  - This is a line with a negative slope $(-9)$, which means it falls steeply as we move to the right. The inclination is 96.4°, so it is slightly more than vertical.
- **For the line joining $(4,6)$ and $(4,8)$:**
  - This is a vertical line since the x-coordinates are the same. Its inclination is 90°.

**2. By means of slopes, show that the following points lie on the same line.**

**(i)** $A(-1,-3)$ ; $B(1,5)$ ; $C(2,9)$

**(ii)** $P(4,-5)$ ; $Q(7,5)$ ; $R(10,15)$

**(iii)** $L(-4,6)$ ; $M(3,8)$ ; $N(10,10)$

**(iv)** $X(a,2b)$; $Y(c,a+b)$; $Z(2c-a,2a)$

To determine if the points lie on the same straight line, we calculate the slopes of the line segments formed by consecutive pairs of points. If all slopes are equal, the points are collinear.

**(i) Points: $A(-1,-3), B(1,5), C(2,9)$**

*Slope of AB:*

$$\text{Slope of } AB = \frac{5-(-3)}{1-(-1)} = \frac{8}{2} = 4$$

*Slope of BC:*

$$\text{Slope of } BC = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

Since Slope of $AB$ = Slope of $BC = 4$, the points are collinear.

**(ii) Points: $P(4,-5), Q(7,5), R(10,15)$**

*Slope of PQ:*

$$\text{Slope of } PQ = \frac{5-(-5)}{7-4} = \frac{10}{3}$$

*Slope of QR:*

$$\text{Slope of } QR = \frac{15-5}{10-7} = \frac{10}{3}$$

Since Slope of $PQ$ = Slope of $QR = \frac{10}{3}$, the points are collinear.

**(iii) Points: $L(-4,6), M(3,8), N(10,10)$**

*Slope of LM:*

$$\text{Slope of } LM = \frac{8-6}{3-(-4)} = \frac{2}{7}$$

*Slope of MN:*

$$\text{Slope of } MN = \frac{10-8}{10-3} = \frac{2}{7}$$

Since Slope of $LM$ = Slope of $MN = \frac{2}{7}$, the points are collinear.

**(iv) Points: $X(a,2b), Y(c,a+b), Z(2c-a,2a)$**

*Slope of XY:*

$$\text{Slope of } XY = \frac{(a+b)-2b}{c-a} = \frac{a-b}{c-a}$$

*Slope of YZ:*

$$\text{Slope of } YZ = \frac{2a-(a+b)}{(2c-a)-c} = \frac{a-b}{c-a}$$

Since Slope of $XY$ = Slope of $YZ = \frac{a-b}{c-a}$, the points are collinear.

**3. Find $k$ so that the line joining $A(7,3)$; $B(k,-6)$ and the**

line joining $C(-4, 5)$ ; $D(-6, 4)$ are:

(i) parallel (ii) perpendicular.

To find $k$ such that the line joining points $A(7,3)$ and $B(k,-6)$ is parallel or perpendicular to the line joining points $C(-4,5)$ a D(−6,4), we will use the concepts of **slope**.

**Step 1:** **Find the slope of the line joining points $C(-4,5)$ and $D(-6,4)$**

The slope $m_{CD}$ of the line joining points $C(x_1,y_1)$ and $D(x_2,y_2)$ is given by:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Substitute the coordinates of $C(-4,5)$ and $D(-6,4)$:

$$m_{CD}=\frac{4-5}{-6-(-4)}=\frac{-1}{-2}=\frac{1}{2}$$

**(i) To make the lines parallel**

For two lines to be **parallel**, their slopes must be equal. The slope of the line joining $A(7,3)$ and $B(k,-6)$ is:

$$m_{AB}=\frac{-6-3}{k-7}=\frac{-9}{k-7}$$

For the lines to be parallel, we set $m_{AB}=m_{CD}$:

$$\frac{-9}{k-7}=\frac{1}{2}$$

Now, solve for $k$:

$$-9\times 2=1\times(k-7)$$

$$-18=k-7$$

$$k=-18+7=-11$$

So, for the lines to be parallel, $k=-11$.

