Exercise 2.3

1. **Without solving, find the sum and the product of the following quadratic equation.**

$$Sum\ of\ roots = S = \alpha + \beta = -\frac{b}{a} = -\frac{\text{Co} - \text{efficient of x}}{\text{Co} - \text{efficient of } x^2}$$

$$\text{Product of roots} = P = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Co}-\text{efficient of } x^2}$$

(i) $x^2 - 5x + 3 = 0$

Here $a = 1, \quad b = -5, \quad c = 3$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{b}{a} = -\frac{(-5)}{1} = 5$$

$$\text{Product of roots } P = \alpha\beta = -\frac{c}{a} = \frac{3}{1} = 3$$

(ii) $3x^2 + 7x - 11 = 0$

$3x^2 + 7x - 11 = 0$

$-\frac{b}{a}$

Here $a = 3,\ b = 7,\ c = 7$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{b}{c} = -\frac{7}{3}$$

$$\text{Product of roots } P = \alpha\beta = \frac{c}{a} = \frac{-11}{3} = -\frac{11}{3}$$

$\frac{c}{a}$.

(iii) $px^2 - qx + r = 0$

Here, $a = p,\ b = -q,\ c = r$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{b}{a} = -\frac{(-q)}{p} = \frac{q}{p}$$

$$\text{Product of roots } P = \alpha\beta = -\frac{c}{a} = \frac{r}{p}$$

(iv) $(a+b)x^2 - ax + b = 0$

Here, $\acute{a} = a + b,\ \acute{b} = -a,\ \acute{c} = b$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{\acute{b}}{\acute{a}} = -\frac{(-a)}{(a+b)} = \frac{a}{a+b}$$

$$\text{Product of roots } P = \alpha\beta = -\frac{\acute{c}}{\acute{a}} = \frac{b}{a+b}$$

(v) $(l+m)x^2 + (m+n)x + n - 1 = 0$

Here, $a = l + m,\ b = m + n,\ c = n - l$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{\acute{b}}{\acute{a}} = -\frac{m+n}{l+m}$$

$$\text{Product of roots } P = \alpha\beta = -\frac{c}{a} = \frac{n-l}{l+m}$$

(vi) $7x^2 - 5mx + 9n = 0$

Here, $a = 7,\ b = -5m,\ c = 9n$

Let α and β be its roots, then:

$$Sum\ of\ roots\ S = \alpha + \beta = -\frac{b}{a} = -\frac{(-5m)}{(7)} = \frac{5m}{7}$$

$$Product\ of\ roots\ P = \alpha\beta = -\frac{c}{a} = \frac{9n}{7}$$

2. Find the sum value of k, if

(i) sum of the roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots.

Solution: $2kx^2 - 3x + 4k = 0$

Here, $a = 2k, \quad b = -3, c = 4k$

Let α and β be its roots, then:

$$Sum\ of\ roots\ S = \alpha + \beta = -\frac{b}{a} = -\frac{(-3)}{(2k)} = \frac{3}{2k}$$

$$Product\ of\ roots\ P = \alpha\beta = -\frac{c}{a} = \frac{4k}{2k} = 2$$

Given that sum of the roots is twice the product of the roots, therefore

$$Sum\ of\ roots = 2 \times Product\ of\ roots$$

$$\alpha + \beta = 2 \times \alpha\beta$$

$$\frac{3}{2k} = 2 \times 2$$

$$\frac{3}{2k} = 4$$

$$8k = 3$$

$$k = \frac{3}{8}$$

(ii) sum of the roots of the equation $x^2 + (3k - 7)x + 5k = 0$ is $\frac{3}{2}$ **times the product of the roots.**

