Review Exercise 13

1. **Four options are given against each statement. Encircle the correct option.**

(i) Each element of the sample space is called:

(a) event (b) experiment

(c) sample point (d) outcomes

(ii) An outcome which represents how many times we expect the things to be happened is called:

(a) outcomes (b) favourable outcome

(c) sample space (d) sample point

(iii) Which one tells us how often a specific event oc relative to the total number of frequency event or trials:

(a) expected frequency
(b) sum of relative frequency
(c) relative frequency (d) frequency

(iv) Estimated probability of an event occurring is also known as:
(a) relative frequency (b) expected frequency
(c) class boundaries
(d) sum of expected frequency

(v) The sum of all expected frequencies is equal to the fixed number of:
(a) trials (b) relative frequencies
(c) outcomes (d) events

(vi) The chance of occurrence of a particular event is called:
(a) sample space (b) estimated probability
(c) probability (d) expected frequency

(vii) An event which will probably occur. It has greater chance to occur is called:
(a) equally likely event (b) likely event
(c) unlikely event (d) certain event

(viii) Find out the total number of possible sample space when 4 dice are rolled:
(a) 6^2 (b) 6^3
(c) 6^4 (d) 6^6

(ix) While rolling a pair of dice, what will be the probability of double 2?
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{5}{6}$ (d) $\frac{1}{36}$

(x) A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:
(a) $\frac{2}{13}$ (b) $\frac{11}{13}$

(c) $\frac{2}{52}$ (d) $\frac{11}{52}$

Answers:

(i)	c	(ii)	b	(iii)	c	(iv)	a	(v)	a
(vi)	c	(vii)	b	(viii)	c	(ix)	d	(x)	b

2. Define the following:

(i) relative frequency

(ii) expected frequency

(i) Relative Frequency

Relative frequency is the ratio of the number of times an event occurs to the total number of trials or observations in an experiment. It is used to estimate the likelihood of an event based on experimental data.

Formula:

$$\text{Relative Frequency} = \frac{\text{Frequency of the Event}}{\text{Total Number of Trials}}$$

Example: If a die is rolled 50 times and the number 6 appears 10 times, the relative frequency of rolling a 6 is:

$$\frac{10}{50} = 0.2.$$

(ii) Expected Frequency

Expected frequency is the predicted number of times an event should occur based on its theoretical probability in a given number of trials. It is calculated by multiplying the probability of the event by the total number of trials.

Formula:

Expected Frequency = Probability of the Event × Total Number of Trials

Example: If a coin is tossed 100 times, the probability of getting heads is 0.5. The expected frequency of heads is:

$$0.5 \times 100 = 50.$$

3. An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.

(i) a green ball (ii) a red ball

(iii) a blue ball (iv) not a red ball

(v) not a green ball

Solution: Given Data:

- Number of red balls: 10
- Number of green balls: 5
- Number of blue balls: 8
- Total balls in the urn:

$$10 + 5 + 8 = 23.$$

(i) Probability of selecting a green ball:

The probability is:

$$P(\text{green ball}) = \frac{\text{Number of green balls}}{\text{Total balls}} = \frac{5}{23}.$$

(ii) Probability of selecting a red ball:

The probability is:

$$P(\text{red ball}) = \frac{\text{Number of red balls}}{\text{Total balls}} = \frac{10}{23}.$$

(iii) Probability of selecting a blue ball:

The probability is:

$$P(\text{blue ball}) = \frac{\text{Number of blue balls}}{\text{Total balls}} = \frac{8}{23}.$$

(iv) Probability of selecting **not a red ball**:

The probability of not selecting a red ball is:

$$P(\text{not a red ball}) = 1 - P(\text{red ball}) = 1 - \frac{10}{23} = \frac{13}{23}.$$

(v) Probability of selecting **not a green ball**:

The probability of not selecting a green ball is:

$$P(\text{not a green ball}) = 1 - P(\text{green ball}) = 1 - \frac{5}{23} = \frac{18}{23}.$$

4. Three coins are tossed together, what is the probability of getting:

(i) exactly three heads (ii) at least two tails

(iii) not at least two heads

(iv) not exactly two heads

We will calculate the required probabilities step by step. Tossing 3 coins results in $2^3 = 8$ possible outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

(i) Probability of **exactly three heads**

Only one outcome has exactly three heads: HHH.

$$P(\text{exactly three heads}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{8}.$$