**(ii) To make the lines perpendicular**

For two lines to be **perpendicular**, the product of their slopes must be $-1$. Therefore, we require:

$$m_{AB}\times m_{CD}=-1$$

Substitute $m_{CD} = \frac{1}{2}$ and $m_{AB} = \frac{-9}{k-7}$:

$$\frac{-9}{k-7} \times \frac{1}{2} = -1$$

ow, solve for $k$:

$$\frac{-9}{2(k-7)} = -1$$

Multiply both sides by $2(k-7)$ to eliminate the denominator:

$$-9 = -2(k-7)$$

Simplify:

$$-9 = -2k + 14$$

Solve for $k$:.

$$-9 - 14 = -2k$$

$$-23 = -2k$$

$$k = \frac{23}{2}$$

So, for the lines to be perpendicular, $k = \frac{23}{2}$.

**Final Answer:**

- For the lines to be **parallel**, $k = -11$.
- For the lines to be **perpendicular**, $k = \frac{23}{2}$.

4. **Using slopes, show that the triangle with its vertices $A(6, 1)$, $B(2, 7)$ and $C(-6, -7)$ is a right triangle.**

To show that the triangle with vertices $A(6,1)$, $B(2,7)$, and $C(-6,-7)$ is a right triangle using slopes, we need to check if two of the sides of the triangle are **perpendicular**. Two lines are perpendicular if the product of their slopes is equal to $-1$.

**Step 1:** **Find the slopes of the sides of the triangle.**

SLOPE OF SIDE $AB$ (BETWEEN POINTS $A(6,1)$ AND $B(2,7)$):

The formula for the slope of a line joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For points $A(6,1)$ and $B(2,7)$:

$$m_{AB} = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

SLOPE OF SIDE $BC$ (BETWEEN POINTS $B(2,7)$ AND $C(-6,-7)$):

For points $B(2,7)$ and $C(-6,-7)$:

$$m_{BC} = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

SLOPE OF SIDE $AC$ (BETWEEN POINTS $A(6,1)$ AND $C(-6,-7)$):

For points $A(6,1)$ and $C(-6,-7)$:

$$m_{AC} = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$

**Step 2:** **Check if any two sides are perpendicular.**

Two lines are perpendicular if the product of their slopes is $-1$.

We check the slopes of different pairs of sides:

CHECKING $AB$ AND $BC$:

$$m_{AB} \times m_{BC} = \left(-\frac{3}{2}\right) \times \frac{7}{4} = -\frac{21}{8}$$

Since this is not equal to $-1$, the lines $AB$ and $BC$ are not perpendicular.

CHECKING $AB$ AND $AC$:

$$m_{AB} \times m_{AC} = \left(-\frac{3}{2}\right) \times \frac{2}{3} = -1$$

Since this is equal to $-1$, the lines $AB$ and $AC$ are **perpendicular**.

**Conclusion:**

Since the lines $AB$ and $AC$ are perpendicular, the triangle with vertices $A(6,1)$, $B(2,7)$, and $C(-6,-7)$ is a right triangle, with the right angle at vertex $A$.

**5. Two pairs of points are given. Find whether the two lines determined by these points are:**

**(i) parallel (ii) perpendicular (iii) none.**

**(a) (1, –2), (2, 4) and (4, 1), (–8, 2)**

**(b) (–3, 4 ), (6, 2) and (4, 5), (–2, –7)**

To determine whether the two lines determined by the given pairs of points are **parallel**, **perpendicular**, or **neither**, we will calculate the slopes of the lines and compare them.

**Formula for slope:**

The slope $m$ of the line joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**(a) Points:** $(1, -2), (2, 4)$ **and** $(4, 1), (-8, 2)$

SLOPE OF LINE 1 (BETWEEN $(1, -2)$ AND $(2,4)$):

$$m_1 = \frac{4 - (-2)}{2 - 1} = \frac{4 + 2}{1} = \frac{6}{1} = 6$$

SLOPE OF LINE 2 (BETWEEN $(4,1)$ AND $(-8,2)$):

$$m_2 = \frac{2 - 1}{-8 - 4} = \frac{1}{-12} = -\frac{1}{12}$$

CHECK FOR PARALLELISM:

For the lines to be parallel, their slopes must be equal. Since $m_1 = 6$ and $m_2 = -\frac{1}{12}$, the lines are **not parallel**.

CHECK FOR PERPENDICULARITY:

For the lines to be perpendicular, the product of their slopes must be $-1$:

$$m_1 \times m_2 = 6 \times -\frac{1}{12} = -\frac{6}{12} = -\frac{1}{2}$$

Since the product is not $-1$, the lines are **not perpendicular**.

