Exercise 2.7

Solve the following simultaneous equations.

1. $x + y = 5$; $x^2 - 2y - 14 = 0$

Solution:

$$x + y = 5 \quad \ldots\ldots\ldots\ldots (i)$$

$$x^2 - 2y - 14 = 0 \quad \ldots\ldots\ldots\ldots (ii)$$

From (i) we get $x = 5 - y \quad \ldots\ldots\ldots (iii)$

Putting $x = 5 - y$ *in* (ii), *we get*

$$(5 - y)^2 - 2y - 14 = 0$$

$$25 + y^2 - 10y - 2y - 14 = 0$$
$$y^2 - 12y + 25 - 14 = 0$$
$$y^2 - 12y + 11 = 0$$
$$y^2 - y - 11y + 11 = 0$$
$$y(y-1) - 11(y-1) = 0$$
$$(y-1)(y-11) = 0$$
$y - 1 = 0$ gives $y = 1$

or $y - 11 = 0$ gives $y = 11$

Case i:

Put $y = 1$ in (i), we get
$$x + 1 = 5$$
$$= x = 5 - 1 = 4$$
therefore first order pair is $(4, 1)$

Case ii:

When $y = 11$

Put $y = 11$ in (i) we get
$$x + 11 = 5$$
$$x = 5 - 11 = -6$$
therefore Second order pair is $(-6, 11)$

Hence Solution set $= \{(4, 1), (-6, 11)\}$

2. $3x - 2y = 1$; $x^2 + xy - y^2 = 1$

Solution: $3x - 2y = 1$ ....(i)

$x^2 + xy - y^2 = 1$ ....(ii)

From (i) $3x = 1 + 2y$
$$x = \frac{1 + 2y}{3} \quad \ldots\ldots\ldots\ldots (iii)$$

By Putting this value of x in equation (ii) we get
$$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)(y) - y^2 = 1$$
$$\frac{1+4y^2+4y}{9} + \frac{y+2y^2}{3} - y^2 = 1$$

Multiplying by 9, both sides we get
$$1 + 4y^2 + 4y + 3(y + 2y^2) - 9y^2 = 9$$
$$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 - 9 = 0$$
$$y^2 + 7y - 8 = 0$$
$$y^2 - y + 8y - 8 = 0$$
$$y(y-1) + 8(y-1) = 0$$
$$(y-1)(y+8) = 0$$
$y - 1 = 0$ gives $y = 1$

$y+8=0$ gives $y=-8$

Case i:

Put $y=1$ in equation (i) we get

$$3x-2(1)=1$$
$$3x-2=0$$
$$3x=1+2$$
$$3x=3$$
$$x=1$$

therefore first order pair is $(1,1)$

Case ii:

When $y=-8$, put $y=-8$ in equation (i) we get

$$3x-2(-8)=1$$
$$3x+16=1$$
$$3x=1-16$$
$$3x=-15$$
$$x=\frac{-15}{3}=-5$$

therefore first order pair is $(-5,-8)$

Solution set $=\{(1,1);(-5,-8)\}$

3. $x-y=7;\quad \frac{2}{x}-\frac{5}{y}=2$

Solution: $x-y=7$ ....(i)

$\frac{2}{x}-\frac{5}{y}=2$ .... (ii)

Multiplying equation (ii) by xy we get

$2y-5x=2xy$ .... (iii)

From equation (i) we get

$$x-y=7$$
$$x=7+y$$

Putting $=x=7+y$ in (iii), we get

$$2y-5(7+y)=2(7y+y)(y)$$
$$2y-35-5y=2(7y+y^2)$$
$$-3y-35=14y+2y^2$$
$$2y^2+14y+3y+35=0$$
$$2y^2++17y+35=0$$
$$2y^2+7y+10y+35=0$$
$$y(2y+7)+5(2y+7)=0$$
$$(2y+7)(y+5)=0$$

