Solve the following equations.

1. $\boxed{2x^4 - 11x^2 + 5 = 0}$

Solution:

$$2x^4 - 11x^2 + 5 = 0 \quad \text{.....................(i)}$$

Let $x^2 = y$ *then* $x^4 = y^2$, *then equation* (i) *becomes*

$$2y^2 - 11y + 5 = 0$$

$$2y^2 - 10y - y + 5 = 0$$

$$2y(y-5) - 1(y-5) = 0$$

$$(2y-1)(y-5) = 0$$

$$2y - 1 = 0 \quad or \quad y - 5 = 0$$

$$2y = 1 \quad or \quad y = 5$$

$$y = \frac{1}{2}$$

Put $y = \frac{1}{2}$ in $x^2 = y$

then $x^2 = \frac{1}{2}$

$$x = \pm\frac{1}{\sqrt{2}}$$

Put $y = 5$ in $x^2 = y$

then $x^2 = 5$

Thus, $x = \pm\sqrt{5}$

$$\textit{Solution set} = \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$$

2. $2x^4 = 9x^2 - 4$

Solution:

$$2x^4 = 9x^2 - 4$$

$$2x^4 - 9x^2 + 4 = 0$$

$$2x^4 - x^2 - 8x^2 + 4 = 0 \quad (\because -9x^2 = -x^2 - 8x^2)$$

$$x^2(2x^2 - 1) - 4(2x^2 - 1) = 0$$

$$(2x^2 - 1)(x^2 - 4) = 0$$

$$2x^2 - 1 = 0 \quad or \quad x^2 - 4 = 0$$

$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \pm\frac{1}{\sqrt{2}}$$

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

Solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

3. $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution:

$$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$$
$$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \quad \text{....................(i)}$$

Let $x^{\frac{1}{4}} = y$ *then* $x^{\frac{1}{2}} = y^2$, *then equation* (i) *becomes*

$$5y^2 - 7y + 2 = 0$$
$$5y^2 - 5y - 2y + 2 = 0$$
$$5y^2 - 5y - 2y + 2 = 0$$
$$5y(y-1) - 2(y-1) = 0$$
$$(y-1)(5y-2) = 0$$
$$y - 1 = 0 \quad or \quad 5y - 2 = 0$$

$$y = 1$$

Put $y = 1$ in $x^{\frac{1}{4}} = y$

then $x^{\frac{1}{4}} = 1$

$$x^{\frac{1}{4}\times 4} = (1)^4$$
$$\boxed{x = 1}$$

$$5y = 2$$
$$y = \frac{2}{5}$$

Put $y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$

then $x^{\frac{1}{4}} = \frac{2}{5}$

$$x^{\frac{1}{4}\times 4} = \left(\frac{2}{5}\right)^4$$
$$x = \frac{16}{625}$$

Solution set $= \left\{\frac{16}{625}, 1\right\}$

4. $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution:

$$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$

$$x^{\frac{2}{3}} - 15x^{\frac{1}{3}} + 54 = 0 \quad \text{.............. (i)}$$

$Let\ x^{\frac{1}{3}} = y\ then\ x^{\frac{2}{3}} = y^2, then\ equation\ (\text{i}) becomes$

$$y^2 - 15y + 54 = 0$$

$$y^2 - 9y - 6y + 54 = 0$$

$$y(y-9) - 6(y-9) = 0$$

$$(y-9)(y-6) = 0$$

$$y - 9 = 0 \quad or \quad y - 6 = 0$$

$$y = 9 \qquad\qquad y = 6$$

Put $y = 9$ in $x^{\frac{1}{3}} = y$	Put $y = 6$ in $x^{\frac{1}{3}} = y$
then $x^{\frac{1}{3}} = 9$	*then* $x^{\frac{1}{3}} = 6$
$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$	$\left(x^{\frac{1}{3}}\right)^3 = (6)^3$
$x = 729$	$x = 216$

$$Solution\ set = \{216, 729\}$$

5. $3x^{-2} + 5 = 8x^{-1}$

Solution:

$$3x^{-2} + 5 = 8x^{-1}$$

$$3x^{-2} - 8x^{-1} + 5 = 0 \text{.....................(i)}$$

Let $x^{-1} = y\ then\ x^{-2} = y^2, then\ equatin\ (\text{i})\ becomes$

$$3y^2 - 8y + 5 = 0$$

$$3y^2 - 3y - 5y + 5 = 0$$

$$3y(y-1) - 5(y-1) = 0$$

$$(y-1)(3y-5) = 0$$

$$y - 1 = 0 \quad or \quad 3y - 5 = 0$$

$$y = 1$$

Put $y = 1$ in $x^{-1} = y$

then $\quad x^{-1} = 1$

$$\frac{1}{x} = 1$$

$$x = 1$$

$$3y = 5$$

$$y = \frac{5}{3}$$

Put $y = \frac{5}{3}$ in $x^{-1} = y$

then $\quad x^{-1} = \frac{5}{3}$

$$\boxed{x = \frac{3}{5}}$$

$x = 5/3$ -

Solution set $= \left\{\frac{3}{5}, 1\right\}$

6. $\quad (2x^2 + 1) + \frac{3}{2x^2+1} = 4$

Solution: $\quad (2x^2 + 1) + \frac{3}{2x^2+1} = 4$ .... **(i)**

Let $2x^2 + 1 = y$*, then equation* (i)*becomes*

$y + \frac{3}{y} = 4$ $\quad$ *(multiplying by y on both sides)*

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$y^2 - y - 3y + 3 = 0$

$y(y - 1) - 3(y - 1) = 0$

$(y - 1)(y - 3) = 0$

$y - 1 = 0 \quad$ *or* $\quad y - 3 = 0$

$$y = 1$$

Put $y = 1$ in $2x^2 + 1 = y$

$$2x^2 + 1 = 1$$

$$2x^2 = 0$$

$$x^2 = 0$$

$$x = 0$$

$$y = 3$$

Put $y = 3$ in $2x^2 + 1 = y$

$$2x^2 + 1 = 3$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x = 1, -1$$

Solution set $= \{-1, 0, 1\}$

7. $$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$$

Solution: $$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4 \quad \text{..............(i)}$$

Let $\frac{x}{x-3} = y$, *then* $\frac{x-3}{x} = \frac{1}{y}$, *then equation* (i) *becomes*

$y + \frac{4}{y} = 4$ (*Multiplying both sides by y*)

$y^2 + 4 = 4y$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$

$y - 2 = 0$

$y = 2$

Put $y = 2$ in $\frac{x}{x-3} = y$

then $\frac{x}{x-3)} = 2$

$x = 2(x-3)$

$x = 2x - 6$

$x - 2x = -6$

$x = 6$

Solution set $= \{6\}$

8. $$\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$$

Solution: $$\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6} \quad \text{..............(i)}$$

Let $\frac{4x+1}{4x-1} = y$, *then* $\frac{4x-1}{4x+1} = \frac{1}{y}$, *then equation* (i) *becomes*

$y + \frac{1}{y} = \frac{13}{6}$ (*Multiplying both sides by 6y*)

$6y^2 + 6 = 13y$

$$6y^2-13y+6=0$$

$$6y^2-4y-9y+6=0$$

$$2y(3y-2)-3(3y-2)=0$$

$$(3y-2)(2y-3)=0$$

$3y-2=0$ *or* $2y-3=0$

$3y=2$	$2y=3$
$y=\frac{2}{3}$	$y=\frac{3}{2}$
Put $y=\frac{2}{3}$ in $\frac{4x+1}{4x-1}=y$	Put $y=\frac{3}{2}$ in $\frac{4x+1}{4x-1}=y$
then $\frac{4x+1}{4x-1}=\frac{2}{3}$	*then* $\frac{4x+1}{4x-1}=\frac{3}{2}$
$3(4x+1)=2(4x-1)$	$2(4x+1)=3(4x-1)$
$12x+3=8x-2$	$8x+2=12x-3$
$12x-8x=-2-3$	$12x-8x=2+3$
$4x=-5$	$4x=5$
$x=\frac{-5}{4}$	$x=\frac{5}{4}$