**Conclusion for (a): The lines are neither parallel nor perpendicular.**

**(b) Points: $(-3, 4), (6, 2)$ and $(4, 5), (-2, -7)$**

SLOPE OF LINE 1 (BETWEEN $(-3,4)$ AND $(6,2)$):

$$m_1 = \frac{2-4}{6-(-3)} = \frac{-2}{6+3} = \frac{-2}{9}$$

SLOPE OF LINE 2 (BETWEEN $(4,5)$ AND $(-2,-7)$):

$$m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

CHECK FOR PARALLELISM:

For the lines to be parallel, their slopes must be equal. Since $m_1 = \frac{-2}{9}$ and $m_2 = 2$, the lines are **not parallel.**

CHECK FOR PERPENDICULARITY:

For the lines to be perpendicular, the product of their slopes must be $-1$:

$$m_1 \times m_2 = \frac{-2}{9} \times 2 = \frac{-4}{9}$$

Since the product is not $-1$, the lines are **not perpendicular**.

**Conclusion for (b): The lines are neither parallel nor perpendicular.**

**Final Answer:**

- For (a): The lines are **neither parallel nor perpendicular**.
- For (b): The lines are **neither parallel nor perpendicular**.

6. **Find an equation of:**
   (a) **the horizontal line through (7, –9)**
   (b) **the vertical line through (–5, 3)**

(c) through A(−6, 5) having slope 7

(d) through (8, −3) having slope 0

(e) through (−8, 5) having slope undefined

(f) through (−5, −3) and (9, −1)

(g) *y*-intercept: −7 and slope: −5

(h) *x*-intercept: −3 and *y*-intercept: 4

(i) *x*-intercept: −9 and slope: −4

To find the equations of lines, we use the general forms:

- **Horizontal line**: $y = c$, where $c$ is the $y$-coordinate.
- **Vertical line**: $x = c$, where $c$ is the $x$-coordinate.
- **Line with slope $m$ through point** $(x_1, y_1)$: Use the **point-slope form**:

$$y - y_1 = m(x - x_1)$$

- **Line through two points** $(x_1, y_1)$ **and** $(x_2, y_2)$: First calculate the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, then use the point-slope form.

**(a) Horizontal line through $(7, -9)$:**

A horizontal line has constant $y$-value:

$$y = -9$$

**(b) Vertical line through $(-5, 3)$:**

A vertical line has constant $x$-value:

$$x = -5$$

**(c) Through $A(-6, 5)$ having slope 7:**

Using point-slope form:

$$y - 5 = 7(x + 6)$$

Simplify:

$$y = 7x + 42 + 5 \Rightarrow y = 7x + 47$$

**(d) Through $(8, -3)$ having slope 0:**

A slope of 0 indicates a horizontal line:

$$y = -3$$

**(e) Through $(-8, 5)$ having undefined slope:**

An undefined slope indicates a vertical line:

$$x = -8$$

**(f) Through $(-5, -3)$ and $(9, -1)$:**

First, find the slope:

$$m = \frac{-1-(-3)}{9-(-5)} = \frac{-1+3}{9+5} = \frac{2}{14} = \frac{1}{7}$$

Now, use the point-slope form with point $(-5, -3)$:

$$y-(-3) = \frac{1}{7}(x-(-5))$$

Simplify:

$$y + 3 = \frac{1}{7}(x+5)$$

$$= \frac{1}{7}x + \frac{5}{7} - 3 \Rightarrow y = \frac{1}{7}x - \frac{16}{7}$$

**(g) $y$-intercept: $-7$, slope: $-5$:**

Use the slope-intercept form $y = mx + b$:

$$y = -5x - 7$$

**(h) $x$-intercept: $-3$, $y$-intercept: 4:**

Using the two intercepts, we can find the slope:

$$m = \frac{4-0}{0-(-3)} = \frac{4}{3}$$

Using slope-intercept form:

$$y = \frac{4}{3}x + 4$$

**(i) $x$-intercept: $-9$, slope: $-4$:**

At the $x$-intercept, $y = 0$, so the point is $(-9,0)$. Use the point-slope form:

$$y - 0 = -4(x+9)$$

Simplify: $$y = -4x - 36$$

**7. Find an equation of the perpendicular bisector of the segment joining the points $A(3, 5)$ and $B(9, 8)$.**

To find the equation of the perpendicular bisector of the segment joining points $A(3,5)$ and $B(9,8)$, follow these steps:

**Step 1: Find the midpoint of the segment $AB$**

The **midpoint** $M$ of the segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $A(3,5)$ and $B(9,8)$ into the formula:

$$M = \left(\frac{3+9}{2}, \frac{5+8}{2}\right) = \left(\frac{12}{2}, \frac{13}{2}\right) = (6, 6.5)$$

**Step 2: Find the slope of the line $AB$**

The **slope** $m_{AB}$ of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the coordinates of $A(3,5)$ and $B(9,8)$:

$$m_{AB} = \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$$

**Step 3: Find the slope of the perpendicular bisector**

The **slope** of the perpendicular bisector is the negative reciprocal of the slope of $AB$. Since the slope of $AB$ is $\frac{1}{2}$, the slope of the perpendicular bisector is:

$$m_{\text{perp}} = -\frac{1}{\frac{1}{2}} = -2$$

**Step 4: Use the point-slope form to find the equation of the perpendicular bisector**

The **point-slope form** of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

Where $m$ is the slope, and $(x_1, y_1)$ is the point through which the line passes.

Substitute $m = -2$ and the midpoint $M(6,6.5)$ into the point-slope form:

$$y - 6.5 = -2(x - 6)$$

Simplify the equation:

$$y - 6.5 = -2x + 12$$
$$y = -2x + 12 + 6.5$$
$$y = -2x + 18.5$$

**Final Answer:**

The equation of the perpendicular bisector is:

$$y = -2x + 18.5$$

**8. Find an equation of the line through (–4, –6) and perpendicular to a line having slope $\frac{-3}{2}$.**

To find the equation of the line through $(-4, -6)$ and perpendicular to a line with slope $m = \frac{-3}{2}$, follow these steps:

Step 1: Slope of the perpendicular line

The slope of a line perpendicular to a given line is the negative reciprocal of the original slope.

$$m_{\text{perpendicular}} = -\frac{1}{m} = -\frac{1}{\frac{-3}{2}} = \frac{2}{3}$$

Step 2: Equation of the line

Use the **point-slope form** of a line:

$$y - y_1 = m(x - x_1)$$

Here, $(x_1, y_1) = (-4, -6)$ and $m = \frac{2}{3}$.

Substitute these values:

$$y - (-6) = \frac{2}{3}(x - (-4))$$

$$y + 6 = \frac{2}{3}(x + 4)$$

Step 3: Simplify the equation

Distribute $\frac{2}{3}$:

$$y + 6 = \frac{2}{3}x + \frac{8}{3}$$

Subtract 6 from both sides:

$$y = \frac{2}{3}x + \frac{8}{3} - 6$$

Express 6 as a fraction with a denominator of 3:

$$y = \frac{2}{3}x + \frac{8}{3} - \frac{18}{3}$$

Combine the constants:

$$y = \frac{2}{3}x - \frac{10}{3}$$

**9. Find an equation of the line through (11, -5) and parallel to a line with slope -24.**

To find the equation of the line through the point $(11, -5)$ and parallel to a line with slope $-24$, follow these steps:

**Step 1: Find the slope of the parallel line**

Since the lines are parallel, they have the same slope. Therefore, the slope of the new line is the same as the given slope, which is $-24$.

**Step 2: Use the point-slope form of the equation**

The **point-slope form** of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

where $m$ is the slope and $(x_1, y_1)$ is a point on the line.

Substitute $m = -24$ and the point $(11, -5)$ into the point-slope form:

$$y - (-5) = -24(x - 11)$$

Simplify:

$$y + 5 = -24(x - 11)$$

Now, expand the right side:

$$y + 5 = -24x + 264$$

Subtract 5 from both sides to solve for $y$:

$$y = -24x + 264 - 5$$

$$y = -24x + 259$$

**Final Answer:**

The equation of the line through $(11, -5)$ and parallel to a line with slope $-24$ is:

$$y = -24x + 259$$

**10. Convert each of the following equations into slope intercept form, two intercept form and normal form:**

(a) $2x - 4y + 11 = 0$ (b) $4x + 7y - 2 = 0$

(c) $15y - 8x + 3 = 0$

To convert the given equations into **slope-intercept form**, **two-intercept form**, and **normal form**, follow these steps for each equation:

1. Forms of the Equation:

- **Slope-Intercept Form:**

$$y = mx + c$$

Rearrange to solve for $y$.

