1. Write the quadratic equations having following roots.

(a) 1, 5

Solution:

$S = Sum\ of\ roots = 1 + 5 = 6$

$P = Product\ of\ the\ roots = (1)(5) = 5$

Quadratic Equation is:

$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$

$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 6x + 5 = 0$$

(b) 4, 9

Solution:

$S = Sum\ of\ roots = 4 + 9 = 13$

$P = Product\ of\ the\ roots = 4 \times 9 = 36$

Quadratic Equation is:

$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$

$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 13x + 36 = 0$$

(c) $-2, 3$

Solution:

$$S = Sum\ of\ roots = -2 + 3 = 1$$
$$P = Product\ of\ the\ roots = (-2)(3) = -6$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - (1)x + (-6) = 0$$
$$x^2 - x - 6 = 0$$

(d) $0, -3$

Solution:

$$S = Sum\ of\ roots = 0 + (-3) = 0 - 3 = -3$$
$$P = Product\ of\ the\ roots = (0)(-3) = 0$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 3x + 0 = 0$$
$$x^2 + 3x = 0$$

(e) $2, -6$

Solution:

$$S = Sum\ of\ roots = 2 + (-6) = 2 - 6 = -4$$
$$P = Product\ of\ the\ roots = (2)(-6) = -12$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 4x + (-12) = 0$$
$$x^2 + 4x - 12 = 0$$

(f) $-1, -7$

Solution:

$$S = Sum\ of\ roots = (-1)(-7) = 1 - 7 = -8$$
$$P = Product\ of\ the\ roots = (-1)(-7) = 7$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - (-8)x + 7 = 0$$
$$x^2 + 8x + 7 = 0$$

(g) $1+i,\ 1-i$

Solution:

$$S = Sum\ of\ roots = 1+i+1-i = 2$$
$$P = Product\ of\ the\ roots = (1+i)(1-i)$$
$$= (1)^2 - (i)^2$$
$$= (1)^2 - (\sqrt{-1})^2 \quad \because i = \sqrt{-1}$$
$$= 1-(-1) = 1+1 = 2$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 2x + 2 = 0$$

(h) $3+\sqrt{2},\ 3-\sqrt{2}$

$$S = Sum\ of\ roots = 3+\sqrt{2}+3-\sqrt{2} = 6$$
$$P = Product\ of\ the\ roots = (3+\sqrt{2})(3-\sqrt{2})$$
$$= (3)^2 - (\sqrt{2})^2$$
$$= 9-2 = 7$$

Quadratic Equation is:

$$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$$
$$x^2 - Sx + P = 0$$

By Putting values of S and P, we get

$$x^2 - 6x + 7 = 0$$

2. If α, β are the roots of the equation $x^2 - 3x + 6 = 0$ From equation whose roots are

(a) $2\alpha+1, 2\beta+1$ **(b)** α^2, β^2

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$ **(d)** $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(e) $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$

Solution: (a) $2\alpha+1, 2\beta+1$

$$x^2 - 3x + 6 = 0$$
$$a = 1, \quad b = -3, \quad c = 6$$

Sum of roots $\alpha+\beta = -\frac{b}{c} = -\frac{-3}{1} = 3$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Now, Roots of the required equation are: $2\alpha+1, 2\beta+1$

therefore,

$$S = Sum\ of\ the\ roots = 2\alpha+1+2\beta+1$$
$$= 2\alpha+2\beta+2$$
$$S = 2(\alpha+\beta)+2$$

by Putting values of $\alpha+\beta$, we get

$= 2(3) + 2 = 6 + 2 = 8 \quad \ldots\ldots\ldots (i)$

$P = \text{Product of the roots} = (2\alpha+1)(2\beta+1)$

$= 4\alpha\beta + 2\alpha + 2\beta + 1$

$= 4\alpha\beta + 2(\alpha+\beta) + 1$

Putting values of $\alpha\beta$ and $\alpha+\beta$ we get

$= 4(6) + 2(3) + 1$

$= 24 + 6 + 1 = 31 \quad \ldots\ldots\ldots\ldots (ii)$

Equation of roots is: $x^2 - Sx + P = 0$

By putting value of S and P from (i) and (ii) we get

$$x^2 - 8x + 31 = 0$$

(b) α^2, β^2

$$x^2 - 3x + 6 = 0$$

$$a = 1, \quad b = -3, \quad c = 6$$

Sum of roots $\alpha+\beta = -\frac{b}{c} = -\frac{-3}{1} = 3$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Now, Roots of the required equation are: α^2, β^2

$S = \text{Sum of the roots} = \alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta \quad \ldots\ldots\ldots\ldots (i)$

by Putting values of $\alpha+\beta$, we get

$= (3)^2 - 2(6)$

$= 9 - 12 = -3 \quad (i)$

$P = \text{Product of the roots} = \alpha^2, \beta^2$

$= (\alpha\beta)^2$

$= (6)^2$

$= 36 \quad \ldots\ldots\ldots\ldots (ii)$

Required equation roots is

$x^2 - Sx + P = 0$

Putting the values of S and P from (i)and (ii) we get

$x^2 - (-3)x + 36 = 0$

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

$$x^2 - 3x + 6 = 0$$

$$a = 1, \quad b = -3, \quad c = 6$$

Sum of roots $\alpha+\beta = -\frac{b}{c} = -\frac{-3}{1} = 3$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Now, Roots of the required equation are: $\frac{1}{\alpha}, \frac{1}{\beta}$

$$S = \text{Sum of the roots} = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

