Exercise 2.6

1. Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

Solution: $P(x) = x^2 + 7x - 1$

From divisor $x - a = x - (-1)$

$\therefore \quad a = -1$

Now write co – efficient of divided in a row and a = −1 on the left side

-1	1	7		-1
	↓	-1		-6
	1	6		-7

Depressed equation is $x + 6$

$$Quotient = Q(x) = x + 6$$

$$Remainder = R = -7$$

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

Solution:

$$P(x) = 4x^3 - 5x + 15$$

$$4x^3 + 0x^2 - 5x + 15$$

From divisor $x - a = x - (-3)$

$$a = -3$$

Now write co – efficient of divided in a row and $a = -3$ *on the left side*

-3	4	0	-5	15
	↓	-12	36	-93
	4	-12	31	-78

Depressed equation is $4x^2 - 12x + 31$

$$Quotient = Q(x) = 4x^2 - 12x + 31$$

$$Remainder = R = -78$$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

Solution:

$$P(x) = x^3 + x^2 - 3x + 2$$

From divisor $x - a = x - 2$

$$a = 2$$

Now write co – efficient of divided in a row and $a = 2$ *on the left side*

2	1	1	-3	2
	↓	2	6	6
	1	3	3	8

Depressed equation is $x^2 + 3x + 3$

$$Quotient = Q(x) = x^2 + 3x + 3$$

$$Remainder = R = 8$$

2. Find the value of h using synthetic division, if

(i) *3 is the zero of the polynomial* $2x^3 - 3hx^2 + 9$

Solution:

$$P(x) = 2x^3 - 3hx^2 + 9$$

$$= 2x^3 - 3hx^2 + 0x + 9$$

Here $a = 3$

3	2	$-3h$	0	9
	↓	6	$-9h+18$	$-27h+54$
	2	$-3h+6$	$-9h+18$	$-27h+63$

If 3 is zero of P(x), then Reminder $R = 0$

Here, $R = -27h + 63 = 0$

$$27h + 63 = 0$$

$$-27h = -63$$

$$h = \frac{-63}{-27} = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

Solution:

$$= P(x) = x^3 - 2hx^2 + 11$$

$$= x^3 - 2hx^2 + 0x11$$

Here $a = 1$

1	1	$-2h$	0	11
	↓	1	$-2h+1$	$-2h+1$
	1	$-2h+1$	$-2h+1$	$-2h+12$

If 1 is zero of the polynomail, then Reminder $R = 0$

therefore $R = -2h + 12 = 0$

$$-2h + 12 = 0$$

$$-2h = -12$$

$$h = \frac{-12}{-12} = 6$$

(iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$

Solution:

$$P(x) = 2x^3 + 5hx - 23$$

$$= 2x^3 + 0x^2 + 5hx - 23$$

Here $a = -1$

-1	2	0	$5h$	-23
	↓	-2	2	$-5h-2$
	2	-2	$5h+2$	$-5h-25$

If -1 *is zero of the polynomial, then Reminder* $R = 0$

therefore, $R = -5h - 25 = 0$

$$-5h - 25 = 0$$

$$h = \frac{25}{-5} = -5$$

3. **Use synthetic division to find the values of l and m, if**

(i) $(x+3)$ **and** $(x-2)$ **are the factors of the polynomial** $x^3+4x^2+2lx+m$

Solution:

Let $P(x) = x^3 + 4x^2 + 2lx + m$

and $x - a = x - (-3)$

therefore $a = -3$

By synthetic division

	1	4	$2l$	m
-3	↓	-3	-3	$-6l+9$
	1	1	$2l-3$	$m-6l+9$

Since -3 *is zero of the polynomial, therefore,* $R = 0$

$$R = m - 6l + 9 = 0$$

$-6l + m + 9 = 0 \ldots\ldots\ldots\ldots (i)$

Now Again $x - a = x - (-2)$

therefore $a = -3$

By synthetic division

	1	4	$2l$	m
2	↓	2	12	$4l+24$
	1	6	$2l+12$	$m+4l+24$

Since 2 is zero of the polynomial, therefore, $R = 0$

$$R = 4l + m + 24 = 0$$

$4k + m + 24 = 0 \ldots\ldots\ldots\ldots (ii)$

By Subtracting (ii) from (i) we get

$-10l - 15 = 0$

$-10l = 15$

$l = \dfrac{15}{-10} = -\dfrac{3}{2}$

by Putting $l = -\dfrac{3}{2}$ *in (i), we get*

$-6\left(-\dfrac{3}{2}\right) + m + 9 = 0$

$9 + m + 9 = 0$

$m = -9 - 9$

$m = -18$

therefore, $l = -\dfrac{3}{2},\ m = -18$

$4l + m + 24$
$-6l \pm m \pm 9$
$10l$ [illegible] 15
$10l - 15 = 0$
$-10l = 15$
$l = \dfrac{15}{-10} = -\dfrac{3}{2}$

(ii) $(x-1)$ **and** $(x+1)$ **are the factors of the polynomial** $x^3 - 3lx^2 + 2mx + 6$

Solution:

Let $P(x) = x^3 - 3lx^2 + 2mx + 6$

$x - a = x - 1$

$\therefore \quad a = 1$

Using synthetic division we get

	1	$-3l$	$2m$	6
1	↓	1	$-3l+1$	$2m-3l+1$
	1	$-3l+1$	$2m-3l+1$	$2m-3l+7$

Since 1 is zero of the polynomial, therefore, $R = 0$

$R = 2m - 3l + 7 = 0$

Thus $2m - 3l + 7 = 0 \ldots\ldots\ldots\ldots(i)$

$P(x) = x^3 - 3lx^2 + 2mx + 6$

Again $x - a = x - (-1)$

$\therefore \quad a = -1$

Using synthetic division

	1	$-3l$	$2m$	6
-1	↓	-1	$3l+1$	$-2m-3l-1$
	1	$-3l-1$	$2m+3l+1$	$-2m-3l+5$

Since -1 *is zero of the polynomial, therefore,* $R = 0$

$R = -2m - 3l + 5 = 0$

Thus $-2m - 3l + 5 = 0$

By Subtracting (ii) from (i), we get

$4m + 2 = 0$

$= m = -\frac{2}{4} = -\frac{1}{2}$

Putting $m = -\frac{1}{2}$ *in (i) we get*

$= 2\left(-\frac{1}{2}\right) - 3l + 7 = 0$

$-1 - 3l + 7 = 0$

$-3l = -7 + 1$

$-3l = -6$

$l = \frac{-6}{-3} = 2$

therefore, $l = 2, \; m = -\frac{1}{2}$

4. Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

Solution:

Let, $P(x) = x^3 - 28x + 48$

$= x^3 + 0x^2 - 28x + 48$

and $a = 2$

Using synthetic equation

2	1	0	−28	48
	↓	2	4	−48
	1	2	−24	0

Therefore depressed equation is

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x + 6) - 4(x + 6) = 0$

$(x + 6)(x - 4) = 0$

$x + 6 = 0$ *gives* $x = -6$

or $x - 4 = 0$ *gives* $x = 4$

Hence $2, -6$ *and* 4 *are the roots of the given equation*

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

Solution:

Let, $P(x) = 2x^3 - 3x^2 - 11x + 6$

and $a = 3$

By synthetic division

3	2	−3	−11	6
	↓	6	9	−6
	2	3	−2	0

Therefore depressed equation is:

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0$

$2x(x + 2)(2x - 1) = 0$

$x + 2 = 0$ *gives* $x = -2$

or $2x - 1 = 0$ *gives* $x = \frac{1}{2}$

Hence $3, -2$ *and* $\frac{1}{2}$ *are the roots of the given equation*

(iii) −1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

Solution:

Let, $P(x) = 4x^3 - x^2 - 11x - 6 = 0$

and $a = -1$

By synthetic division

	4	−1	−11	−6
−1	↓	−4	5	6
	4	−5	−6	0

Depressed equation is:

$$4x^2 - 5x - 6 = 0$$
$$4x^2 - 8x + 3x - 6 = 0$$
$$4x(x-2) + 3(x-2) = 0$$
$$(x-2)(4x+3) = 0$$

$x - 2 = 0$ *gives* $x = 2$

or $4x + 3 = 0$ *gives* $x = -\frac{3}{4}$

Hence $-1, 2$ *and* $-\frac{3}{4}$ *are the roots of the given equation*

5. Solve by using synthetic division, if

(i) 1 and 3 are the roots of equation $x^4 - 10x^2 + 9 = 0$

Solution:

Let, $P(x) = x^4 - 10x^2 + 9$

$= x^4 + 0x^3 - 10x^2 + 0x + 9$

and $a = 1$

By synthetic division

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0

Depressed equation is

$$x^3 + x^2 - 9x - 9 = 0$$

3 is again a root of the given equation therefore,

	1	1	−9	−9
3	↓	3	12	9
	1	4	3	0

Depressed equation is

$$x^2 + 4x + 3 = 0$$
$$x^2 + x + 3x + 3 = 0$$
$$x(x+1) + 3(x+1) = 0$$
$$(x+1)(x+3) = 0$$

$= 0$ *gives* $x = -1$

or $\quad x+3=0 \qquad$ gives $\quad x=-3$

Hence 1, 3, −1 and −3 are the roots of the given equation

(ii) 3 and −4 are the roots of the equation

$$x^4+2x^3-13x^2-14x+24=0$$

Solution:

Let, $\qquad P(x)=x^4+2x^3-13x^2-14x+24$

and $\qquad \alpha=3$

By synthetic division

1	1	2	−13	−14	24
	↓	3	15	6	−24
	1	5	2	−8	0

Depressed equation is:

$$x^3+5x^2+2x-8=0$$

Again $\qquad \alpha=-4$

Therefore,

−4	1	5	2	−8
	↓	−4	−4	8
	1	1	−2	0

Depressed equation is:

$$x^2+x-2=0$$
$$x^2+2x-x-2=0$$
$$x(x+2)-1(x+2)=0$$
$$(x+2)(x-1)=0$$

$x+2=0 \qquad$ gives $\quad x=-2$

or $\quad x-1=0 \qquad$ gives $\quad x=1$

Hence 3, −4, −2 and 1 are the roots of the given equation