- **Two-Intercept Form:**

$$\frac{x}{a} + \frac{y}{b} = 1$$

Divide by the constant to express the intercepts.

- **Normal Form:**

$$x\cos\theta + y\sin\theta = p$$

Normalize the equation so that the coefficient of $x^2 + y^2$ is 1.

**(a) Equation: $2x - 4y + 11 = 0$**

(i) Slope-Intercept Form:

$$2x - 4y + 11 = 0 \Rightarrow -4y = -2x - 11 \Rightarrow y = \frac{1}{2}x + \frac{11}{4}$$

**(ii) Two-Intercept Form:**

Divide the equation by 11 to express it in intercept form:

$$\frac{2x}{11} - \frac{4y}{11} = 1 \Rightarrow \frac{x}{\frac{11}{2}} - \frac{y}{\frac{11}{4}} = 1$$

Rewriting:

$$\frac{x}{\frac{11}{2}} + \frac{y}{-\frac{11}{4}} = 1$$

(iii) Normal Form:

Rewrite the equation in standard form $Ax + By + C = 0$, where $A, B,$ and $C$ are divided by $\sqrt{A^2 + B^2}$:

$$\sqrt{2^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

Normalize:

$$\frac{2x - 4y + 11}{2\sqrt{5}} = 0 \Rightarrow \frac{x}{\sqrt{5}} - \frac{2y}{\sqrt{5}} + \frac{11}{2\sqrt{5}} = 0$$

**(b) Equation: $4x + 7y - 2 = 0$**

(i) Slope-Intercept Form:

$$4x + 7y - 2 = 0 \Rightarrow 7y = -4x + 2 \Rightarrow y = -\frac{4}{7}x + \frac{2}{7}$$

(ii) Two-Intercept Form:

Divide by 2 to express intercepts:

$$\frac{4x}{2} + \frac{7y}{2} = 1 \Rightarrow \frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$$

(iii) Normal Form:

$$\sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$$

Normalize:

$$\frac{4x + 7y - 2}{\sqrt{65}} = 0 \Rightarrow \frac{4x}{\sqrt{65}} + \frac{7y}{\sqrt{65}} - \frac{2}{\sqrt{65}} = 0$$

**(c) Equation: $15y - 8x + 3 = 0$**

(i) Slope-Intercept Form:

$$15y - 8x + 3 = 0 \Rightarrow 15y = 8x - 3 \Rightarrow y = \frac{8}{15}x - \frac{1}{5}$$

(ii) Two-Intercept Form:

Divide by 3: $\frac{-8x}{3} + \frac{15}{1y}$

**11. In each of the following check whether the two lines are**

**(i) parallel (ii) perpendicular**

**(iii) neither parallel nor perpendicular**

**(a) $2x + y - 3 = 0$ ; $4x + 2y + 5 = 0$**

**(b) $3y = 2x + 5$ ; $3x + 2y - 8 = 0$**

**(c) $4y + 2x - 1 = 0$ ; $4x + 2y + 5 = 0$**

***Solution:*** To determine whether the given lines are **parallel**, **perpendicular**, or **neither**, we compare their slopes.

**Key Steps:**

- Rewrite each line in **slope-intercept form** $y = mx + c$ to identify the slope $m$.
- Check:
  - **Parallel lines**: Slopes are equal $m_1 = m_2$.
  - **Perpendicular lines**: Slopes are negative reciprocals $m_1 \cdot m_2 = -1$.
  - **Neither**: If neither condition is satisfied.

**(a) Lines:**

- $2x + y - 3 = 0$
  Rewrite in slope-intercept form:

$$y = -2x + 3 \quad \text{(slope: } m_1 = -2\text{)}.$$

- $4x + 2y + 5 = 0$
  Rewrite in slope-intercept form:

$$2y = -4x - 5 \Rightarrow y = -2x - \frac{5}{2} \quad (\text{slope}: m_2 = -2).$$

**Comparison:**
Since $m_1 = m_2 = -2$, the lines are **parallel**.

**(b)** **Lines:**

- $3y = 2x + 5$
  Rewrite in slope-intercept form:

$$y = \frac{2}{3}x + \frac{5}{3} \quad (\text{slope: } m_1 = \frac{2}{3}).$$

- $3x + 2y - 8 = 0$
  Rewrite in slope-intercept form:

$$2y = -3x + 8 \Rightarrow y = -\frac{3}{2}x + 4 \quad \left(\text{slope}: m_2 = -\frac{3}{2}\right)$$

