Exercise 13.1

1. **Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?**

Solution: The sides of the dice are labeled: **L, M, N, O, P, U**.

Step 1: Identify the consonants and vowels

- **Consonants:** L, M, N, P
- **Vowels:** O, U

Step 2: Count the consonants and vowels

- Total consonants = 4 (L, M, N, P)
- Total sides on the dice = 6

Step 3: Calculate the probability

The probability of landing on a consonant is the ratio of the number of consonants to the total number of sides:

$$P(\text{consonant}) = \frac{\text{Number of consonants}}{\text{Total sides}} = \frac{4}{6} = \frac{2}{3}$$

Final Answer:

The probability that the dice lands on a consonant is $\frac{2}{3}$.

2. **Shazia throws a pair of fair dice. What will be the probability of getting:**
   (i) **sum of dots is at least 4.**
   (ii) **product of both dots is between 5 to 10.**
   (iii) **the difference between both the dots is equal to 4.**
   (iv) **number at least 5 on the first dice and the number at least 4 on the second dice.**

Solution: To solve these probability problems, we'll consider that Shazia is throwing **two fair six-sided dice**. Each die has faces numbered from 1 to 6, and the total number of outcomes is:

$$6 \times 6 = 36$$

Now, let's evaluate each part:

(i) The sum of dots is at least 4

The sum of dots S ranges from 2 (1+1) to 12 (6+6). For $S \geq 4$, we exclude $S = 2$ and $S = 3$.

- $S = 2$: $(1,1) \rightarrow 1$ outcome.
- $S = 3$: $(1,2), (2,1) \rightarrow 2$ outcomes.
- Total outcomes where $S < 4 = 1 + 2 = 3$.
- Favorable outcomes where $S \geq 4 = 36 - 3 = 33$.

Probability:

$$P(S \geq 4) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{33}{36} = \frac{11}{12}.$$

(ii) The product of both dots is between 5 and 10

We look for outcomes where $5 \leq P \leq 10$. Let the dots on the dice be (x, y), and the product is $P = x \cdot y$.

- $P = 5$: $(1,5), (5,1)$.
- $P = 6$: $(1,6), (2,3), (3,2), (6,1)$.
- $P = 8$: $(2,4), (4,2)$.
- $P = 9$: $(3,3)$.
- $P = 10$: $(2,5), (5,2)$.

Total favorable outcomes:

$$(1,5), (5,1), (1,6), (2,3), (3,2), (6,1), (2,4), (4,2), (3,3), (2,5), (5,2) \quad (11 \text{ outcomes}).$$

Probability:

$$P(5 \leq P \leq 10) = \frac{11}{36}.$$

(iii) The difference between both dots is equal to 4

Let the dots on the dice be (x, y), and the difference is $|x - y| = 4$.

- Possible pairs:
  - $x - y = 4$: $(5,1), (6,2)$.
  - $y - x = 4$: $(1,5), (2,6)$.

Total favorable outcomes:

$$(5,1), (6,2), (1,5), (2,6) \quad (4 \text{ outcomes}).$$

Probability:

$$P(|x - y| = 4) = \frac{4}{36} = \frac{1}{9}.$$

(iv) Number at least 5 on the first die and the number at least 4 on the second die

Let the dots on the dice be (x, y), where $x \geq 5$ and $y \geq 4$.

- For $x = 5$: $y = 4,5,6 \rightarrow 3$ outcomes.
- For $x = 6$: $y = 4,5,6 \rightarrow 3$ outcomes.

Total favorable outcomes:

$$3 + 3 = 6.$$

Probability:

$$P(x \geq 5 \text{ and } y \geq 4) = \frac{6}{36} = \frac{1}{6}.$$

3. One alphabet is selected at random from the word "MATHEMATICS". Find the probability of getting:

(i) vowel (ii) consonant (iii) an E

(iv) an A (v) not M (vi) not T

The word "**MATHEMATICS**" contains 11 letters. Let's break down the problem:

Step 1: Identify vowels and consonants

- Vowels: A, E, I → Total vowels = 3.
- Consonants: M, T, H, M, T, C, S → Total consonants = 8.

