Exercise 3.1

1. Express the following as a ratio a : b and as a fraction in its simplest (lowest) form.

(i) $Rs.750, Rs.1250$

Solution: $= 750 : 1250$ *Dividing by* 10

$= 75 : 125$

$= 3 : 5$ *Dividing by* 25

$= 3 : 5$ *or* $\frac{3}{5}$

(ii) $450cm, 3m$

Solution: $3m = 300\ cm.$

$= 450 : 300 \quad \because 1m = 100\ cm$

$= \frac{450}{10} : \frac{300}{10}$ **(Dividing by 10)**

$= 45 : 30$

$= \frac{45}{5} : \frac{30}{5}$ **(Dividing by 5)**

$= 9 : 6$

$= \frac{9}{3} : \frac{6}{3}$ **(Dividing by 3).**

$= 3 : 2$

$= \frac{3}{2}$

(iii) $4kg, 2kg\ 750\ gm$

Solution: $4kg : 2kg\ 750\ gm$

$= 4000\ g : 2750\ g \quad \because 1kg = 1000\ gm$

$= 4000 : 2750$

$= \frac{400}{10} : \frac{2750}{10}$ **(Dividing by 10)**

$= 400 : 275$

$= \frac{400}{5} : \frac{275}{5}$ **(Dividing by 5)**

$= 80 : 55$

$= \frac{80}{5} : \frac{55}{5}$ **(Dividing by 5)**

$= 16 : 11$

$= \frac{16}{11}$

(iv) $27min.30sec, 1\ hour$

Solution: $27\ \text{min}\ 30\ sec : 1\ hour$

$= (27 \times 60 + 30)\ \text{sec} : (1 \times 60 \times 60) sec$

$= 1650 : 3600$

$= \frac{1650}{10} : \frac{3600}{10}$ **(Dividing by 10)**

$= 165 : 360$

$= \frac{165}{5} : \frac{360}{5}$ **(Dividing by 5)**

$= 33 : 72$

$= \frac{33}{3} : \frac{72}{3}$ **(Dividing by 3)**

$= 11 : 24$

$= \frac{11}{24}$

(v) $75^o, 225^o$

Solution: $75^o : 225^o = \frac{75}{5} : \frac{225}{5}$ *(Dividing by 5)*

$= 15 : 45$

$= \frac{15}{5} : \frac{45}{5}$ *(Dividing by 5)*

$= 3 : 9$

$= \frac{3}{3} : \frac{9}{3}$ *(Dividing by 3)*

$= 1 : 3$

$= \frac{1}{3}$

2. In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

(i) *boys to total students*

Solution: *Total student* $= 60$

Number of girls students $= 25$

Number of boys students $= 60 - 25 = 35$

$25 : 60 = \frac{25}{5} : \frac{60}{5}$ *(Dividing by 5)*

$7 : 12$

(ii) ***boys to girls***

Solution:

Number of boys $= 35$

Number of girls $= 25$

Ratio boy to girls $35 : 25 = \frac{35}{5} : \frac{25}{5}$ Dividing by 5

$= 7 : 5$

3. *If* $3(4x-5y)=2x-7y$, *find the ratio* $x:y$.

Solution:

$$3(4x-5y)=2x-7y$$
$$12x-15y=2x-7y$$
$$12x-2x=15-7y$$
$$10x=8y$$

$$\frac{10x}{y}=\frac{8y}{y}$$ *(Dividing both sides by y, we get)*

$$\frac{10x}{y}=8$$

$$\frac{10}{10}\times\frac{x}{y}=\frac{8}{10}$$ *(Dividing both sides by 10, we get)*

$$\frac{x}{y}=\frac{4}{5}$$

or $x:y=4:5$

4. **Find the value of p, if the ratio** $2p+5:3p+4$ *and* $3:4$ *are equal.*

Solution: *As the given ratios are equal, so*

$$2p+5:3p+4=3:4$$

$$\frac{2p+5}{3p+4}=\frac{3}{4}$$

$$3(3p+4)=4(2p+5)$$
$$9p+12=8p+20$$
$$9p-8p=20-12$$
$$-p=-8$$
Thus $p=8$

5. ***If the ratio*** $3x+1:6+4x$ ***and*** $2:5$ ***are equal. Find the value of*** x.

Solution: *As the given ratios are equal, so*

$$3x+1:6+4x=2:5$$

$$\frac{3x+1}{6+4x}=\frac{2}{5}$$

$$5(3x+1)=2(6+4x)$$

$$15x+5=12+8x$$
$$15x-8x=12-5$$
$$7x=$$

Dividing both sides by 7, we get

$$x = \frac{7}{7}$$

$$x = 1$$

6. Two numbers are in the ratio 5 : 8. If 9 is added to each number, we get a new ratio 8 : 11. Find the numbers.

Solution: *Let the required numbers be* $5x, 8x$

Therefore according to given condition

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$11(5x + 9) = 8(8x + 9)$$
$$55x + 99 = 64x + 72$$
$$55x - 64x = 72 - 99$$
$$-9x = -27$$
$$x = \frac{-27}{-9}$$
$$x = 3$$

Therefore required numbers are: $5x, 8x$

$$= 5(3), (8)(3) = 15, 24$$

7. If 10 is added in each number of the ratio 4 : 13, we get a new ratio 1 : 2. What are numbers?

Solution: *Let the required numbers be* $4x, 13x$

Therefore according to given condition

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$$2(4x + 10) = 1(13x + 10)$$
$$8x + 20 = 13x + 10$$
$$8x - 13 = 10 - 20$$
$$-5x = -10$$
$$x = 2$$