Solution: $x^2 + (3k - 7)x + 5k = 0$

Here, $a = 1, \ b = 3k - 7, \ c = 5k$

Let α and β be its roots, then:

$$Sum\ of\ roots\ S = \alpha + \beta = -\frac{b}{a} = -\frac{(3k-7)}{1} = -3k + 7$$

$$Product\ of\ roots\ P = \alpha\beta = -\frac{c}{a} = \frac{5k}{1} = 5k$$

Given that sum of the roots is $\frac{3}{2}$ times the product of the roots, therefore

$$Sum\ of\ roots = \frac{3}{2} \times Product\ of\ roots$$

$$\alpha+\beta=\frac{3}{2}\times\alpha\beta$$

$-3k+7=\frac{3}{2}(5k)$

$-3k+7=\frac{15k}{2}$

Multiplying both side by 2 we get

$-6k+14=15k$

$-6k-15k=-14$

$-21k=-14$

$k=\frac{-14}{-21}=\frac{2}{3}$

3. **Find k, if**

(i) **sum of the squares of the roots of the equation** $4kx^2+3kx-8=0$ is 2

Solution: $a=4k,\quad b=3k,\quad c=-8$

Let α and β be its roots, then:

Sum of roots $S=\alpha+\beta=-\frac{b}{a}=-\frac{3k}{4k}=-\frac{3}{4}$

Product of roots $P=\alpha\beta=\frac{c}{a}=\frac{-8}{4k}$

Given that sum of square of the roots is 2, therefore

Sum of square of roots $=2$

Therefore $\alpha^2+\beta^2=2$

$(\alpha+\beta)^2-2\alpha\beta=2$

Putting values of $\alpha+\beta$, and $\alpha\beta$ we get

$=\left(-\frac{3}{4}\right)^2-2\left(\frac{-8}{4k}\right)=2$

$\frac{9}{16}+\frac{16}{4k}=2$

$\frac{16}{4k}=2-\frac{9}{16}$

$\frac{16}{4k}=\frac{32-9}{16}$

$\frac{16}{4k}=\frac{23}{16}$

$23\times4k=16\times16$

$k=\frac{16\times16}{23\times4}$

$a^2+\beta^2=2$

$(a+\beta)^2-2\alpha\beta=2$

$$k = \frac{64}{23}$$

(ii) **sum of the squares of the roots of the equation.** $x^2 - 2kx + (2k+1) = 0$ is 6.

Solution: $a = 1, \quad b = -2k, \quad c = 2k+1$

Let α and β be its roots, then:

$$\text{Sum of roots } S = \alpha + \beta = -\frac{b}{a} = -\frac{(-2k)}{1} = 2k$$

$$\text{Product of root } P = \alpha\beta = \frac{c}{a} = \frac{2k+1}{1} = 2k+1$$

Given that sum of square of the roots is 6, therefore

$$\text{Sum of square of roots} = 6$$

Therefore $\alpha^2 + \beta^2 = 6$

$$(\alpha+\beta)^2 - 2\alpha\beta = 6$$

Putting values of $\alpha+\beta$, and $\alpha\beta$ we get

$$(2k)^2 - 2(2k+1) = 6$$

$$4k^2 - 4k - 2 = 6$$

$$4k^2 - 4k - 2 - 6 = 0$$

$$4k^2 - 4k - 8 = 0$$

(Dividingboth sides by 4 we get)

$$k^2 - k - 2 = 0$$

$$k^2 + k - 2k - 2 = 0$$

$$k(k+1) - 2(k+1) = 0$$

$$(k+1)(k-2) = 0$$

$$k+1 = 0 \quad \text{or} \quad k-2 = 0$$

$$k = -1 \quad \text{or} \quad k = 2$$

4. **Find p, if**

(i) **the roots of the equation $x^2 - x + p^2 = 0$ differ by unity.**

Solution: $a = 1, \quad b = -1, \quad c = p^2$

Let α, and $\alpha - 1$ be the required roots of $x^2 - x + p^2 = 0$,

$$\text{Then Sum of roots } \alpha + \alpha - 1 = -\frac{b}{a} = -\frac{(-1)}{1} = 1$$

$$2\alpha - 1 = 1 \quad \Longrightarrow 2\alpha = 1 + 1$$

$$2\alpha = 2$$

$$\alpha = 1 \quad \text{..........} \quad \text{(i)}$$

$$\text{Now product of roots } \alpha(\alpha-1) = \frac{c}{a} = \frac{p^2}{1} = p^2 \quad \text{or}$$

$$\alpha(\alpha-1) = p^2 \quad \text{..........} \quad \text{(ii)}$$

Putting value of α from equation (i) in equation (ii), we get

$$\boxed{\begin{aligned} 1(1-1) &= p^2 \\ 1(0) &= p^2 \\ p^2 &= 0 \\ p &= 0 \end{aligned}}$$

(ii) the roots of the equation $x^2 - 3x + p - 2 = 0$ differ by 2.