Step 2: Solve each part

(i) Probability of getting a vowel

The favorable outcomes are A, E, I (3 vowels). Total letters = 11.

$$P(\text{vowel}) = \frac{\text{Number of vowels}}{\text{Total letters}} = \frac{3}{11}.$$

(ii) Probability of getting a consonant

The favorable outcomes are M, T, H, M, T, C, S (8 consonants). Total letters = 11.

$$P(\text{consonant}) = \frac{\text{Number of consonants}}{\text{Total letters}} = \frac{8}{11}.$$

(iii) Probability of getting an E

The letter E appears **once** in the word.

$$P(\text{E}) = \frac{\text{Number of E's}}{\text{Total letters}} = \frac{1}{11}.$$

(iv) Probability of getting an A

The letter A appears **twice** in the word.

$$P(\text{A}) = \frac{\text{Number of A's}}{\text{Total letters}} = \frac{2}{11}.$$

(v) Probability of not getting an M

The letter M appears **twice** in the word, so the remaining letters = $11 - 2 = 9$.

$$P(\text{not M}) = \frac{\text{Number of non-M letters}}{\text{Total letters}} = \frac{9}{11}.$$

(vi) **Probability of not getting a T**

The letter T appears **twice** in the word, so the remaining letters = $11 - 2 = 9$.

$$P(\text{not T}) = \frac{\text{Number of non-T letters}}{\text{Total letters}} = \frac{9}{11}.$$

5. Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.

Aslam rolls a standard six-sided die, where each face is numbered 1,2,3,4,5,6.

Step 1: Total possible outcomes

The total number of outcomes when rolling a die is:

$$6$$

Step 2: Probability of getting the numbers 3 or 4

- Favorable outcomes: $\{3, 4\} \rightarrow$ **2 outcomes**.
- Total outcomes: 6.

The probability of getting 3 or 4 is:

$$P(3 \text{ or } 4) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}.$$

Step 3: Probability of not getting the numbers 3 or 4

If the outcomes 3 or 4 are excluded, the remaining outcomes are $\{1, 2, 5, 6\} \rightarrow$ **4 outcomes**.

The *probability* of not getting 3 or 4 is:

$$P(\text{not } 3 \text{ or } 4) = \frac{\text{Remaining outcomes}}{\text{Total outcomes}} = \frac{4}{6} = \frac{2}{3}.$$

Final Answer:

- Probability of getting 3 or 4: $\frac{1}{3}$.
- Probability of not getting 3 or 4: $\frac{2}{3}$.

6. Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:

(i) the number 25

(ii) number between 17 to 22

(iii) **number at least 20**
(iv) **number not 27 and 29**
(v) **number not between 12 to 15**

Abdul Hadi has **30 cards labeled from 1 to 30**, so the total number of outcomes is:

$$\text{Total outcomes} = 30$$

Let's calculate each part:

(i) Probability that the selected card contains the number 25

The number 25 appears on **1 card**.

$$P(\text{number } 25) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{30}.$$

(ii) Probability that the selected card contains a number between 17 and 22

The numbers between 17 and 22 are: **17, 18, 19, 20, 21, 22 → 6 cards.**

$$P(\text{number between 17 and 22}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{6}{30} = \frac{1}{5}.$$

(iii) Probability that the selected card contains a number at least 20

The numbers that are at least 20 are: **20, 21, 22, ..., 30 → 11 cards.**

$$P(\text{number at least 20}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{11}{30}.$$

(iv) Probability that the selected card is not 27 and not 29

The numbers 27 and 29 are excluded, so the remaining numbers are $30 - 2 = 28$ cards.

$$P(\text{not 27 and not 29}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{28}{30}$$

$$= \frac{14}{15}.$$

(v) Probability that the selected card is not between 12 and 15

The numbers between 12 and 15 are: **12, 13, 14, 15 → 4 cards.**

The remaining numbers are $30 - 4 = 26$ cards.

$$P(\text{not between 12 and 15}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{26}{30} = \frac{13}{15}.$$

7. **The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?**

Solution: The probability that Ayesha will **not pass** the examination is the complement of the probability that she will pass. The formula for the complement is:

$$P(\text{not passing}) = 1 - P(\text{passing})$$

Given:

$$P(\text{passing}) = 0.85$$

Calculation:

$$P(\text{not passing}) = 1 - 0.85 = 0.15$$

Final Answer:

The probability that Ayesha will not pass the examination is **0.15**.