$Solution\ set=\{\pm\frac{5}{4}\}$

9. $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution: $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ ....(i)

Let $\frac{x-a}{x+a}=y$, *then* $\frac{x+a}{x-a}=\frac{1}{y}$, *then equation* (i) *becomes*

$y-\frac{1}{y}=\frac{7}{12}$ (*Multiplying both sides by* $12y$)

$$12y^2-12=7y$$

$$12y^2-7y-12=0$$

$$12y^2-16y+9y-12=0$$

$$4y(3y-4)+3(3y-4)=0$$

$$(3y-4)(4y+3)=0$$

$3y-4=0$ or $4y+3=0$

$$3y=4$$

$$y=\frac{4}{3}$$

Put $y=\frac{4}{3}$ in $\frac{x-a}{x+a}=y$

then $$\frac{x-a}{x+a}=\frac{4}{3}$$

$$3(x-a)=4(x+a)$$

$$3x-3a=4x+4a$$

$$3x-4x=4a+3a$$

$$-x=7a$$

$$x=-7a$$

$$4y=-3$$

$$y=-\frac{3}{4}$$

Put $y=-\frac{3}{4}$ in $\frac{x-a}{x+a}=y$

then $$\frac{x-a}{x+a}=-\frac{3}{4}$$

$$4(x-a)=-6(x+a)$$

$$4x-4a=-3x-3a$$

$$4x+3x=-3a+4a$$

$$7x=a$$

$$x=\frac{a}{7}$$

Solution set $=\left\{-7a,\ \frac{a}{7}\right\}$

10. $x^4-2x^3-2x^2+2x+1=0$

Solution: $x^4-2x^3-2x^2+2x+1=0$

Dividing each term by x^2*, we get*

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$\left(x-\frac{1}{x}\right)^2$

by rearranging, we get

$x^2+\frac{1}{x^2}-2$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad \ldots\ldots\ldots\ldots\text{(i)}$$

Let $x-\frac{1}{x}=y$ *(squaring on both sides)*

$$x^2+\frac{1}{x^2}-2=y^2$$

$x^2+\frac{1}{x^2}=y^2+2$ *then equation* (i) *becomes*

$$y^2+2-2y-2=0$$

$$y^2-2y=0$$

$$y(y-2)=0$$

$y = 0$ or $y - 2 = 0$

$y = 2$

Put $y = 0$ in $x - \frac{1}{x} = y$

$$x - \frac{1}{x} = 0$$
$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm 1$$

Put $y = 2$ in $x - \frac{1}{x} = y$

$$x - \frac{1}{x} = 2$$
$$x^2 - 1 = 2x$$
$$x^2 - 2x - 1 = 0$$

Here $a = 1, b = -2, c = -1$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2a}$$
$$x = \frac{2 \pm \sqrt{4+4}}{2}$$
$$x = \frac{2 \pm \sqrt{8}}{2}$$
$$x = \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$$
$$x = 1 \pm \sqrt{2}$$

Solution set $= \{\pm 1, 1 \pm \sqrt{2}\}$

11. $2x^4 + x^3 - 6x^2 + x + 2 = 0$ $2x^2$

Solution: $2x^4 + x^3 - 6x^2 + x + 2 = 0$ $2x^2$

Dividing both sides by x^2 *we get*

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

by rearranging, we get

$$\left(2x^2 + \frac{2}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$$ $(2x^2 +$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \ldots\ldots\ldots\ldots(i)$$ $2(x^2 + \frac{1}{x^2}$

Let $x + \frac{1}{x} = y$ *(by squaring on both sides)*

$x^2 + \frac{1}{x}$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = \boxed{y^2 - 2}$$ *then equation (i) becomes*

$$2(y^2 - 2) + y - 6 = 0$$

$$2y^2 - 4 + y - 6 = 0$$

$$2y^2 + y - 10 = 0$$

$$2y^2 + 5y - 4y - 10 = 0$$

$$y(2y + 5) - 2(2y + 5) = 0$$

$$(2y + 5)(y - 2) = 0$$

$$y - 2 = 0 \quad or \quad 2y + 5 = 0$$

$$y = 2$$

Put $y = 2$ in $x + \frac{1}{x} = y$

then $$x + \frac{1}{x} = 2$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x = 1$$