Therefore required numbers are: $4x, 13x$

$$= (4)(2), (13)(2)$$
$$= 8, 26$$

8. Find the cost of 8kg of mangoes, if 5kg of mangoes cost *Rs.* 250.

Solution: *Let Rs.x be the cost of 8kg of mangoes.*

Then in proportion, we have

$$8kg : 5kg :: Rs\,x : Rs\,250$$

Product of means $=$ *Product of extremes*

$$5x = 8 \times 250$$

$$x = \frac{8 \times 250}{5}$$

$$x = 8 \times 50$$

$$= Rs.400$$

9. **If $a : b = 7 : 6$, find the value of $3a + 5b : 7b - 5a$.**

Solution: *Given that* $a : b = 7 : 6$

$$\frac{a}{b} = \frac{7}{6}$$

Also $= 3a + 5b : 7b - 5a$

Dividing throughout by b.

$$= \frac{3a}{b} + \frac{5b}{b} : \frac{7b}{b} - \frac{5a}{b}$$

$$= 3\left(\frac{a}{b}\right) + 5 : 7 - 5\left(\frac{a}{b}\right)$$

$$= 3\left(\frac{7}{6}\right) + 5 : 7 - 5\left(\frac{7}{6}\right)$$

$$= \left(\frac{7}{2}\right) + 5 : 7 - \left(\frac{35}{6}\right)$$

$$= \left(\frac{7 + 10}{2}\right) : \left(\frac{42 - 35}{6}\right)$$

$$= \frac{17}{2} : \frac{7}{6}$$

Multiplying by 6, we get

$$6\left(\frac{17}{2}\right) : 6\left(\frac{7}{6}\right)$$

$$= 51 : 7$$

Hence $3a + 5b : 7b - 5a = 51 : 7$

10. **Complete the following:**

(i) **If $\frac{24}{7} = \frac{6}{x}$ then $4x =$ _____**

Solution:

$$\frac{24}{7} = \frac{6}{x}$$

$$24x = 6 \times 7$$

$$\frac{24x}{6} = \frac{6 \times 7}{6}$$

$$4x = 7$$

(ii) If $\frac{5a}{3x} = \frac{15b}{y}$ then ay=_____

Solution: $\frac{5a}{3x} = \frac{15b}{y}$

$$5ay = (15b)(3x)$$

$$ay = \frac{(15b)(3x)}{5} = 9bx$$

(iii) If $\frac{9pq}{21m} = \frac{18p}{5m}$ then 5q=_____

Solution: $\frac{9pq}{21m} = \frac{18p}{5m}$ *Dividing by 9p, we get*

$$\frac{q}{21m} = \frac{2}{5m}$$

$$q = \frac{2(21m)}{5m}$$

$$q = \frac{42}{5} \qquad \text{Hence } 5q = 42$$

11. Find x in the following proportions.

(i) $3x - 2 : 4 :: 2x + 3 : 7$

Solution: $3x - 2 : 4 :: 2x + 3 : 7$

Product of extremes = Product of means

$$7(3x - 2) = 4(2x + 3)$$

$$21x - 14 = 8x + 12$$
$$21x - 8x = 12 + 14$$
$$13x = 26$$

$$x = \frac{26}{13}$$

$$x = 2$$

(ii) $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

Solution: $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

Product of extremes = Product of means

$$\left(\frac{7}{5}\right)\left(\frac{3x-1}{7}\right) = \left(\frac{3}{5}\right)\left(\frac{2x}{3}\right)$$

$$\frac{3x-1}{5} = \frac{2x}{5}$$

$$3x - 1 = 2x$$
$$3x - 2x = 1$$
$$x = 1$$

(iii) $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

Solution: $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

Product of extrems = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right) = \left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{2(x-3)}{x+4} = \frac{5}{3}$$

$$2 \times 3(x-3) = 5(x+4)$$
$$6x - 18 = 5x + 20$$
$$6x - 5x = 20 + 18$$
$$x = 38$$

(iv) $p^2 + pq + q^2 : x :: \frac{p^3-q^3}{p+q} : (p-q)^2$

Solution: $x : p^2 + pq + q^2 :: (p-q)^2 : \frac{p^3-q^3}{p+q}$

$$\frac{x}{p^2+pq+q^2} = \frac{(p-q)^2}{p^3-q^3} \times (p+q)$$

$$x = \frac{(p-q)^2(p+q)}{p^3-q^3} \times (p^2 + pq + q^2)$$

$$= \frac{(p-q)(p-q)(p+q)}{(p-q)(p^2+pq+q^2)} \times (p^2 + p + q^2)$$

$$(p-q)(p+q)$$
$$x = p^2 - q^2$$

(v) $8 - x : 11 - x :: 16 - x : 25 - x$

Solution: $8 - x : 11 - x :: 16 - x : 25 - x$

Product of extremes = Product of means

$$(8-x)(25-x) = (11-x)(16-x)$$
$$200 - 3x + x^2 = 176 - 27x + x^2$$
$$x^2 - 33x - x^2 + 27x = 176 - 200$$
$$-6x = -24$$

$$x = \frac{-24}{-6}$$

$$x = 4$$