Exercise 1.4

Solve the following equations.

1. $2x + 5 = \sqrt{7x + 16}$

Solution: $2x + 5 = \sqrt{7x + 16}$,

$$(2x + 5)^2 = (\sqrt{7x + 16})^2 \quad (\textit{squaring on both sides})$$

$$4x^2 + 20x + 25 = 7x + 16$$

$$4x^2 + 20x + 25 - 7x - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 4x + 9x + 9 = 0$$

$$4x(x + 1) + 9(x + 1) = 0$$

$$(x + 1)(4x + 9) = 0$$

$x + 1 = 0$ or $4x + 9 = 0$

$x + 1 = 0$ | $4x = -9$

$x = -1$ | $x = -\frac{9}{4}$

$$\textit{Solution set} = \left\{-1, \frac{9}{4}\right\}$$

2. $\sqrt{x + 3} = 3x - 1$

Solution: $\sqrt{x+3} = 3x - 1$ *(squaring on both sides)*

$$(\sqrt{x+3})^2 = (3x-1)^2$$

$$x + 3 = 9x^2 - 6x + 1$$

$$9x^2 - 6x - x + 1 - 3 = 0$$

$$9x^2 - 7x - 2 = 0$$

$$9x^2 - 9x + 2x - 2 = 0$$

$$9x(x-1) + 2(x-1) = 0$$

$$(x-1)(9x+2) = 0$$

$x - 1 = 0$ *or* $9x + 2 = 0$

$x + 1$ | $9x = -2$

$x = 1$ | $x = -\frac{2}{9}$

Solution set $= \left\{1, -\frac{2}{9}\right\}$

3. $4x = \sqrt{13x + 14} - 3$

Solution: $4x = \sqrt{13x+14} - 3$

$$4x + 3 = \sqrt{13x+14} \quad \textit{(squaring on both sides)}$$

$$(4x+3)^2 = (\sqrt{13x+14})^2$$

$$16x^2 - 24x + 9 - 13x - 14 = 0$$

$$16x^2 - 11x - 5 = 0$$

$$16x^2 + 16x - 5x - 5 = 0$$

$$16x(x+1) - 5(x+1) = 0$$

$$(x+1)(16x-5) = 0$$

$x + 1 = 0$ *or* $16x - 5 = 0$

$x + 1 = 0$ | $16x = 5$

$x = -1$ | $x = \frac{5}{16}$

Solution set $= \left\{-1, \frac{5}{16}\right\}$

4. $\sqrt{3x+100} - x = 4$

Solution: $\sqrt{3x+100} - x = 4$

$\sqrt{3x+100} = 4 + x$

$4 + x = \sqrt{3x+100}$ *(squaring on both sides)*

$(4+x)^2 = (\sqrt{3x+100})^2$

$16 + 8x + x^2 = 3x + 100$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$(x+12)(x-7) = 0$

$x - 7 = 0$ *or* $x + 12 = 0$

$x = 7$ *or* $x = -12$

Solution set $= \{7, 12\}$

5. $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

Solution: $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$ *(squaring both sides)*

$(\sqrt{x+5} + \sqrt{x+21})^2 = (\sqrt{x+60})^2$

$x + 5 + x + 21 + 2\sqrt{(x+5)(x+21)} = x + 60$

$2x + 26 + 2\sqrt{x^2 + 26x + 105} = x + 60$

$2\sqrt{x^1 + 26 + 105} = x + 60 - 2x - 26$

$2\sqrt{x^2 + 26 + 105} = 34 - x$ *(squaring again we get)*

$4(x^2 + 26x + 105) = 1156 - 68x + x^2$

$4x^2 + 104x + 420 - x^2 + 68x - 1156 = 0$

$3x^2 + 172x - 736 = 0$

$= 3x^2 - 12x + 184x - 736 = 0$

$3x(x-4) + 184(x-4) = 0$

$(x-4)(3x+184)=0$

$x-4=0$ or $3x+184=0$

$x=4$ | $3x=-184$

$x=-\frac{184}{3}$

Solution set $=\left\{4,-\frac{184}{3}\right\}$.

