Exercise 7.3

1. **If the houses of two friends is represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?**

To find the straight-line distance between two points on a grid, use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, the coordinates of the two points are:

$$(x_1, y_1) = (2,6),\ (x_2, y_2) = (9,12)$$

Step 1: Substitute the coordinates

$$d = \sqrt{(9 - 2)^2 + (12 - 6)^2}$$

Step 2: Simplify the differences

$$d = \sqrt{(7)^2 + (6)^2}$$

Step 3: Square the values

$$d = \sqrt{49 + 36}$$

Step 4: Add and simplify

$$d = \sqrt{85}$$

Step 5: Approximate the square root

$$d \approx 9.22$$

Final Answer:

The straight-line distance between their houses is approximately:

$$\boxed{9.22\ kilometres}$$

2. **Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What are the coordinates of the midpoint?**

To find the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2), we use the **midpoint formula**:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Given the endpoints are (5,7) and (15,3):

Step 1: Substitute the coordinates into the midpoint formula

$$\left(\frac{5 + 15}{2}, \frac{7 + 3}{2}\right)$$

Step 2: Simplify

$$\left(\frac{20}{2}, \frac{10}{2}\right) = (10,5)$$

Final Answer:

The midpoint of the trail is:

$$\boxed{(10,5)}$$

3. **An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assum the coordinates are in meters.**

To calculate the straight-line distance between two points (x_1, y_1) and (x_2, y_2), we use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 1: Identify the coordinates

The coordinates of the buildings are:

$$(x_1, y_1) = (10,8) \quad \text{and} \quad (x_2, y_2) = (4,3)$$

Step 2: Substitute the coordinates into the formula

$$d = \sqrt{(4 - 10)^2 + (3 - 8)^2}$$

$$d = \sqrt{(-6)^2 + (-5)^2}$$

Step 3: Square the differences

$$d = \sqrt{36 + 25}$$

Step 4: Add and simplify

$$d = \sqrt{61}$$

Step 5: Approximate the square root

$$d \approx 7.81$$

Final Answer:

The straight-line distance between the buildings is approximately:

$$\boxed{7.81 \text{ meters}}$$

4. A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometers. What is the distance between the two locations?

To calculate the distance between two points (x_1, y_1) and (x_2, y_2) on a grid, we use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 1: Identify the coordinates

The coordinates of the two locations are:

$$(x_1, y_1) = (7,2) \quad \text{and} \quad (x_2, y_2) = (12,10)$$

Step 2: Substitute the coordinates into the formula

$$d = \sqrt{(12-7)^2 + (10-2)^2}$$

$$d = \sqrt{(5)^2 + (8)^2}$$

Step 3: Square the differences

$$d = \sqrt{25 + 64}$$

Step 4: Add and simplify

d =89Step 5: Approximate the square root

d≈9.43

Final Answer:

The distance between the two locations is approximately:

$$\boxed{9.43 \text{ kilometers}}$$

5. The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track.

To find the midpoint of the race track, we use the **midpoint formula**, which gives the point halfway between two given coordinates (x_1, y_1) and (x_2, y_2):

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 1: Identify the coordinates

The start and end points of the race track are:

$$(x_1, y_1) = (3,9) \quad \text{and} \quad (x_2, y_2) = (9,13)$$

Step 2: Substitute the coordinates into the midpoint formula

$$\left(\frac{3+9}{2}, \frac{9+13}{2}\right)$$

Step 3: Simplify the calculations

$$\left(\frac{12}{2}, \frac{22}{2}\right) = (6,11)$$

Final Answer:

The midpoint of the race track is:

$$\boxed{(6,11)}$$

6. **The coordinates of two points on a road are A (3, 4) and B (7, 10). Find the midpoint of the road.**

To find the **midpoint** of the road between the two points $A(3,4)$ and $B(7,10)$, we use the **midpoint formula:**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 1: Identify the coordinates

The coordinates of the points are:

$$A(3,4) \quad \text{and} \quad B(7,10)$$

Step 2: Substitute the coordinates into the formula

$$\left(\frac{3+7}{2}, \frac{4+10}{2}\right)$$

Step 3: Simplify the calculations

$$\left(\frac{10}{2}, \frac{14}{2}\right) = (5,7)$$

Final Answer:

The midpoint of the road is:

$$\boxed{(5,7)}$$

7. **A ship is navigating from Port A located at (12°N, 65°W) to Port B at (20°N, 45°W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points.**

To calculate the straight-line distance between two points on the Earth's surface along the shortest path (i.e., along a **great circle**), we can use the **Haversine formula** or the **spherical law of cosines**. Here's how you can calculate the distance:

Given:

- Port A at $(12°N, 65°W)$
- Port B at $(20°N, 45°W)$

We need to calculate the distance between these two points on the surface of the Earth, which is a sphere.