$2y+7=0$ gives $y=-\frac{7}{2}$

$$y+5=0 \qquad \text{gives } y=-5$$

Case i:

Putting $y=-\frac{7}{2}$ in (i), we get

$$x-\left(-\frac{7}{2}\right)=7$$

$$x+\frac{7}{2}=7$$

$$x=7-\frac{7}{2}$$

$$x=\frac{14-7}{2}$$

$$x=\frac{7}{2}$$

therefore first order pair is $\left(\frac{7}{2},-\frac{7}{2}\right)$

Case ii:

Putting $y=-5$ in (i), we get

$$x-5=7$$
$$x+5=7$$
$$x=7-5$$
$$x=2$$

therefore Second order pair is $(2,-5)$

Hence Solution set $=\left\{(2,-5)\left(\frac{7}{2},\frac{-7}{2}\right)\right\}$

4. $x+y=a-b; \frac{a}{x}-\frac{b}{y}=2$

Solution: $x+y=a-b$ .... (i)

$$\frac{a}{x}-\frac{b}{y}=2 \quad \text{.......... (ii)}$$

Multiplying both sides by xy we get

$$ay-bx=2xy \quad \text{...........(iii)}$$

from (i) $\quad x=a-b-y$ ....(iv)

Putting the value of x in (iii), we get.

$$ay-b(a-b-y)=2(a-b-y)(y)$$
$$ay-ab+b^2+by=2ay-2by-2y^2$$
$$2y^2+ay^2+by-2ay+2by-ab+b^2=0$$
$$2y^2-ay+3by-ab+b^2=0$$
$$2y^2-(a-3b)y-(ab-b^2)=0$$

Here $a=2, b=-(a-3b)$ and $c=-(ab-b^2)$

$$y = \frac{(a-3b) \pm \sqrt{[-(a-3b)]^2+4(2)(ab-b^2)}}{2(2)}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2+9b^2-6ab+8ab-8b^2}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2+2ab+b^2}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{(a+b)^2}}{4}$$

$$y = \frac{(a-3b) \pm (a+b)}{4}$$

$$y = \frac{(a-3b) + (a+b)}{4} \quad or \quad y = \frac{(a-3b)-(a+b)}{4}$$

$$y = \frac{a-3b+a+b}{4} \quad or \quad y = \frac{a-3b-a-b}{4}$$

$$y = \frac{2a-2b}{4} \quad or \quad y = \frac{-4b}{4}$$

$$y = \frac{2(a-b)}{4} \quad or \quad y = -b$$

$$y = \frac{a-b}{2}$$

Case i:

when $y = \frac{a-b}{2}$ *Putting it in (iv) we get*

$$x = a - b - \left(\frac{a-b}{2}\right)$$

$$x = \frac{2a-2b-a+b}{2}$$

$$x = \frac{a-b}{2}$$

therefore first order pair is $\left(\frac{a-b}{2}, \frac{a-b}{2}\right)$

Case ii:

when $y = -b$ *Put it in (iv)*

$$x = a - b - (-b)$$

$$x = a - b + b$$

$$x = a$$

therefore second order pair is $(a, -b)$

$$\textbf{Solution set} = \left\{(a, -b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right\}$$

5. $x^2 + (y-1)^2 = 10; \quad x^2 + y^2 + 4x = 1$

Solution:

$$x^2 + (y-1)^2 = 10 \quad \text{..........}(i)$$

$$x^2 + y^2 + 4x = 1 \quad \text{..........}(ii)$$

from (i) $x^2 + y^2 - 2y + 1 = 10$

$$x^2 + y^2 - 2y = 10 - 1$$

$$x^2 + y^2 - 2y = 9 \quad \text{..........}(iii)$$

from (ii) $x^2 + y^2 + 4x = 1$

By Subtracting (ii) from (iii) we get

$$-2y - 4x = 8$$

$$2y + 4x = -8$$

$$y + 2x = -4 \quad (\textit{Dividing by 2 we get})$$

$$y = -4 - 2x \quad \text{..........}(iv)$$

Putting $y = -4 - 2x$ *in (ii) we get*

$$x^2 + (-4 - 2x)^2 + 4x = 1$$

$$x^2 + 16 + 4x^2 + 16x + 4x = 1$$

$$5x^2 + 20x + 15 = 0$$

$$x^2 + 4x + 3 = 0 \quad (\textit{Dividing by 5 we get})$$

$$x^2 + x + 3x + 3 = 0$$

$$x(x+1) + 3(x+1) = 0$$

$$(x+1)(x+3) = 0$$

$$x + 1 = 0 \quad \textit{gives } x = -1$$

$$or = x + 3 = 0 \quad \textit{gives } x = -3$$

Case i:

When $x = -1$ *then from (iv we get)*

$$y = -4 - 2(-3)$$

$$y = -4 + 2 = -2$$

therefore first order pair is $(-1, -2)$

Case ii:

When $x = -3$, *then form (iv) we get*

$$y = -4 - 2(-3)$$

$$y = -4 + 6 = 2$$

therefore second order pair is $(-3, 2)$

Solution set $= \{(-1, -2), (-3, 2)\}$

6. $(x+1)^2+(y+1)^2=5$; $(x+2)^2+y^2=5$

Solution:

$(x+1)^2+(y+1)^2=5$ ....(i)

$(x+2)^2+y^2=5$ ....(ii)

$x^2+2x+1+y^2+2y+1=5$ from (i)

$x^2+y^2+2x+2y+2=5$

$x^2+y^2+2x+2y=3$ ....(iii)

$x^2+4x+4+y^2=5$ from ii

$x^2+y^2+4x=1$ ....(iv)

Subtracting (iv) from (iii) we get

$-2x+2y=2$

$x-y=-1$ (Dividing by -2 we get)

$x=y-1$ ....(v)

Putting this value of x in (iii) we get

$(y-1)^2+y^2+2(y-1)+2y=3$

$y^2-2y+1+y^2+2y-2+2y=3$

$2y^2+2y+1-2-3=0$

$2y^2+2y-4=0$

$y^2+y-2=0$ (Dividing by 2)

$y^2+2y-y-2=0$

$y(y+2)-1(y+2)=0$

$(y+2)(y-1)=0$

$y+2=0$ gives $y=-2$

or $y-1=0$ gives $y=1$

Case i:

When $y=-2$, *then from (v) we get*

$x=-2-1=-3$

therefore first order pair is $(-3,-2)$

Case ii:

When $y=1$, *then from (v) we get*

$x=1-1=0$

therefore second order pair is $\{0,1\}$

Solution set $=\{(0,1),(-3,-2)\}$

7. $x^2+2y^2=22$, $5x^2+y^2=29$

Solution:

$x^2+2y^2=2$ ....(i)

$5x^2+y^2=29$ ....(ii)

Multiplying (i) by 5 we get

$5x^2+10y^2=110$ ....(iii)

Subtaracting (ii) from (iii) we get

$$-9y^2 = -81$$
$$y^2 = 9 \quad (Dividing\ by - 9\ we\ get)$$
$$y = \pm 3$$

Case i:

When $y = 3$, *put it in (i) we get*

$$x^2 + 2(3)^2 = 22$$
$$x^2 + 18 = 22$$
$$x^2 = 22 - 18$$
$$x^2 = 4$$
$$x = \pm 2$$

therefore first order pair is $(\pm 2, 3)$

Case ii:

When $y = -3$, *then from (i) we get*

$$x^2 + 2(-3)^2 = 22$$
$$x^2 + 18 = 22$$
$$x^2 + 18 = 22$$
$$x^2 = 4$$
$$x = \pm 2$$

therefore second order pair is $(\pm 2, -3)$

Solution set $= \{(\pm 2, \pm 3)\}$

8. $4x^2 - 5y^2 = 6$, $3x^2 + y^2 = 14$

Solution:

$$4x^2 - 5y^2 = 6 \quad \ldots\ldots\ldots(i)$$
$$3x^2 + y^2 = 14 \quad \ldots\ldots\ldots(ii)$$