Solution: $a = 1, \quad b = 3, \quad c = p - 2$

Let α, and $\alpha - 2$ be the roots of given equation

$x^2 - 3x + p^2 - 2 = 0,$

Then sum of roots $\alpha + \alpha - 2 = -\frac{b}{a} = -\frac{3}{1} = -3$

$$2\alpha - 2 = -3$$

$$2\alpha = -3 + 2 \quad \Rightarrow \quad 2\alpha = -1$$

$$\alpha = -\frac{1}{2} \ldots\ldots\ldots\ldots (i)$$

Product of roots $\alpha(\alpha - 2) = \frac{c}{a} = \frac{p-2}{1} = p - 2$

or $\alpha(\alpha - 2) = p - 2$ (ii)

Putting values of α from equation (i) in equation (ii) we get

$$-\frac{1}{2}\left(-\frac{1}{2} - 2\right) = p - 2$$

$$-\frac{1}{2}\left(\frac{-1-4}{2}\right) = p = 2$$

$$-\frac{1}{2}\left(-\frac{5}{2}\right) = p - 2$$

$$\frac{5}{4} = p - 2$$

$$p = \frac{5}{4} + 2$$

$$p = \frac{5+8}{4}$$

$$p = \frac{13}{4}$$

$x^2 - 3x + p - 2 = 0.$

$a, \ a+2.$

$(a) + (a+2) = -b/a.$

$-\frac{-3}{1} = 0.$

$(a) + (a-2) = -3.$

$(a)(a-2) = c/a = \frac{p-}{1}$

$(a)(a-2) = p - 2.$

$a + a - 2.$

$2a - 2 = -3.$

$2a = -3 + 2.$

$2a = -1, \ a = -\frac{1}{2}$

5. Find m. if

(i) the roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$

$\left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 2\right) = p - 2.$

Solution: $x^2 - 7x + 3m - 5 = 0$

$a = 1, \quad b = -7, \quad c = 3m - 5,$

Let α *and* β *be the roots of given equation; then*

sum of roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{(-7)}{2} = 7 \ldots\ldots$ (i)

product of roots $P = \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5 \ldots\ldots$ (ii)

From (i) $\beta = 7 - \alpha$ ....(iii)

Now, given that $3\alpha + 2\beta = 4$

Put $\beta = 7 - \alpha$ *in it* we get

$3\alpha + 2(7 - \alpha) = 4$

$3\alpha + 14 - 2\alpha = 4$

$3\alpha - 2\alpha = 4 - 14$

$\alpha = -10$

Now Put $\alpha = -10$ *in (iii) we get*

$-10 + \beta = 7$

$\beta = 7 + 10$

$\beta = 17$

Now from (ii) $\alpha\beta = 3m - 5$

Putting value of α *and* β *in it we get*

$(-10)(17) = 3m - 5$

$-170 = 3m - 5$

$3m = -170 + 5$

$3m = -165$

$m = -\frac{165}{3}$

$m = -55$

(ii) The roots of the equation $x^2 + 7x + 3m - 5 = 0$ **satisfy the relation** $3\alpha - 2\beta = 4$

Solution: $x^2 + 7x + 3m - 5 = 0$

$a = 1, \quad b = 7, \quad c = 3m - 5$

Let α *and* β *be the roots of given equation; then*

sum of roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{(7)}{2} = -7 \ldots\ldots$ (i)

product of roots $P = \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5 \ldots\ldots$ (ii)

