Exercise 2.4

1. If α, are the roots of the equation $x^2 + px + q = 0$, then evaluate
   (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3\beta + \alpha\beta^3$
   (iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution: (i) $\alpha^2 + \beta^2$

$x^2 + px + q = 0$

$$a = 1, \quad b = p, \quad c = q$$

$$sum\ of\ roots\ S = \alpha + \beta = -\frac{-b}{a} = \frac{-p}{1} = -p$$

$$product\ of\ roots\ P = \alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

Since $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

By *Putting the values of* $\alpha + \beta$ *and* $\alpha\beta$, *we get*

$$\alpha^2 + \beta^2 = (-p)^2 - 2q$$
$$\alpha^2 + \beta^2 = p^2 - 2q$$

Solution: (ii) $\alpha^3\beta + \alpha\beta^3$

$$x^2 + px + q = 0$$
$$a = 1, \quad b = p, \quad c = q$$

$$sum\ of\ roots\ S = \alpha + \beta = -\frac{-b}{a} = \frac{-p}{1} = -p$$

$$product\ of\ roots\ P = \alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

$$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$$
$$= \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$$

By *Putting the values of* $\alpha\beta$ *and* $\alpha +$

$$\alpha^3\beta + \alpha\beta^3 = q[(-p)^2 - 2q]$$
$$\alpha^3\beta + \alpha\beta^3 = q(p^2 - 2q)$$

Solution: **(iii)** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$x^2 + px + q = 0$$
$$a = 1, \quad b = p, \quad c = q$$

$$\text{sum of roots } S = \alpha + \beta = -\frac{-b}{a} = \frac{-p}{1} = -p$$

$$\text{product of roots } P = \alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$$
$$= \frac{1}{\alpha\beta}[\alpha^2 + \beta^2]$$
$$= \frac{1}{\alpha\beta}[(\alpha + \beta)^2 - 2\alpha\beta]$$

By Putting values of $\alpha + \beta$ *and* $\alpha\beta$*, we get*

$$= \frac{1}{q}[(-p)^2 - 2q]$$
$$= \frac{1}{q}[p^2 - 2q]$$

2. **If α, β are the roots of the equation $4x^2 - 5x + 6 = 0$, then find the values of**

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ **(ii)** $\alpha^2\beta^2$

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ **(iv)** $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution: **(i)** $\frac{1}{\alpha} + \frac{1}{\beta}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$4x^2 - 5x + 6 = 0$$
$$a = 4, \quad b = -5, \quad c = 6$$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{(-5)}{4} = \frac{5}{4}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

by Putting the values of $\alpha + \beta$ *and* $\alpha\beta$ *we get*

$$= \frac{\frac{5}{4}}{\frac{3}{2}} = \frac{5}{4} \times \frac{2}{3}$$

$$= \frac{5}{6}$$

Solution: **(ii)** $\alpha^2\beta^2$

$$4x^2 - 5x + 6 = 0$$

$$a = 4, \quad b = -5, \quad c = 6$$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

$\alpha^2\beta^2 = (\alpha\beta)^2$

$= \left(\frac{3}{2}\right)^2$ *Putting the value of $\alpha\beta$, we get.*

$= \frac{9}{4}$

Solution: **(iii)** $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$

$$4x^2 - 5x + 6 = 0$$

$$a = 4, \quad b = -5, \quad c = 6$$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{(-5)}{4} = \frac{5}{4}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{\beta+\alpha}{\alpha^2\beta^2} = \frac{(\alpha+\beta)}{(\alpha\beta)^2}$

By Putting values of $\alpha + \beta$ and $\alpha\beta$ we get

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \left(\frac{\frac{5}{4}}{\left(\frac{3}{2}\right)^2}\right) = \left(\frac{\frac{5}{4}}{\frac{9}{4}}\right)$$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{5}{4} \times \frac{4}{9} = \frac{5}{9}$$

Solution: **(iv)** $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ $4x^2 - 5x + 6 = 0$

$$4x^2 - 5x + 6 = 0$$

$$a = 4, \quad b = -5, \quad c = 6$$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{(-5)}{4} = \frac{5}{4}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$$= \frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

By Putting values of $\alpha+\beta$, and $\alpha\beta$ we get

$$= \frac{\left(\frac{5}{4}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{4}\right)}{\frac{3}{2}} = \frac{\frac{125}{54} - \frac{45}{8}}{\frac{3}{2}}$$

$$= \frac{125-360}{64} \times \frac{2}{3} = -\frac{235}{64} \times \frac{2}{3}$$

$$= -\frac{235}{32} \times \frac{1}{3} = -\frac{235}{96}$$

(a+β)³

(a+b)³

3. **If α, β are the roots of the equation $lx^2 + mx + n = 0$ $(l \neq 0)$, then find the value of**

(i) $\alpha^3\beta^2 + \alpha^2\beta^3$ **(ii)** $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

Solution: **(i)** $\alpha^3\beta^2 + \alpha^2\beta^3$

$lx^2 + mx + n = 0$

$a = l, \quad b = m, \quad c = n$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{m}{l}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{n}{l}$

$\alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha+\beta)$

$= (\alpha\beta)^2(\alpha+\beta)$

By*Putting the values of $\alpha+\beta$ and $\alpha\beta$, we get*

$$= \left(\frac{n}{l}\right)^2\left(\frac{-m}{l}\right) = \left(\frac{n^2}{l^2}\right)\left(\frac{-m}{l}\right)$$

$$= -\frac{mn^2}{l^3}$$

Solution: **(ii)** $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$lx^2 + mx + n = 0$

$a = l, \quad b = m, \quad c = n$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{m}{l}$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{n}{l}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{\alpha^2\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

By Putting values of $\alpha+\beta, \alpha\beta$ *we get*

$$= \frac{\left(\frac{-m}{l}\right)^2-2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \left(\frac{m^2}{l^2}-\frac{2n}{l}\right)\times\left(\frac{l}{n}\right)^2$$

$$= \left(\frac{m^2-2nl}{l^2}\right)\times\frac{l^2}{n^2}$$

$$= \frac{1}{n^2}(m^2-2nl)$$