Exercise 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Solution: *Let the two numbers be* $x, x+1$

According to the given condition

$(x)(x+1) = 182$

$x^2 + x - 182 = 0$

$x(x+14) - 13(x+14) = 0$

$(x-13)(x+14) = 0$

$x - 13 = 0$ *gives* $x = 13$

Now, numbers are: $x = 13$

and $x + 1 = 13 + 1 = 14$

Now, $x + 14 = 0$ *gives* $x = -14$

Neglecting − ve value.

2. The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution: *Let the three numbers be* $x, x+1, x+2$

According to the given condition

$(x)^2 + (x+1)^2 + (x+2)^2 = 77$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 77$

$3x^2 + 6x + 5 = 77$

$3x^2 + 6x - 72 = 0$

$x^2 + 2x - 24 = 0$ *(Dividing by 3)*

$x^2 + 6x - 4x - 24 = 0$

$x(x+6) - 4(x+6) = 0$

$(x-4)(x+6) = 0$

$x - 4 = 0$ *gives* $x = 4$

Numbers are when: $x = 4$

$x + 1$ $\quad 4 + 1 = 5$

and $x + 2$ $\quad 4 + 2 = 6$

Therefore Numbers are $4, 5, 6$

Now, $x + 6 = 0$

$x = -6$

Neglecting − ve value.

3. The sum of five times a number and the square of the number is 204. Find the number.

Solution: *Let the number be* x

According to the given condition.

$(x)^2 + 5 \times x = 204$

$x^2 + 5x - 204 = 0$

$x^2 + 17x - 12x - 204 = 0.$

$x(x + 17) - 12(x + 17) = 0$

$(x + 17)(x - 12) = 0$

$x - 12 = 0$ *gives* $x = 12$

Therefore Number is **12**

and $x + 17 = 0$ *gives* $x = -17$

Neglecting − ve value.

4. The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution: *Let the number be x*

According the given condition.

$(3x - 5)(4x - 1) = 7$

$12x^2 - 23x + 5 = 7$

$12x^2 - 23x + 5 - 7 = 0$

$12x^2 - 24x + x - 2 = 0$

$12x(x - 2) + 1(x - 2) = 0$

$(12x + 1)(x - 2) = 0$

$x - 2 = 0$ *gives* $x = 2$

Number is $x = 2$

and $12x + 1 = 0$ *gives* $x = -\frac{1}{12}$

Therefore Numbers are $2, -\frac{1}{12}$

5. The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution: *Let the number be x*

According to the given condition

Then, $x - \frac{1}{x} = \frac{15}{4}$

$4x^2 - 4 = 15x$ (*Multiplying by* $4x$)

$4x^2 - 15x - 4 = 0$

$4x^2 - 16x + x - 4 = 0$

$4x(x - 4) + 1(x - 4) = 0$

$(x - 4)(4x + 1) = 0$

$x - 4 = 0$ *gives* $x = 4$

Number is $x = 4$

and $\quad 4x+1=0 \quad$ gives $\quad x=-\frac{1}{4}$

$$\text{Therefore Numbers are } 4, -\frac{1}{4}$$

6. The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution: Let xy be the number, where unit digit is y and tens digit is x.

According the given condition.

$x^2+y^2=65 \quad \ldots\ldots (i)$

Two digit number $= y+10x$

According to the given condition

$y+10x = 9(x+y)$

$y+10x = 9x+9y$

$10x-9x = 9y-y$

$x = 8y \quad \ldots\ldots\ldots (ii)$

By Putting $x=8y$ in (i) we get

$(8y)^2+y^2 = 65$

$64y^2+y^2 = 65$

$65y^2+y^2 = 65$

$y^2 = 1$

$\therefore \quad y = \pm 1$

$y = 1$ (taking + ve value only)

Put $\quad y = 1$ in (ii) we get

$x = 8(1) = 8$

Therefore, number $= xy = 81$

7. The sum of the co ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinate of the point.

Solution: Let $P(x,y)$ be the co – ordinate of the point.

According to the given conditions.

$x+y=9 \quad \ldots\ldots\ldots\ldots (i)$

and $\quad x^2+y^2=45 \quad \ldots\ldots\ldots\ldots (ii)$

From (i) $x=9-y \quad \ldots\ldots\ldots\ldots (iii)$

Putting $x=9-y$ in (ii) we get

$(9-y)^2+y^2=45$

$81-18y+y^2+y^2-45=0$

$2y^2-18y+36=0$

$2y^2-18y+36=0$

$$y^2 - 9y + 18 = 0 \quad \text{(Dividing by 2)}$$
$$y^2 - 6y - 3y + 18 = 0$$
$$y(y-6) - 3(y-6) = 0$$
$$(y-6)(y-3) = 0$$
$$y - 6 = 0 \quad gives \quad y = 6$$

Then form (iii) we get

$$x = 9 - 6 = 3$$

co – ordinate of the point is $P(3, 6)$

When $y - 3 = 0$ *then* $y = 3$

From (iii) we get

$$x = 9 - 3 = 6$$

co – ordinate of the point is $P(6, 3)$

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution: *Let the two integers be x, y.*

According to the given condition

Then $x + y = 9$ ....(i)

and $x^2 - y^2 = 9$ ....(ii)

From (i) we get

$$x = 9 - y$$

Putting $x = 9 - y$ *in (ii) we get*

$$(9-y)^2 - y^2 = 9$$
$$81 - 18y + y^2 = 9$$
$$-18y = 9 - 81$$
$$-18 = -72$$
$$y = -\frac{72}{18}$$
$$y = 4$$

Putting $y = 4$ *in (i) we get*

$$x + 4 = 9$$
$$x = 9 - 4 = 5$$

Therefore Integers are 5, 4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Solution: *Let the integers by x and y.*

According to the given conditions.

$x - y = 4$ ....(i)

and $x^2 - y^2 = 72$ ....(ii)

$x - y = 4$ *from (i) we get*

$\therefore x = 4 + y$

By Putting $= x = 4 + y$ in (ii), we get

$$(4+y)^2 - y^2 = 72$$
$$16 + y^2 + 8y - y^2 = 72$$
$$8y = 72 - 16$$
$$8y = 56 \quad \Rightarrow \quad y = \frac{56}{8}$$
$$y = 7$$

Put $y = 7$ in (i) we get

$$x - 7 = 4$$
$$x = 4 + 7 = 11$$

Therefore Integers are: 11 and 7

10. Find the dimensions (length and width) of a rectangle, whose perimeter is 80cm and its area is 375cm^2.

Solution: Let the length of the rectangle $= x$

Let the width of the rectangle $= y$

Perimeter $= 2$(length + width)

$$2x + 2y = 80$$
$$x + y = 40 \quad \ldots\ldots\ldots\ldots(i)$$

Area $=$ length $\times$ width

$$xy = 375 \quad \ldots\ldots\ldots\ldots(ii)$$
$$x = 40 - y \quad \text{from (i) we get}$$

Putting $x = 40 - y$ in (ii) we get

$$(40 - y)y = 375$$
$$40y - y^2 = 375$$
$$y^2 - 40y + 375 = 0$$
$$y^2 - 25y - 15y + 375 = 0$$
$$y(y - 25) - 15(y - 25) = 0$$
$$(y - 15)(y - 25) = 0$$
$$y - 15 = 0 \quad \text{gives} \quad y = 15$$

Putting $y = -15$ in (i) we get

$$x + 15 = 40$$
$$x = 40 - 15 = 25$$

Therefore Length $= x = 25cm$

width $= y = 15cm$

Hence dimensions are 25 cm by 15 cm or 15 cm by 25 cm