Exercise 1.2

1. Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution:

$$2 - x^2 = 7x$$
$$-x^2 - 7x + 2 = 0$$
$$x^2 + 7x - 2 = 0$$

Here $a = 1, b = 7, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

$$Solution\ set = \left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$$

(ii) $5x^2 + 8x + 1 = 0$

Solution:

$$5x^2 + 8x + 1 = 0$$

Here $a = 5,\ b = 8,\ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{2\times2\times11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{2(-4 \pm \sqrt{11})}{10}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

$$\textit{Solution set} = \left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution:

$$\sqrt{3}x^2 + x = 4\sqrt{3}$$

$$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$$

Here $a = \sqrt{3},\ b = 1,\ c = -4\sqrt{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\times3}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1+7}{2\sqrt{3}}$$

$$x = \frac{-1+7}{2\sqrt{3}} \quad \textit{or} \quad x = \frac{-1-7}{2\sqrt{3}}$$

$$x = \frac{6}{2\sqrt{3}} \quad \textit{or} \quad x = \frac{-8}{2\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}} \quad \textit{or} \quad x = \frac{-4}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}.\sqrt{3}}{\sqrt{3}}$$

$x = \sqrt{3}$

$Solution\ set = \left\{\sqrt{3}, \frac{-4}{\sqrt{3}}\right\}$

(iv) $4x^2 - 14 = 3x$

Solution:

$4x^2 - 14 = 3x$

$4x^2 - 3x - 14 = 0$

Here $a = 4,\ b = -3, c = -14$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

$$x = \frac{3 \pm \sqrt{233}}{8}$$

$$Solution\ set = \left\{\frac{3 \pm \sqrt{233}}{8}\right\}$$

(v) $6x^2 - 3 - 7x = 0$

Solution:

$6x^2 - 3 - 7x = 0$

$6x^2 - 7x - 3 = 0$

Here $a = 6,\ b = -7,\ c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$x = \frac{7+11}{12} \quad or \quad x = \frac{7-11}{12}$$

$$x = \frac{18}{12} \quad or \quad x = \frac{-4}{12}$$

$$x = \frac{3}{2} \quad or \quad x = \frac{-1}{3}$$

$$Solution\ set = \left\{\frac{-1}{3}, \frac{3}{2}\right\}$$

(vi) $3x^2 - 8x + 2 = 0$

Solution:

$$3x^2 + 8x + 2 = 0$$

Here $a = 3,\ b = 8,\ c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{2[-4 \pm \sqrt{10}]}{6}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

$$Solution\ set = \left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$$

(vii) $\frac{3}{x-6} - \frac{4}{x-5} = 1$

Solution:

$$\frac{3}{x-6} - \frac{4}{x-5} = 1$$

Multiplying by $(x-6)(x-5)$ *on both sides*

$$(x-6)(x-5) \times \left(\frac{3}{x-6}\right) - (x-6)(x-5) \times \left(\frac{4}{x-5}\right) = 1(x-6)(x-5)$$

$$3(x-5) - 4(x-6) = (x-6)(x-5)$$

$$3x - 15 - 4x + 24 = x^2 - 11x + 30$$

$$9 - x = x^2 - 11x + 30$$

$$0 = x^2 - 11x + x + 30 - 9$$

$$x^2 - 10x + 21 = 0$$

Here $a = 1,\ b = -10,\ c = 21$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{10+4}{2} \quad Or \quad x = \frac{10-4}{2}$$

$$x = \frac{14}{2} \quad Or \quad x = \frac{6}{2}$$

$$x = 7 \quad Or \quad x = 3$$

Solution set $= \{3, 7\}$

(viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

Solution:

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$$

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$$

Multiplying both sides by $3(x-1)(2x)$, *we get*

$$3(x-1)(2x) \times \left(\frac{x+2}{x-1}\right) - 3(x-1)(2x) \times \left(\frac{4-x}{2x}\right) = \frac{7}{3} \times 3(x-1)(2x)$$

$$3(2x)(x+2) - 3(x-1)(4-x) = 7(x-1)(2x)$$

$$6x(x+2) - (3x-3)(4-x) = 14x(x-1)$$

$$6x^2 + 12x - (12x - 3x^2 - 12 + 3x) = 14x^2 - 14x$$

$$6x^2 + 12x - 12x + 3x^2 + 12 - 3x = 14x^2 - 14x$$

$$9x^2 - 3x + 12 - 14x^2 + 14x = 0$$

$$-5x^2 + 11x + 12 = 0$$

$$5x^2 - 11x - 12 = 0$$

Here $a = 5,\ b = -11,\ c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-12)}}{2(5)}$$

$$x = \frac{11 \pm \sqrt{121+240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10}$$

$$x = \frac{11 \pm 19}{10}$$

$$x = \frac{11+19}{10} \quad or \quad \frac{11-19}{10}$$

$$x = \frac{30}{10} \quad or \quad \frac{-8}{10}$$

$$x = 3 \quad or \quad -\frac{4}{5}$$

$$\text{Solution set} = \left\{3, -\frac{4}{5}\right\}$$

(ix) $\frac{a}{x-b} + \frac{b}{x-a} = 2$

Solution:

$$\frac{a}{x-b} + \frac{b}{x-a} = 2$$

Multiplying by $(x-b)(x-a)$ *on both sides*

$$(x-b)(x-a)\left(\frac{a}{x-b}\right) + (x-b)(x-a)\left(\frac{b}{x-a}\right) = 2(x-b)(x-a)$$

$$a(x-a) + b(x-b) = 2(x^2 - ax - bx + ab)$$

$$ax - a^2 + bx - b^2 = 2x^2 - 2ax - 2bx + 2ab$$

$$-2x^2 + ax + bx + 2ax - 2ab - a^2 - b^2 = 0$$

$$-2x^2 + 3ax + 3bx - 2ab - a^2 - b^2 = 0$$

$$2x^2 - 3ax - 3bx + 2ab + a^2 + b^2 = 0$$

$$2x^2 - (3a + 3b)x + (a + b)^2 = 0$$

Here $a = 2,\ b = -(3a + 3b),\ c = (a + b)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-(3a+3b)] \pm \sqrt{[-(3a+3b)]^2 - 4(2)(a+b)^2}}{2(2)}$$

$$x = \frac{3(a+b) \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$x = \frac{3(a+b) \pm \sqrt{(a+b)^2}}{4}$$

$$x = \frac{3(a+b) \pm (a+b)}{4}$$

$$x = \frac{3(a+b)+(a+b)}{4} \quad or \quad x = \frac{3(a+b)-(a+b)}{4}$$

$$x = \frac{3a+3b+a+b}{4} \quad or \quad x = \frac{3a+3b-a-b}{4}$$

$$x = \frac{4a+4b}{4} \quad or \quad x = \frac{2a+2b}{4}$$

$$x = \frac{4(a+b)}{4} \quad or \quad x = \frac{2(a+b)}{4}$$

$$x = a+b \quad or \quad x = \frac{a+b}{2}$$

$$x = a+b \quad or \quad x = \frac{1}{2}(a+b)$$

$$Solution\ set = \left\{a+b, \frac{1}{2}(a+b)\right\}$$

(x) $-(l+m) - lx^2 + (2l+m)x = 0, \quad l \neq 0$

Solution:

$$-(l+m) - lx^2 + (2l+m)x = 0$$

$$lx^2 - (2l+m)x + (l+m) = 0$$

Here $a = l,\ b = -(2l+m),\ c = (l+m)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-[-(2l+m)] \pm \sqrt{[-(2l+m)]^2 - 4l(l+m)}}{2l}$$

$$x = \frac{(2l+m)) \pm \sqrt{4l^2+4lm+m^2-4l^2-4lm}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{m^2}}{2l}$$

$$x = \frac{(2l+m) \pm m}{2l}$$

$$x = \frac{2l+m+m}{2l} \quad or \quad x = \frac{2l+m-m}{2l}$$

$$x = \frac{2l+2m}{2l} \quad or \quad x = \frac{2l}{2l}$$

$$x = \frac{2(l+m)}{2l} \quad or \quad x = l$$

$$x = \frac{l+m}{l} \quad or \quad x = l$$

$$\text{Solution set} = \left\{\frac{l+m}{l}, l\right\}$$