6. $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$.

Solution: $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ *(Squaring both sides)*

$(\sqrt{x+1}+\sqrt{x+2})^2=(\sqrt{x+6})^2$

$(x+1)(x-2)+2\sqrt{(x+1)(x-2)}=x+6$

$x+1+x-2+2\sqrt{x^2-x-2}=x+6$

$2x-1+2\sqrt{x^2-x-2}=x+6$

$2\sqrt{x^2-x-2}=x+6-2x+1$

$2\sqrt{x^2-x-2}=7-x$ *(again squaring both sides)*

$4(x^2-x-2)=49-14x+x^2$

$4x^2-4x-8=49-14x+x^2$

$4x^2-x^2-4x+14x-8-49=0$

$3x^2+10x-57=0$

$3x^2-9x+19x-57=0$

$3x(x-3)+19(x-3)=0$

$(x-3)(3x+19)=0$

$x-3=0$ or $3x=-19$

$x=3$ | $x=-\frac{19}{3}$

Solution set $=\left\{3,-\frac{19}{3}\right\}$

7. $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

Solution: $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$(\sqrt{11-x}-\sqrt{6-x})^2=(\sqrt{27-x})^2$

(squaring on both sides)

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$11-x+6-x-2\sqrt{66-17x+x^2}=27-x$

$17-2x-2\sqrt{66-17x+x^2}=27-x$

$-2\sqrt{66-17x+x^2}=27-x-17+2x$

$-2\sqrt{66-17x+x^2}=10+x$ *(squaring again we get)*

$4(66-17x+x^2)=100+20x+x^2$

$264-68x+4x^2=100+20x+x^2$

$4x^2-x^2-68x-20x+264-100=0$

$3x^2-88x+164=0$

$3x^2-6x-82x+164=0$

$3x(x-2)-82(x-2)=0$

$(x-2)(3x-82)=0$

$x-2=0$ *or* $3x-82=0$

$x=2$ | $3x=82$

$x=\frac{82}{3}$

Solution set $=\left\{2, \frac{82}{3}\right\}$

8. $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution: $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ *(squaring both sides)*

$(\sqrt{4a+x}-\sqrt{a-x})^2=(\sqrt{a})^2$

$(4a+x)+(a-x)-2\sqrt{(4a+x)(a-x)}=a$

$4a+x-a+x-2\sqrt{4a^2-3ax-x^2}=a$

$3a+2x-2\sqrt{4a^2-3ax-x^2}=a$

$$3a + 2x - 2\sqrt{4a^2 - 3ax - x^2} = a$$

$$-2\sqrt{4a^2 - 3ax - x^2} = a - 3a - 2x$$

$$-2\sqrt{4a^2 - 3ax - x^2} = -2a - 2x$$

$$-2\sqrt{4a^2 - 3ax - x^2} = -2(a + x)$$

$$\sqrt{4a^2 - 3ax - x^2} = (a + x)$$

(squaring again we get)

