Exercise 9.3

1. The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Sol: $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = k^3, \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

or $\frac{V_1}{V_2} = k^3$

$\frac{r_1}{r_2} = \frac{3}{4}$

$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

$\therefore \quad V_1 : V_2 = 27 : 64$

2. Two regular tetrahedrons have volumes in the ratio 8:27. What is the ratio of their sides?

Sol: $V_1 : V_2 = 8 : 27$

$$\frac{V_1}{V_2} = \frac{8}{27}$$

If l_1 and l_2 are the sides of tetrahedrons then

$$\left(\frac{l_1}{l_2}\right)^3 = \frac{8}{27} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{2}{3}$$

$l1 : l_2 = 2 : 3.$

3. Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(i) their heights (ii) their base areas?

Sol: $V_1 : V_2 = 64 : 125$

Volume of right circular cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.

$$\text{Ratio of height to } = h = \frac{3}{\pi r^2}$$

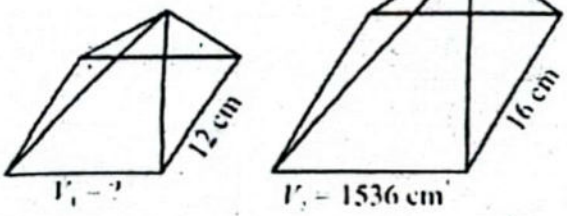
$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

(i) $$\text{or } \frac{h_1}{h_2} = \left(\frac{V_1}{V_2}\right)^{1/3} = \left(\frac{64}{125}\right)^{1/3} = \frac{4}{5}$$

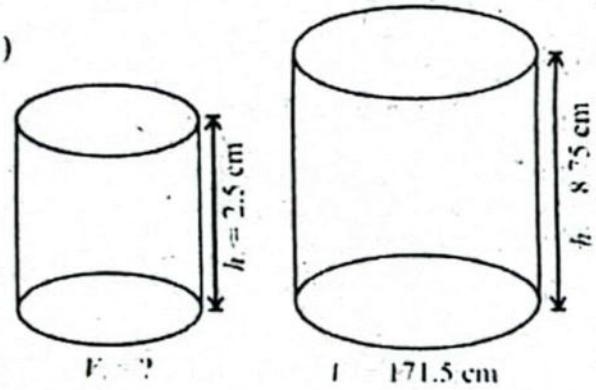
$$\frac{A_1}{A_2} = = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

4. Find the missing value in the following similar solids.

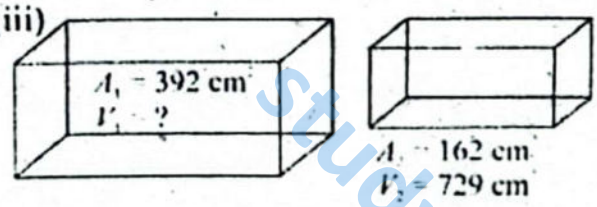
(i)



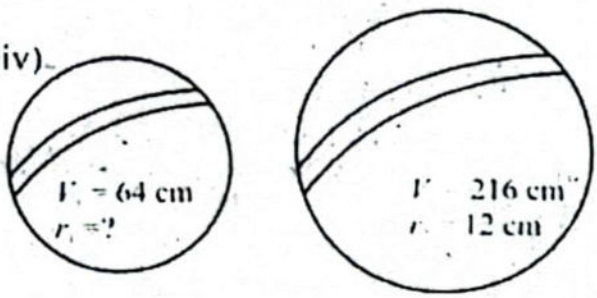
(ii)



(iii)



(iv)



Sol: (i) $V_1 = ?$ $l_1 = 12$

$V_2 = 1536$ $l_2 = 16$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$V_1 = \left(\frac{l_1}{l_2}\right)^3 \times V_2 = \left(\frac{12}{16}\right)^3 \times 1536$$

$$V_2 = \frac{(12)^3}{(16)^3} \times 1536 = \frac{1728 \times 1536}{4036} = 657.63$$

(ii) $l_1 = 2.5$ $l_2 = 8.75$

$V_1 = ?$ $V_2 = 171.5$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$V_1 = \left(\frac{l_1}{l_2}\right)^3 \times V_2 = \left(\frac{2.5}{8.75}\right)^3 \times 171.5$$

$$= \frac{15.625 \times 171.5}{669.922} = \frac{2679.6875}{669.922} = 4$$

(iii) $A_1 = 392$ $V_1 = ?$

$A_2 = 162$ $V_2 = 729$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \text{or} \quad \frac{l_1}{l_2} = \left(\frac{A_1}{A_2}\right)^{1/2}$$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{A_1}{A_2}\right)^{3/2} = \left(\frac{392}{162}\right)^{3/2} = 2$$

$$V_1 = \left(\frac{392}{162}\right)^{3/2} \times 729 = 2744 \quad \text{Ans.}$$

(iv) $V_1 = 64$ $r_1 = ?$

$V_2 = 216$ $r_2 = 12$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{216}$$

$$\frac{r_1}{r_2} = \left(\frac{64}{216}\right)^{1/3} = \frac{4}{6} = \frac{2}{3}$$

$$r_1 = \frac{2}{\cancel{3}} \times \cancel{12}^{4} = 8$$

5. The ratio of the corresponding lengths of two similar canonical cans is 3 : 2.

(i) The larger canonical can have surface area of 96 m^2.

Find the surface area of the smaller canonical can.

(ii) The smaller canonical can have a volume of 240 m^3.

Find the volume of larger canonical can.

Sol: (i) $l_1 : l_2 = 3 : 2$ $\quad \frac{l_1}{l_2} = \frac{3}{2}$

$A_1 = 90$; $\quad A_2 = ?$

$$\left(\frac{3}{2}\right)^2 = \frac{A_1}{A_2} = A_2 = A_1 \times \left(\frac{2}{3}\right)^2$$

$$A_2 = {}^{10}\cancel{90} \times \frac{4}{\cancel{9}} = 40$$

(ii) $V_2 = 250$, $V_1 = ?$

$$\frac{V_1}{V_2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8} = \frac{7 \times 250}{8}$$

$$V_1 = 843.75$$

6. The ratio of the heights of two similar cylindrical water tanks is 5 : 3.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.

(ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.

Sol: (i) $h : h = 5 : 3$ or $\frac{h_1}{h_2} = \frac{5}{3}$

$A_1 = 250$, $A_2 = ?$

$$\frac{A_1}{A_2} = \left(\frac{h_2}{h_1}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$A_2 = \frac{9}{25} \times 250 = 90$$

$V_2 = 270$ $V_1 = ?$

$$\frac{V_1}{V_2} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$$

$$V_1 = \frac{125}{\cancel{27}} \times {}^{10}\cancel{270} = 1250$$