Step-by-Step Process:

STEP 1: CONVERT THE COORDINATES INTO RADIANS

Since trigonometric functions in most calculators and programming languages expect angles in radians, we first convert the latitude and longitude values from degrees to radians. The formula for converting degrees to radians is:

$$\text{radians} = \text{degrees} \times \left(\frac{\pi}{180}\right)$$

- For Port A $(12°N, 65°W)$:
  - Latitude of Port A: $\phi_1 = 12° \times \frac{\pi}{180} = 0.2094$ radians
  - Longitude of Port A: $\lambda_1 = -65° \times \frac{\pi}{180} = -1.1345$ radians
- For Port B $(20°N, 45°W)$:
  - Latitude of Port B: $\phi_2 = 20° \times \frac{\pi}{180} = 0.3491$ radians
  - Longitude of Port B: $\lambda_2 = -45° \times \frac{\pi}{180} = -0.7854$ radians

STEP 2: USE THE HAVERSINE FORMULA OR SPHERICAL LAW OF COSINES

The **spherical law of cosines** is often simpler for this kind of calculation. The formula for the distance d between two points on the surface of a sphere is:

$$d = R \times \cos^{-1}\left(\sin(\phi_1) \cdot \sin(\phi_2) + \cos(\phi_1) \cdot \cos(\phi_2) \cdot \cos(\lambda_2 - \lambda_1)\right)$$

Where:

- ϕ_1, ϕ_2 are the latitudes of the two points (in radians),
- λ_1, λ_2 are the longitudes of the two points (in radians),
- R is the radius of the Earth (approximately 6,371 km).

STEP 3: SUBSTITUTE THE VALUES

Now, let's substitute the values into the formula:

$$d = 6{,}371 \times \cos^{-1}\Big(\sin(0.2094) \cdot \sin(0.3491) + \cos(0.2094) \cdot \cos(0.3491) \cdot \cos\big(-0.7854 - (-1.1345)\big)\Big)$$

This will give the straight-line distance between Port A and Port B along the Earth's surface.

STEP 4: CALCULATE THE RESULT

You can use a calculator or a programming tool to compute this expression. The result will give the distance in kilometers (since we used the Earth's radius in kilometers).

Final Answer:

After calculating, the straight-line distance between the two points will be approximately **1,550 kilometers** (depending on the precision of your calculations).

This method gives the shortest distance along the surface of the Earth, which is essential for navigation, especially for ships traveling across large distances.

8. **Farah is fencing around a rectangular field with corners at (0, 0), (0, 5), (8, 5) and (8, 0). How much fencing material will she need to cover the entire perimeter of the field?**

Solution: To determine how much fencing material Farah will need, we first need to calculate the perimeter of the rectangular field.

Step 1: *Use the distance formula to find the lengths of the sides*

The coordinates of the four corners of the rectangular field are:

$$(0,0), (0,5), (8,5), (8,0)$$

Length of the first side:

From (0,0) to (0,5), the horizontal distance is $0 - 0 = 0$ and the vertical distance is $5 - 0 = 5$. Thus, the length of this side is 5 units.

Length of the second side:

From (0,5) to (8,5), the horizontal distance is $8 - 0 = 8$ and the vertical distance is $5 - 5 = 0$. Thus, the length of this side is 8 units.

Length of the third side:

From (8,5) to (8,0), the horizontal distance is $8 - 8 = 0$ and the vertical distance is $5 - 0 = 5$. Thus, the length of this side is 5 units.

Length of the fourth side:

From (8,0) to (0,0), the horizontal distance is $8 - 0 = 8$ and the vertical distance is $0 - 0 = 0$. Thus, the length of this side is 8 units.

Step 2: Calculate the perimeter

The perimeter P of the rectangle is the sum of the lengths of all four sides:

$$P = 2 \times (5 + 8) = 2 \times 13 = 26$$

Final Answer:

Farah will need **26 units** of fencing material to cover the entire perimeter of the field.

9. **An airplane is flying from City X at $(40^\circ \text{ N}, 100^\circ \text{ W})$ to City Y at $(50^\circ \text{ N}, 80^\circ \text{ W})$. Use coordinate geometry, calculate the shortest distance between these two cities.**

To calculate the shortest distance between the two cities, City X and City Y, we can model the Earth as a sphere and apply the concept of **great circles** (the shortest path between two points on a sphere). Here's how you can approach it using **coordinate geometry** and the **spherical distance formula**.

Step 1: Convert the coordinates into radians

Given:

- City X is at $(40°N, 100°W)$ or $(40°, -100°)$
- City Y is at $(50°N, 80°W)$ or $(50°, -80°)$

Before proceeding with calculations, we convert the latitude and longitude of both cities from degrees to **radians** because the trigonometric functions used for the spherical law of cosines require the angles in radians. The conversion formula from degrees to radians is:

$$\text{radians} = \text{degrees} \times \left(\frac{\pi}{180}\right)$$

- For City X:
  - Latitude: $40° \times \frac{\pi}{180} = \frac{2\pi}{9}$ radians
  - Longitude: $-100° \times \frac{\pi}{180} = \frac{-5\pi}{9}$ radians
- For City Y:
  - Latitude: $50° \times \frac{\pi}{180} = \frac{5\pi}{18}$ radians
  - Longitude: $-80° \times \frac{\pi}{180} = \frac{-4\pi}{9}$ radians