$$\left(\sqrt{4a^2 - 3ax - x^2}\right)^2 = (a + x)^2$$

$$4a^2 - 3ax - x^2 = a^2 + x^2 + 2ax$$

$$-x^2 - x^2 - 3ax - 2ax + 4a^2 - a^2 = 0$$

$$-(2x^2 + 5ax - 3a^2) = 0$$

$$2x^2 + 5ax - 3a^2 = 0$$

$$2x^2 + 6ax - ax - 3a^2 = 0$$

$$2x(x + 3a) - a(x + 3a) = 0$$

$$(x + 3a)(2x - a) = 0$$

$x + 3a = 0$ or $2x - a = 0$

$x = -3a$ or $2x = a$

$x = \frac{a}{2}$

$$\textbf{Solution set} = \left\{-3a, \frac{a}{2}\right\}$$

9. $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1$

Solution: $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1$

$\sqrt{x^2 + x + 1} = 1 + \sqrt{x^2 + x - 1}$ *(squaring both sides)*

$$x^2 + x + 1 = (1)^2 + (x^2 + x - 1) + 2(1)\sqrt{x^2 + x - 1}$$

$$x^2 + x + 1 = 1 + x^2 + x - 1 + 2\sqrt{x^2 + x - 1}$$

$$x^2 + x + 1 - x^2 - x = 2\sqrt{x^2 + x - 1}$$

$$1 = 2\sqrt{x^2 + x - 1}$$

(squaring again on both sides we get)

$$1 = 4(x^2 + x - 1)$$

$$1 = 4x^2 + 4x - 4$$

$$0 = 4x^2 + 4x - 4 - 1$$

$$0 = 4x^2 + 4x - 5$$

or $4x^2 + 4x - 5 = 0$

Here $a = 4,\ b = 4, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16 + 80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{4 \times 4 \times 6}}{8}$$

$$x = \frac{-4 \pm \sqrt{6}}{8}$$

$$x = \frac{4(-1 \pm \sqrt{6})}{8}$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

Solution set $= \left\{-\frac{-1 \pm \sqrt{6}}{2}\right\}$

10. $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$

Solution: $\sqrt{x^2 + 3x + 8} + \sqrt{x^2 + 3x + 2} = 3$

$\sqrt{x^2 + 3x + 8} = 3 - \sqrt{x^2 + 3x + 2}$ *(squaring both sides)*

$$x^2 + 3x + 8 = 9 + (x^2 + 3x + 2) - 6\sqrt{x^2 + 3x + 2}$$

$x^2 + 3x + 8 = 9 + (x^2 + 3x + 2) - 6\sqrt{x^2 + 3x + 2}$

$x^2 + 3x + 8 - 9 - x^2 - 3x - 2 = -6\sqrt{x^2 + 3x + 2}$

$-3 = -6\sqrt{x^2 + 3x + 2}$ *(dividing by 3)*

$-1 = -2\sqrt{x^2 + 3x + 2}$

(squaring again on both sides we get)

$1 = 4(x^2 + 3x + 2)$

$1 = 4x^2 + 12x + +8$

$0 = 4x^2 + 12x + 8 - 1$

$0 = 4x^2 + 12x + 7$

or $4x^2 + 12x + 7 = 0$

Here $a = 4,\ b = 12,\ c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(7)}}{2(4)}$$

$$x = \frac{-12 \pm \sqrt{144 - 112}}{8}$$

$$x = \frac{-12 \pm \sqrt{32}}{8}$$

$$x = \frac{-12 \pm \sqrt{4 \times 4 \times 2}}{8}$$

$$x = \frac{-12 \pm 4\sqrt{2}}{8}$$

$$x = \frac{4(-3 \pm \sqrt{2})}{8}$$

$$x = \frac{-3 \pm \sqrt{2}}{2}$$

$$\textit{Solution set} = \left\{-\frac{-3 \pm \sqrt{2}}{2}\right\}$$

11. $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution: $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

$$\sqrt{x^2+3x+9}=5-\sqrt{x^2+3x+4}$$

(squaring on both sides)

$$x^2+3x+9=25+(x^2+3x+4)-10\sqrt{x^2+3x+4}$$

$$x^2+3x+9-25-x^2-3x-4=-10\sqrt{x^2+3x+4}$$

$$-20=-10\sqrt{x^2+3x+4}$$

(dividing by -10 on both sides)

$$2=\sqrt{x^2+3x+4}$$

(squaring again on both sides we get)

$$4=(x^2+3x+4)$$

$$0=x^2+3x+4-4$$

$$0=x^2+3x$$

$$x^2+3x=0$$

$$x(x+3)=0$$

$x=0$ or $x+3=0$

$x=0$ or $x=-3$

Solution set $=\{0,-3\}$