Multiplying (ii) by 5 we get

$$15x^2 + 5y^2 = 70 \quad \ldots\ldots\ldots(iii)$$

Adding (i) and (iii) we get

$$19x^2 = 76$$
$$x^2 = 4$$
$$x = \pm 2$$

Case i:

When $x = 2$, *(ii) gives*

$$3(-2)^2 + y^2 = 14$$
$$12 + y^2 = 14$$
$$y^2 = 14 - 12$$
$$y^2 = 2$$
$$y = \pm\sqrt{2}$$

therefore first order pair is $(2, \pm\sqrt{3})$

Case ii:

When $x = -2$, *(ii) gives*

$$3(-2)^2 + y^2 = 14$$
$$12 + y^2 = 14$$
$$y^2 = 14 - 12$$
$$y^2 = 2$$
$$y = \pm\sqrt{2}$$

therefore second order pair is $(-2, \pm\sqrt{2})$

Solution set $= \{(\pm 2, \pm\sqrt{2})\}$

9. $7x^2 - 3y^2 = 4$, $2x^2 + 5y^2 = 7$

Solution:

$$7x^2 - 3y^2 = 4 \quad \text{..........(i)}$$
$$2x^2 + 5y^2 = 7 \quad \text{..........(ii)}$$

Multiplying (i) by 5 and (ii) by 3. we get

$$35x^2 - 15y^2 = 20 \quad \text{..........(iii)}$$
$$6x^2 + 15y^2 = 21 \quad \text{..........(iv)}$$

Subtaracting (iii) from (iv) we get

$$41x^2 = 41$$
$$x^2 = 1$$
$$x = \pm 1$$

Case i:

When $x = 1$, *then (ii) gives*

$$2(1)^2 + 5y^2 = 7$$
$$2 + 5y^2 = 7$$
$$5y^2 = 7 - 2$$
$$5y^2 = 5$$
$$y^2 = 1$$
$$y = \pm 1$$

therefore first order pair is $(1, \pm 1)$

Case ii:

When $x = -1$, *then (ii) gives*

$$2(-1)^2 + 5y^2 = 7$$
$$2 + 5y^2 = 7$$
$$5y^2 = 7 - 2$$
$$5y^2 = 5$$
$$y^2 = 1$$
$$y = \pm 1$$

therefore second order pair is $(-1, \pm 1)$

Solution set $= \{(\pm 1, \pm 1)\}$

10. $x^2 + 2y^2 = 3$, $x^2 + 4xy - 5y^2 = 0$

Solution:

$$x^2 + 2y^2 = 3 \quad \text{...........} (i)$$
$$x^2 + 4xy - 5y^2 = 0 \quad \text{...........}(ii)$$
$$x^2 + 5xy - xy - 5y^2 = 0 \quad \textit{from (ii) we get}$$
$$x(x+5y) - y(x+5y) = 0$$
$$(x+5y)(x-y) = 0$$
$$x - y = 0 \quad \text{gives} \quad x = y \quad \text{...........}(iii)$$
$$x + 5y = 0 \quad \text{gives} \quad x = -5y \quad \text{...........}(iv)$$

Case i:

When $x = y$, *then* (i) *gives*

$$(y)^2 + 2y^2 = 3$$
$$y^2 + 2y^2 = 3$$
$$3y^2 = 3$$
$$y^2 = 1$$
$$y = \pm 1$$

then from (iii), *put* $y = \pm 1$ *in it we get*

$$x = \pm 1$$

therefore first order pair is $(\pm 1, \pm 1)$

Case ii:

When $x = -5y$ *then* (i), *gives*

$$(-5y)^2 + 2y^2 = 3$$
$$25y^2 + 2y^2 = 3$$
$$27y^2 = 3$$
$$y^2 = \frac{3}{27}$$
$$y^2 = \frac{1}{9} \quad \textit{(By taking square root on both sides)}$$
$$y = \pm\frac{1}{3}$$

Put $y = \pm\frac{1}{3}$ *in* (iv) *we get*

$$x = -5\left(\pm\frac{1}{3}\right)$$
$$= \left(\mp\frac{5}{3}\right)$$

therefore second order pair is $\left(\mp\frac{5}{3}, \pm\frac{1}{3}\right)$

Hence Solution set $= \left\{(\pm 1, \pm 1), \left(\mp\frac{5}{3}, \pm\frac{1}{3}\right)\right\}$

Or

Solution set $= \left\{\left(\frac{5}{3}, \frac{-1}{3}\right), \left(\frac{-5}{3}, \frac{1}{3}\right), (1, 1), (-1, -1)\right\}$

11. $3x^2 - y^2 = 26,\qquad 3x^2 - 5xy - 12y^2 = 0$

Solution:

$$3x^2 - y^2 = 26 \quad \ldots\ldots\ldots..(i)$$
$$3x^2 - 5xy - 12y^2 = 0 \quad \ldots\ldots\ldots..(ii)$$
$$3x^2 - 9xy - 4xy - 12y^2 = 0 \quad from\ (ii)$$
$$3x(x - 3y) + 4y(x - 3y) = 0$$
$$(x - 3y)(3x + 4y) = 0$$
$$x - 3y = 0 \quad gives \quad x = 3y \ldots\ldots\ldots..(iii)$$
$$3x + 4y = 0\ gives\ \ x = -\frac{4y}{3} \ldots\ldots\ldots(iv)$$

Case i:

When $x = 3y$, *then* (i) *becomes*

$$3(3y)^2 - y^2 = 26$$
$$3(9y^2) - y^2 = 26$$
$$27y^2 - y^2 = 26$$
$$26y^2 = 26$$
$$y^2 = 1$$
$$y = \pm 1$$

Put $y = \pm 1$ *in* (iii) *we get*

$$x = 3(\pm 1) = \pm 3$$

therefore first order pair is $(\pm 3, \pm 1)$

Case ii:

When $x = \frac{-4y}{3}$ *then* (i) *becomes*

$$3\left(-\frac{4y}{3}\right)^2 - y^2 = 26$$
$$3\left(\frac{16y^2}{9}\right) - y^2 = 26$$
$$\frac{16y^2}{3} - y^2 = 26$$

Multiplying both sides by 3 we get

$$16y^2 - 3y^2 = 78$$
$$13y^2 = 78$$
$$y^2 = \frac{78}{13}$$
$$y^2 = 6$$
$$y = \pm\sqrt{6}$$

Put $y = \pm\sqrt{6}$ *in* (iv) we get

$$= x = \mp\frac{4(\pm\sqrt{6})}{3}$$

therefore second order pair is $\left(\mp\frac{4\sqrt{6}}{3}, \pm\sqrt{6}\right)$

$$\textbf{Solution set} = \left\{(\pm 3, \pm 1), \left(\mp\frac{4\sqrt{6}}{3}, \pm\sqrt{6}\right)\right\}$$

Or

$$\textbf{Solution set} = \left\{(3,1), (-3,-1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$$

12. $x^2 + xy = 5$, $y^2 + xy = 3$

Solution:

$x^2 + xy = 5$ ....(i)

$y^2 + xy = 3$ ....(ii)

from (i) $x(x+y) = 5$ ....(iii)

from (ii) $y(y+x) = 3$

or $y(x+y) = 3$ ....(iv)