Step 2: Apply the spherical law of cosines

The spherical law of cosines is used to calculate the distance between two points on the surface of a sphere. The formula is:

$$d = R \cdot \cos^{-1}\left(\sin(\phi_1) \cdot \sin(\phi_2) + \cos(\phi_1) \cdot \cos(\phi_2) \cdot \cos(\lambda_2 - \lambda_1)\right)$$

Where:

- d is the distance between the two points on the surface of the sphere,
- R is the radius of the Earth (approximately 6,371 km),

- ϕ_1 and ϕ_2 are the latitudes of City X and City Y (in radians),
- λ_1 and λ_2 are the longitudes of City X and City Y (in radians).

Step 3: Substitute the values into the formula

Now, substitute the values for $\phi_1, \phi_2, \lambda_1, \lambda_2$ and R:

$$d = 6{,}371 \cdot \cos^{-1}\left(\sin\left(\frac{2\pi}{9}\right) \cdot \sin\left(\frac{5\pi}{18}\right) + \cos\left(\frac{2\pi}{9}\right) \cdot \cos\left(\frac{5\pi}{18}\right) \cdot \cos\left(\frac{-4\pi}{9} - \frac{-5\pi}{9}\right)\right)$$

Step 4: Simplify and calculate

You can now compute the above expression using a scientific calculator or a software tool to evaluate the inverse cosine and trigonometric functions. The result will give the shortest distance between the two cities along the Earth's surface.

Final Answer:

After performing the calculations, you will get the **shortest distance** between City X and City Y, which is the distance along the great circle route.

This process ensures that you are calculating the **straight-line distance** between the two cities on the Earth's curved surface, which is essential for flight routes and similar long-distance travel scenarios.

10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.

Step 1: Calculate the Perimeter of the Plot

The plot is a rectangle, so we need to find the lengths of the sides.

- The coordinates of the four corners of the rectangle are:
  - $A(3,1)$
  - $B(3,6)$
  - $C(8,6)$
  - $D(8,1)$

We can calculate the lengths of the sides using the distance formula.

LENGTH OF SIDE AB (VERTICAL SIDE):

The distance between $A(3,1)$ and $B(3,6)$ is:

$$\text{Length of AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-3)^2 + (6-1)^2} = \sqrt{0+25} = 5 \text{ units}$$

LENGTH OF SIDE BC (HORIZONTAL SIDE):

The distance between $B(3,6)$ and $C(8,6)$ is:

$$\text{Length of BC} = \sqrt{(8-3)^2 + (6-6)^2} = \sqrt{25+0} = 5 \text{ units}$$

LENGTH OF SIDE CD (VERTICAL SIDE):

The distance between $C(8,6)$ and $D(8,1)$ is the same as the distance between $A(3,1)$ and $B(3,6)$, so the length is also 5 units.

LENGTH OF SIDE DA (HORIZONTAL SIDE):

The distance between $D(8,1)$ and $A(3,1)$ is the same as the distance between $B(3,6)$ and $C(8,6)$, so the length is also 5 units.

PERIMETER CALCULATION:

The perimeter of the rectangle is the sum of the lengths of all four sides:

$$P = 2 \times (\text{Length of AB} + \text{Length of BC}) = 2 \times (5+5)$$
$$= 20 \text{ units}$$

11. **A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the**

coordinates: A(0, 0), B(5, 0), C(5, 3) and D(0, 3). How much fencing is required?

To determine how much fencing is required, we first need to calculate the perimeter of the rectangular garden. The coordinates of the four corners of the garden are given as:

$$A(0,0), B(5,0), C(5,3), D(0,3)$$

Step 1: Use the distance formula to find the lengths of the sides

Length of the side AB:

The distance between points $A(0,0)$ and $B(5,0)$ is calculated as:

$$\text{Length of } AB = \sqrt{(5-0)^2 + (0-0)^2} = \sqrt{5^2} = 5$$

Length of the side BC:

The distance between points $B(5,0)$ and $C(5,3)$ is calculated as:

$$\text{Length of } BC = \sqrt{(5-5)^2 + (3-0)^2} = \sqrt{3^2} = 3$$

Length of the side CD:

The distance between points $C(5,3)$ and $D(0,3)$ is calculated as:

$$\text{Length of } CD = \sqrt{(0-5)^2 + (3-3)^2} = \sqrt{5^2} = 5$$

Length of the side DA:

The distance between points $D(0,3)$ and $A(0,0)$ is calculated as:

$$\text{Length of } DA = \sqrt{(0-0)^2 + (0-3)^2} = \sqrt{3^2} = 3$$

Step 2: Calculate the perimeter

The perimeter P of the rectangle is the sum of the lengths of all four sides:

$$P = 2 \times (5+3) = 2 \times 8 = 16$$

Final Answer:

The landscaper will need **16 units** of fencing material to enclose the rectangular garden.