From (i) $\beta = -7 - \alpha$ ....(iii)

Now, given that $3\alpha - 2\beta = 4$

Put $\beta = -7 - \alpha$ *in it* we get

$3\alpha - 2(-7 - \alpha) = 4$

$3\alpha + 14 + 2\alpha = 4$

$3\alpha + 2\alpha = 4 - 14$

Putting values of α, and β in it we get.

$$2\left(-\frac{4}{3}\right) = \frac{7m+2}{3}$$

$$\frac{-8}{3} = \frac{7m+2}{3}$$

$$7m + 2 = -8$$

$$7m = -8 - 2$$

$$7m - 10$$

$$m = -\frac{10}{7}$$

6. Find m, if sum and product of the roots of the following equations is equal to a given number?

(i) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Solution: $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

$a = 2m+3,\ b = 7m-5,\ c = 3m-10$

Let α and β be the roots of given equation; then

Sum of roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{(7m-5)}{(2m+3)} = \lambda$ ………..(i)

Product of roots $P = \alpha\beta = \frac{c}{a} = \frac{3m-10}{2m+3} = \lambda$ ……….(ii)

Given that **Sum and product of the roots are equal, so**

$\alpha + \beta = \alpha\beta$

$$-\frac{(7m-5)}{2m+3} = \frac{3m-10}{2m+3}$$

$$-(7m-5) = 3m - 10$$

$$-7m + 5 = 3m - 10$$

$$-7m - 3m = -10 - 5$$

$$-10m = -15$$

$$m = \frac{-15}{-10}$$

$$m = \frac{3}{2}$$

(ii) $4x^2 - (3+5m)x - (9m-17) = 0$

Solution: $4x^2 - (3+5m)x - (9m-17) = 0$

$a = 4,\ b = -(3+5m),\ c = -(9m-17)$

Let α and β be the roots of the equation, then

Sum of roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{-(3+5m)}{4} = \lambda$

$5\alpha = -10$

$\alpha = -2$

Now Put $\alpha = -2$ *in (iii) we get*

$\beta = -7 - (-2)$

$\beta = -7 + 2$

$\beta = -5$

Now from (ii) $\alpha\beta = 3m - 5$

Putting value of α and β in it we get

$$(-2)(-5) = 3m - 5$$
$$10 = 3m - 5$$
$$3m = 10 + 5$$
$$3m = 15$$
$$m = \frac{15}{3}$$
$$m = 5$$

(iii) the roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$

Solution: $3x^2 - 2x + 7m + 2 = 0$

$a = 3, \quad b = -2, \quad c = 7m + 2$

Let α and β be the roots of given equation; then

sum of roots $S = \alpha + \beta = -\frac{b}{a} = -\frac{(-2)}{3} = \frac{2}{3}$ ……. (i)

product of roots $P = \alpha\beta = \frac{c}{a} = \frac{3m + 2}{3}$ ……. (ii)

From (i) $\beta = \frac{2}{3} - \alpha$………. (iii)

Given relation is $7\alpha - 3\beta = 18$

Putting $\beta = \frac{2}{3} - \alpha$ *in it we get (iii)*

$$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$$

$$7\alpha - 2 + 3\alpha = 18$$
$$10\alpha = 18 + 2$$
$$10\alpha = 20$$
$$\alpha = 2$$

Putting $\alpha = 2$ in (iii) we get

$$\beta = \frac{2}{3} - 2 = \frac{2-6}{3} = \frac{-4}{3}$$

Now $\alpha\beta = \frac{7m+2}{3}$

$$Product\ of\ roots\ P = \alpha\beta = \frac{c}{a} = \frac{-(9m-17)}{4} = \lambda$$

Given that Sum and product of the roots are equal so

$\boldsymbol{\alpha + \beta = \alpha\beta}$

$$-\frac{-(3+5m)}{4} = \frac{-(9m-17)}{4}$$

$$3 + 5m = -9m + 17$$

$$5m + 9m = 17 - 3$$

$$14m = 14$$

$$m = 1$$