By dividing (iii) and (iv) we get

$$\frac{x}{y} = \frac{5}{3}$$

$$x = \frac{5}{3}y \quad ....(v)$$

By Putting $x = \frac{5}{3}y$, *in (i) we get*

$$\left(\frac{5}{3}y\right)^2 + \left(\frac{5}{3}y\right)y = 5$$

$$\frac{25}{9}y^2 + \frac{5}{3}y^2 = 5$$

(*Multiplying both side by 9 we get*)

$$25y^2 + 15y^2 = 45$$

$$5y^2 + 3y^2 = 9$$

(*Dividing both side by 5 we get*)

$$8y^2 = 9$$

$$y^2 = \frac{9}{8}$$

$$y = \pm\sqrt{\frac{9}{8}} = \pm\frac{3}{2\sqrt{2}}$$

$$y = \pm\frac{3}{2\sqrt{2}}$$

Case i:

When $y = +\frac{3}{2\sqrt{2}}$ *then (v) gives*

$$x = \left(\frac{5}{3}\right)\left(\frac{3}{2\sqrt{2}}\right) = \frac{5}{2\sqrt{2}}$$

therefore first order pair is $\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$

Case ii:

When $x = \frac{-3}{2\sqrt{2}}$ *then (v) gives*

$$x = \frac{5}{3}\left(-\frac{3}{2\sqrt{2}}\right)$$

$$-\frac{5}{2\sqrt{2}}$$

therefore second order pair is $\left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)$

Solution set $= \left\{\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right), \left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)\right\}$

13. $x^2 - 2xy = 7$, $xy + 3y^2 = 2$

Solution:

$x^2 - 2xy = 7$ ....(i)

$xy + 3y^2 = 2$ ....(ii)

Multiplying (i) by 2 and (ii) by 7 we get

$2x^2 - 4xy = 14$ ....(iii)

$7xy + 21y^2 = 14$ ....(iv)

By subtracting (iv) from (iii) we get

$2x^2 - 11xy - 21y^2 = 0$

$2x^2 - 14xy + 3xy - 21y^2 = 0$

$2x(x - 7y) + 3y(x - 7y) = 0$

$(x - 7y)(2x + 3y) = 0$

$x - 7y = 0$ *gives* $x = 7y$ ....(v)

$2x + 3y = 0$ *gives* $x = \frac{-3y}{2}$ ....(vi)

Put $x = 7y$ *in (i), then we get*

$(7y)^2 - 2(7y)y = 7$

$49y^2 - 14y^2 = 7$

$35y^2 = 7$

$y^2 = \frac{7}{35}$

$y^2 = \frac{1}{5}$

$y = \pm\frac{1}{\sqrt{5}}$

Putting $y = \pm\frac{1}{\sqrt{5}}$ *in* (v) *we get*

$x = 7\left(\pm\frac{1}{\sqrt{5}}\right)$

$x = \pm\frac{7}{\sqrt{5}}$

therefore first order pair is $\left(\pm\frac{7}{\sqrt{5}}, \pm\frac{1}{\sqrt{5}}\right)$

From (vi), $x = -\frac{3y}{2}$ *Putting this value in* (i) *we get*

$\left(-\frac{3y}{2}\right)^2 - 2\left(\frac{-3y}{2}\right)(y) = 7$

$\frac{9y^2}{4} + 3y^2 = 7$

$9y^2 + 12y^2 = 28$

(Multiplying both sides by 4)

$21y^2 = 28$

$y^2 = \frac{28}{21}$

$y^2 = \frac{4}{3}$

$y = \pm\frac{2}{\sqrt{3}}$

Putting $y = \pm\frac{2}{\sqrt{3}}$ *in* (vi) *we get*

$x = \left(-\frac{3}{2}\right)\left(\pm\frac{2}{\sqrt{3}}\right) \quad \Rightarrow \quad = x = \mp\sqrt{3}$

therefore second order pair is $\left(\mp\sqrt{3}, \pm\frac{2}{\sqrt{3}}\right)$

Solution set $= \left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)\left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)\right\}$