$$x + 2 = 0$$

$$x = -2$$

$$2y + 5 = 0$$

$$2y = -5$$

$$y = \frac{-5}{2}$$

Put $y = \frac{-5}{2}$ in $x + \frac{1}{x} = y$

then $$x + \frac{1}{x} = \frac{-5}{2}$$

$$2x^2 + 2 = -5x$$

$$2x^2 + 5x + 2 = 0$$

$$2x^2 + 4x + x + 2 = 0$$

$$2x(x + 2) + 1(x + 2) = 0$$

$$(x + 2)(2x + 1) = 0$$

Either $x + 2 = 0$ *or* $2x + 1 = 0$

$$x = -2 \text{ or } 2x = -1$$

$$x = -\frac{1}{2}$$

Solution set $= \left\{1, -2, -\frac{1}{2}\right\}$

12. $4.2^{2x+1} - 9.2^x + 1 = 0$ $\quad 4.2^{2x+1}$

Solution: $\quad 4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}.2 - 9.2^x + 1 = 0$ $\quad 4.2^{2x}.2 - 9.2^x +$

$8.2^{2x} - 9.2^x + 1 = 0$ ....(i)

Let $\quad 2^x = y$, *then* $2^{2x} = y^2$, *then equation* (i) *becomes*

$8y^2 - 9y + 1 = 0$ $\quad 4(y^2).2-(9)(y)+1$

$8y^2 - y - 8y + 1 = 0$

$y(8y-1) - 1(8y-1) = 0$ $\quad 8y^2 - 9y + 1 = 0.$

$(8y-1)(y-1) = 0$

$y - 1 = 0 \quad$ *or* $\quad 8y - 1 = 0$

$y - 1 = 0$

$y = 1$

Put $y = 1$ in $2^x = y$

then $\quad 2^x = 1$

$2^x = 2^0$

$x = 0$

$8y - 1 = 0$

$8y = 1$

$y = \dfrac{1}{8}$

Put $y = \frac{1}{8}$ in $2^x = y$

$2^x = \dfrac{1}{8}$

$2^x = \dfrac{1}{2^3}$

$2^x = 2^{-3}$

$x = -3$

Solution set $= \{-3, 0\}$

13. $3x^{2x+2} = 12.3^x - 3$ $\quad 3^{2x}.9 = 12.3^x.3$

Solution: $\quad 3^{2x+2} = 12.3^x - 3$ $\quad 3x^{2x+2} = 12.3^x - 3$

$3^{2x}.3^2 = 12.3^x - 3$ $\quad 3^{2x}.3^2 = 3^x - 3.$

$9.3^{2x} = 12.3^x - 3$ ....(i)

Let $\quad 3^x = y$, *then* $3^{2x} = y^2$, *then equation* (i)*becomes*

$9y^2 = 12y - 3$

$9y^2 = 12y + 3 = 0$ $\qquad$ *(Dividing both sides by 3)*

$3y^2 - 4y + 1 = 0$

$$3y^2 - 3y - y + 1 = 0$$
$$3y(y-1) - 1(y-1) = 0$$
$$(y-1)(3y-1) = 0$$
$y - 1 = 0$ *or* $3y - 1 = 0$

$y - 1 = 0$	$3y = 1$
$y = 1$	$y = \frac{1}{3}$
Put $y = 1$ in $3^x = y$	Put $y = \frac{1}{3}$ in $3^x = y$
$3^x = 1$	$3^x = \frac{1}{3}$
$3^x = 3^0$	$3^x = 3^{-1}$
$x = 0$	$x = -1$

Solution set $= \{0, -1\}$

14. $2^x + 64.2^{-x} - 20 = 0$

Solution: $2^x + 64.2^{-x} - 20 = 0$ ....(i)

Let $2^x = y$, *then* $2^{-x} = y^{-1} = \frac{1}{y}$, *then equation* (i) *becomes*