**Comparison:**
Since $m_1 \cdot m_2 = \frac{2}{3} \cdot -\frac{3}{2} = -1$, the lines are **perpendicular**.

**(c)** **Lines:**

- $4y + 2x - 1 = 0$
  Rewrite in slope-intercept form:

$$4y = -2x + 1 \Rightarrow y = -\frac{1}{2}x + \frac{1}{4} \quad \left(\text{slope}: m_1 = -\frac{1}{2}\right)$$

- $4x + 2y + 5 = 0$
  Rewrite in slope-intercept form:

$$2y = -4x - 5 \Rightarrow y = -2x - \frac{5}{2} \quad (\text{slope}: m_2 = -2)$$

**Comparison:**

Since $m_1 \neq m_2$ and $m_1 \cdot m_2 = -\frac{1}{2} \cdot -2 = 1 \neq -1$, the lines are **neither parallel nor perpendicular**.

**12. Find an equation of the line through (–4, 7) and parallel to the line 2x – 7y + 4 = 0.**

To find the equation of the line through $(-4,7)$ and parallel to the line $2x - 7y + 4 = 0$, follow these steps:

Step 1: Find the slope of the given line

Rewrite the given equation $2x - 7y + 4 = 0$ in slope-intercept form $y = mx + c$, where $m$ is the slope.

$$2x-7y+4=0 \Rightarrow -7y=-2x-4 \Rightarrow y=\frac{2}{7}x+\frac{4}{7}$$

Thus, the slope of the given line is $m = \frac{2}{7}$.

Step 2: Use the same slope for the parallel line

Since parallel lines have the same slope, the line through $(-4,7)$ will also have a slope of $m = \frac{2}{7}$.

Step 3: Use the point-slope form to find the equation of the line

The point-slope form of the equation is:

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1) = (-4,7)$ and $m = \frac{2}{7}$.

Substitute these values:

$$y - 7 = \frac{2}{7}(x + 4)$$

Step 4: Simplify the equation

Distribute the slope $\frac{2}{7}$:

$$y - 7 = \frac{2}{7}x + \frac{8}{7}$$

Add 7 to both sides:

$$y = \frac{2}{7}x + \frac{8}{7} + 7$$

Express 7 as $\frac{49}{7}$:

$$y = \frac{2}{7}x + \frac{8}{7} + \frac{49}{7}$$

Combine the constants:

$$y = \frac{2}{7}x + \frac{57}{7}$$

Final Answer:

The equation of the line through $(-4,7)$ and parallel to the line $2x - 7y + 4 = 0$ is:

$$y = \frac{2}{7}x + \frac{57}{7}$$

**13. Find an equation of the line through (5, –8) and perpendicular to the join of *A*(–15, 8), *B*(10, 7).**

***Solution:*** To find the equation of the line through $(5, -8)$ and perpendicular to the line joining points $A(-15,8)$ and $B(10,7)$, follow these steps:

Step 1: Find the slope of the line joining $A(-15,8)$ and $B(10,7)$

The formula for the slope of a line through two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $A(-15,8)$ and $B(10,7)$ into the formula:

$$m_{\text{AB}} = \frac{7-8}{10-(-15)} = \frac{-1}{25}$$

So, the slope of the line joining $A$ and $B$ is $m_{\text{AB}} = \frac{-1}{25}$.

Step 2: Find the slope of the perpendicular line

The slope of a line perpendicular to a given line is the negative reciprocal of the slope of the given line.
Since the slope of the line joining $A$ and $B$ is $m_{\text{AB}} = \frac{-1}{25}$, the slope of the perpendicular line is:

$$m_{\text{perpendicular}} = -\frac{1}{\frac{-1}{25}} = 25$$

Step 3: Use the point-slope form to find the equation of the line

The point-slope form of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1) = (5, -8)$ and $m = 25$.

Substitute these values into the point-slope form:

$$y - (-8) = 25(x - 5)$$

Simplify:

$$y + 8 = 25(x - 5)$$

Distribute the 25:

$$y + 8 = 25x - 125$$

Subtract 8 from both sides:

$$y = 25x - 125 - 8$$

$$y = 25x - 133$$

**Final Answer:**

The equation of the line through $(5, -8)$ and perpendicular to the line joining $A(-15,8)$ and $B(10,7)$ is:

$$y = 25x - 133$$