By Putting values of $\alpha\beta$*, we get*

$$S = \frac{3}{6} = \frac{1}{2} \quad \ldots\ldots\ldots\ldots (i)$$

$$P = \text{Product of the roots} = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$$

$$= \frac{1}{\alpha\beta}$$

$$= \frac{1}{6} \quad \ldots\ldots\ldots (ii)$$

Required equation is:

$$x^2 - Sx + P = 0$$

By *Putting the value of S and P from (i) and (ii)*

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiplying by both sides by 2 we get

$$6x^2 - 3x + 1 = 0$$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$x^2 - 3x + 6 = 0$$

$$a = 1, \quad b = -3, \quad c = 6$$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{-3}{1} = 3$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Now, Roots of the required equation are: $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$S = \text{Sum of the roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

By Putting values of $\alpha + \beta$, *and* $\alpha\beta$ *we get*

$$= \frac{(3)^2 - 2(6)}{6}$$

$$= \frac{9-12}{6}$$

$$= -\frac{3}{6} = -\frac{1}{2} \qquad \text{........}(i)$$

$$P = \text{Product of the roots} = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1 \qquad \text{.......}(ii)$$

Required equation is:

$$x^2 - Sx + P = 0$$

By Putting the values of S and P from (i) and (ii)

$$x^2 - \frac{1}{2}x + 1 = 0$$

$$x + \frac{1}{2}x + 1 = 0$$

Multiplying by both sides by 2 we get

$$2x + x + 2 = 0$$

(e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

$$x^2 - 3x + 6 = 0$$

$$a = 1, \quad b = -3, \quad c = 6$$

Sum of roots $\alpha + \beta = -\frac{b}{c} = -\frac{-3}{1} = 3$

Product of roots $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Now, Roots of the required equation are: $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

$$S = \text{Sum of the roots} = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= (\alpha + \beta) + \frac{\beta + \alpha}{\alpha\beta}$$

$$= (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$$

By Putting values of $\alpha + \beta$ *and* $\alpha\beta$ *we get*

$$S = 3 + \frac{3}{6}$$
$$= 3 + \frac{1}{2} = 3\frac{1}{2} = \frac{7}{2} \quad \text{.........} (i)$$

$$P = Product\ of\ the\ roots = (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$
$$= (\alpha + \beta)\left(\frac{\alpha + \beta}{\alpha\beta}\right) \qquad or$$
$$= (\alpha + \beta)\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$
$$= (3)\left(\frac{3}{6}\right) = \frac{1}{2}$$

Required equation is:

$$x^2 - Sx + P = 0$$

By Putting values of S and P from (i)and (ii)we get

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying by both sides by 2 we get

$$2x^2 - 7x + 3 = 0$$

3. If $\propto, \beta$ are the roots of the equation $x^2 + px + q = 0$. Form equations whose roots are:

(a) $\propto^2, \beta$ **(b)** $\frac{\propto}{\beta}, \frac{\beta}{\propto}$

Solution:

Given equation is $x^2 + px + q = 0$ *whose roots are* $\propto, \beta$.

Sum of the roots $\propto + \beta = -\frac{p}{1} = -p,$

Product of the roots $\propto \beta = \frac{q}{1} = q \quad \text{.............} (i)$

(a) *Roots of the required equation are:* $\propto^2, \beta^2$

$$S = Sum\ of\ the\ roots = \propto^2 + \beta^2$$
$$= (\propto + \beta)^2 - 2 \propto \beta$$
$$= (-p)^2 - 2q \qquad from(i)$$
$$= p^2 - 2q \ldots\ldots\ldots\ldots (ii)$$
$$P = Product\ of\ the\ roots = (\propto)^2(\beta)^2$$
$$= (\propto \beta)^2 \qquad (from\ i)$$
$$= q^2 \ldots\ldots\ldots\ldots (iii)$$

therefore required equation is:

$$x^2 - Sx + P = 0$$

Putting the values of S and P from (ii)and (iii) we get

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

(b) Roots of the required equation are: $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Given equation is $x^2 + px + q = 0$ *whose roots are* $\alpha; \beta$.

Sum of the roots $\alpha + \beta = -\frac{p}{1} = -p \ldots\ldots\ldots\ldots (i)$

Product of the roots $\alpha\beta = \frac{q}{1} = q \quad \ldots\ldots\ldots\ldots (ii)$

Roots of the required equation are: $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$S = Sum\ of\ the\ roots = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

Putting the values of $\alpha + \beta$ *and* $\alpha\beta$ *from (i) and (ii) in (iii) we get*

$$= \frac{(-P)^2 - 2q}{q}$$

$$= \frac{p^2 - 2q}{q} \ldots\ldots\ldots\ldots (iv)$$

$$P = Product\ of\ the\ roots = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1 \ldots\ldots\ldots (v)$$

Required equation is:

$$x^2 - Sx + P = 0$$

Putting the values of S and P from (iv) and (v)

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

multiplying both side by q we get

$$qx^2 - (p^2 - 2q)x + q = 0$$