$y^2 + \frac{64}{y} - 20 = 0$ (*multiplying both sides by* y)

$$y^2 - 20y + 64 = 0$$
$$y^2 - 4y - 16y + 64 = 0$$
$$y(y-4) - 16(y-4) = 0$$
$$(y-4)(y-16) = 0$$
$y - 4 = 0$ *or* $y - 16 = 0$

$y = 4$	$y = 16$
Put $y = 4$ in $2^x = y$	Put $y = 16$ in $2^x = y$
$2^x = 4$	$2^x = 16$
$2^x = 2^2$	$2^x = 2^4$
$x = 2$	$x = 4$

Solution set $= \{2, 4\}$

15. $(x+1)(x+3)(x-5)(x-7) = 192$

Solution: $(x+1)(x+3)(x-5)(x-7) = 192$

by rearranging $(1 - 5 = 3 - 7)$

$$[(x+1)(x-5)][(x+3)(x-7)] = 192$$

$$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \quad \ldots\ldots\ldots\ldots\ldots(i)$$

Let $x^2 - 4x = y$, *then equation* (i) *becomes*

$$(y-5)(y-21) = 192$$

$$y^2 - 26y + 105 - 192 = 0$$

$$y^2 - 26y - 87 = 0$$

$$y^2 - 29y + 3y - 87 = 0$$

$$y(y-29) + 3(y-29) = 0$$

$$(y-29)(y+3) = 0$$

$y - 29 = 0$ *or* $y + 3 = 0$

$$y = 29$$

Put $y = 29$ in $x^2 - 4x = y$

$$x^2 - 4x = 29$$

$$x^2 - 4x - 29 = 0$$

$$a = 1, b = -4, c = -29$$

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 116}}{2}$$

$$x = \frac{4 \pm \sqrt{132}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 33}}{2}$$

$$x = \frac{4 \pm 2\sqrt{33}}{2}$$

$$= \frac{2(2 \pm \sqrt{33})}{2}$$

$$= 2 \pm \sqrt{33}$$

$$y = -3$$

Put $y = -3$

in $x^2 - 4x = y$

$$x^2 - 4x = -3$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - x - 3x + 3 = 0$$

$$x(x - 1) - 3(x - 1) = 0$$

$$(x - 1)(x - 3) = 0$$

$$x - 1 = 0, \quad x - 3 = 0$$

$$x = 1, x = 3$$

Solution set $= \{1, 3, 2 \pm \sqrt{33}\}$

16. $(x - 1)(x - 2)(x - 8)(x + 5) + 360 = 0$

Solution: $(x - 1)(x - 2)(x - 8)(x + 5) + 360 = 0$

by rearranging $(-1 - 2 = -8 + 5)$

$$[(x - 1)(x - 2)][(x - 8)(x + 5)] + 360 = 0$$

$$(x^2 - 3x + 2)(x^2 - 3x - 40) + 360 = 0 \quad \text{........} \quad \text{(i)}$$

Let $x^2 - 3x = y$, *then equation* (i) *becomes*

$$(y + 2)(y - 40) + 360 = 0$$
$$y^2 - 38y - 80 + 360 = 0$$
$$y^2 - 38y + 280 = 0$$

$$y^2 - 10y - 28y + 280 = 0$$
$$y(y - 10) - 28(y - 10) = 0$$
$$y - 10 = 0 \quad or \quad y - 28 = 0$$

$y = 10$	$y = 28$
Put $y = 10$ in $x^2 - 3x = y$	Put $y = 28$ in $x^2 - 3x = y$
$x^2 - 3x = 10$	$x^2 - 3x = 28$
$x^2 - 3x - 10 = 0$	$x^2 - 3x - 28 = 0$
$x^2 - 5x + 2x - 10 = 0$	$x^2 - 7x + 4x - 28 = 0$
$x(x - 5) + 2(x - 5) = 0$	$x(x - 7) + 4(x - 7) = 0$
$(x - 5)(x + 2) = 0$	$(x - 7)(x + 4) = 0$
$x - 5 = 0,\ or\ x + 2 = 0$	$x - 7 = 0\ \ or\ \ x + 4 = 0$
$x = 5, \quad x = -2$	$x = 7, \quad x = -4$

$$Solution\ set = \{-4, -2, 5, 7\}$$