(ii) Probability of **at least two tails**

At least two tails means the number of tails is 2 or 3. The favorable outcomes are:

$$\{HTT, THT, TTH, TTT\} \quad \text{(4 outcomes)}.$$

$$P(\text{at least two tails}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}.$$

(iii) Probability of **not at least two heads**

At least two heads means the number of heads is 2 or 3. The favorable outcomes for at least two heads are:

$$\{HHT, HTH, THH, HHH\} \quad \text{(4 outcomes)}.$$

The unfavorable outcomes (not at least two heads) are the remaining $8 - 4 = 4$ outcomes:

$$\{HTT, THT, TTH, TTT\}.$$

$$P(\text{not at least two heads}) = \frac{\text{Number of unfavorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}.$$

(iv) Probability of **not exactly two heads**

The favorable outcomes for exactly two heads are:

$$\{HHT, HTH, THH\} \quad \text{(3 outcomes).}$$

The unfavorable outcomes (not exactly two heads) are the remaining $8 - 3 = 5$ outcomes:

$$\{HHH, HTT, THT, TTH, TTT\}.$$

$$P(\text{not exactly two heads}) = \frac{\text{Number of unfavorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{5}{8}.$$

5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:

(i) king or jack of red colour

(ii) not "2" of club and spade

Given:

A standard pack of 52 playing cards contains:

- 26 red cards (13 hearts + 13 diamonds).
- 26 black cards (13 clubs + 13 spades).
- Each suit has one king, one jack, one "2", etc.

(i) Probability of getting a **king or jack of red color**

- There are 2 red suits: hearts and diamonds.

- Each suit has 1 king and 1 jack, so there are 2 kings and 2 jacks of red color.
- Total favorable outcomes = 2 + 2 = 4.
- Total possible outcomes = 52.

$$P(\text{king or jack of red color}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Probability of **not "2" of club and spade**

- The "2 of clubs" and "2 of spades" are specific cards, so there are 2 unfavorable outcomes.
- Total favorable outcomes = 52 − 2 = 50.
- Total possible outcomes = 52.

$$P(\text{not "2" of club and spade}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{50}{52} = \frac{25}{26}.$$

6. Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:

No. of tails	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	**110**	**90**	**105**	**80**	**76**	**123**	**16**

Find the relative frequency of given table.

Solution: The **relative frequency** is calculated as:

$$\text{Relative Frequency} = \frac{\text{Frequency of a specific event}}{\text{Total number of trials}}.$$

Given Data:

- Total trials = 600

- Frequency for each number of tails:

No. of tails (x)	Frequency (f)
0	110
1	90
2	105
3	80
4	76
5	123
6	16

Step-by-step Calculation of Relative Frequencies:

For each x, the relative frequency is:

$$\text{Relative Frequency} = \frac{\text{Frequency}}{600}.$$

- For $x = 0$: $\frac{110}{600} = 0.1833$
- For $x = 1$: $\frac{90}{600} = 0.15$
- For $x = 2$: $\frac{105}{600} = 0.175$
- For $x = 3$: $\frac{80}{600} = 0.1333$
- For $x = 4$: $\frac{76}{600} = 0.1267$
- For $x = 5$: $\frac{123}{600} = 0.205$
- For $x = 6$: $\frac{16}{600} = 0.0267$

Final Table:

No. of tails	0	1	2	3	4	5	6	Total
f	110	90	105	80	76	123	16	$\Sigma f = 600$
Relative Frequency	$\frac{11}{60}$	$\frac{3}{20}$	$\frac{7}{40}$	$\frac{2}{15}$	$\frac{19}{150}$	$\frac{41}{200}$	$\frac{2}{75}$	

7. **From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.**

Given Data:

- Total number of items = 25
- Number of defective items = 8
- Number of non-defective items = 25 − 8 = 17

(i) Relative Frequency of Non-Defective Items

The relative frequency is calculated as:

$$\text{Relative Frequency of non-defective items} = \frac{\text{Number of non-defective items}}{\text{Total number of items}} = \frac{17}{25} = 0.68.$$

(ii) Expected Frequency of Non-Defective Items

If we know the relative frequency, we can multiply it by the total *number* of trials (items in this case) to find the expected frequency:

$$\text{Expected Frequency of non-defective items} = \text{Relative Frequency of non-defective items} \times \text{Total number of items} = 0.68 \times 25 = 17.$$