8. **Tabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:**

(i) **tail on coin and at least 4 on dice.**

(ii) **head on coin and the number 2,3 on dice.**

(iii) **head and tail on coin and the number 6 on dice.**

(iv) **not tail on coin and the number 5 on dice.**

(v) **not head on coin and the number 5 and 2 on dice.**

Tabish tosses a fair coin and rolls a fair six-sided die, so we calculate probabilities based on the total outcomes and the favorable outcomes.

Step 1: Total possible outcomes

- Outcomes of the coin: {Head (H), Tail (T)} → 2 outcomes.
- Outcomes of the dice: {1, 2, 3, 4, 5, 6} → 6 outcomes.
- Total outcomes:

$$2 \times 6 = 12$$

Step 2: Solve each part

(i) Tail on the coin and at least 4 on the dice

- **Tail on the coin:** 1 outcome (T).
- **At least 4 on the dice:** Numbers 4, 5, 6 → 3 outcomes.
- Favorable outcomes:

$$(T,4),(T,5),(T,6) \quad \text{(3 outcomes).}$$

Probability:

$$P(\text{Tail and at least 4}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{12} = \frac{1}{4}.$$

(ii) Head on the coin and the number 2 or 3 on the dice

- **Head on the coin:** 1 outcome (H).
- **Number 2 or 3 on the dice:** Numbers 2, 3 → 2 outcomes.
- Favorable outcomes:

$$(H,2),(H,3) \quad \text{(2 outcomes).}$$

Probability:

$$P(\text{Head and number 2 or 3}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{12} = \frac{1}{6}.$$

(iii) Head and Tail on the coin and the number 6 on the dice

This event is **impossible**, as you cannot have both a head and a tail from one coin toss.

Probability:

$$P(\text{Head and Tail on coin and number 6}) = 0.$$

(iv) Not tail on the coin and the number 5 on the dice

- **Not tail on the coin:** This means **Head (H)** → 1 outcome.
- **Number 5 on the dice:** 1 outcome (5).
- Favorable outcome:

$$(H,5) \quad \text{(1 outcome).}$$

Probability:

$$P(\text{Not tail and number 5}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{12}.$$

(v) Not head on the coin and the number 5 and 2 on the dice

- **Not head on the coin:** This means **Tail (T)** → 1 outcome.

- **Number 5 and 2 on the dice:** This is impossible since a single dice roll can result in only one number.

Probability: $P(\text{Not head and number 5 and 2}) = 0$.

10. A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:

(i) a queen (ii) neither a queen nor a jack

A standard deck of 52 playing cards consists of 4 suits (hearts, diamonds, clubs, spades), with 13 cards in each suit. Each suit contains exactly one queen and one jack.

(i) Probability of selecting a queen

There are **4 queens** in the deck (one in each suit).

The total number of cards is 52.

Probability:

$$P(\text{queen}) = \frac{\text{Number of queens}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Probability of selecting neither a queen nor a jack

- Total cards: 52.
- Total queens: 4.
- Total jacks: 4.
- Cards that are either a queen or a jack: $4 + 4 = 8$.
- Cards that are neither a queen nor a jack: $52 - 8 = 44$.

Probability:

$$P(\text{neither queen nor jack}) = \frac{\text{Number of favorable outcomes}}{\text{Total cards}} = \frac{44}{52} = \frac{11}{13}.$$

11. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:

(i) a jack (ii) no diamond

(i) Probability of getting a jack

In a standard deck of 52 playing cards:

- There are **4 jacks** (one in each suit: hearts, diamonds, clubs, spades).
- Total number of cards = 52.

Probability: $P(\text{jack}) = \frac{\text{Number of jacks}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}$.

(ii) Probability of getting no diamond

In a standard deck of 52 playing cards:

- There are **13 diamonds** (one-quarter of the deck).
- Cards that are not diamonds = $52 - 13 = 39$.

Probability:

$$P(\text{no diamond}) = \frac{\text{Number of non-diamond cards}}{\text{Total cards}} = \frac{39}{52} = \frac{3}{4}.$$