Exercise 3.2

1. If y varies directly as x, and $y = 8$ when $x = 2$, find

(i) y *in x terms of* x **(ii)** y *when* $x = 5$

(iii) x *when* $y = 28$

Solution: *Given that y varies directly as x*

Therefore $y \propto x$

$y = kx$ ……………………… (i)

Put $y = 8, x = 2$ *in equation* (i)

$8 = k(2)$

$2k = 8$

$k = 4$

Now $y = kx$ *becomes*

$y = 4x$ $(\because k = 4)$

(ii) y when $x = 5$

Now $y = kx$

Put $x = 5,\ k = 4$ *we get*

$y = 4 \times 5$

$y = 20$

(iii) x when $y = 28$

Now $y = kx$

$28 = 4x$ $(\because k = 4)$

$x = \frac{28}{4}$

$x = 7$

2. If $y \propto x$, and $y = 7$ when $x = 3$ find

(i) y *in terms of* x

(ii) x **when** $y = 35$ **and** y **when** $x = 18$

Solution: *Given that* $y \propto x$

Thus $y = kx$ ……………………… (i)

Put $y = 7$ *and* $x = 3$

Then, $7 = k(3)$

$k = \frac{7}{3}$

Therefore equation (i) becomes

(i) $y = \frac{7}{3}x$

(ii) *Since* $y = \frac{7}{3}x$ ……………………… (ii)

Put $y = 35$

$35 = \frac{7}{3}x$

$x = \frac{35 \times 3}{7}$

$x = 15$

Now put $x = 18$ *in* (ii)

$y = \frac{7}{3}x$

$y = \frac{7}{3}x$

$y = \frac{7}{3}(18)$

$y = 7 \times 6$

$y = 42$

3. If $R \propto T$ and $R = 5$ when $T = 8$, find the equation connecting R and T. Also find R when $T = 64$ and T when $R = 20$

Solution: *Given that* $R \propto T$

Thus, $R = kT$ ……………………… (i)

Put $R = 5$ *and* $T = 8$ *in* (i)

$5 = k(8)$

$k = \frac{5}{8}$

Put $k = \frac{5}{8}$ in (i)

$R = \frac{5}{8}T$

(ii) *Put* $T = 64$ *in* $R = \frac{5}{8}T$ ………. (ii)

$R = \frac{5}{8}(64)$

$R = 5 \times 8 = 40$

Now put $R = 20$ *in* (ii) we get

$R = \frac{5}{8}T$

$$20 = \frac{5}{8}T$$

$$T = \frac{20\times 8}{5}$$

$$T = 4 \times 8$$

$$T = 32$$

4. If $R \propto T^2$ and $R = 8$ when $T = 3$, find R when $T = 6$.

Solution: *Given that* $R \propto T^2$

$R = kT^2$ ……………………… (i)

Put $R = 8, T = 3$ *in* (i)

$8 = k(3)^2$

$8 = 9k$

$k = \frac{8}{9}$

Put $k = \frac{8}{9}$ *in* (i)

$R = \frac{8}{9}T^2$

Now put $T = 6$ *in* $R = \frac{8}{9}T^2$

$R = \frac{8}{9}(6)^2$

$= \frac{8}{9} \times 36$

$= 8 \times 4$

$R = 32$

(5) If $V \propto R^3$ and $V = 5$ when $R = 3$, find R, when $V = 625$.

Solution: *Given that* $V \propto R^3$

Thus, $V = kR^3$

Put $V = 5, R = 3$ *in* (i)

$5 = k(3)^3$

$8 = k(27)$

$k = \frac{5}{27}$

Put $k = \frac{5}{27}$ *in* (i)

$V = \frac{5}{27}R^3$

Now put $V = 625$ *in* $V = \frac{5}{27}R^3$

$$625 = \frac{5}{27}R^3$$

$$R^3 = \frac{625 \times 27}{5}$$

$$R^3 = 125 \times 27$$

$$R^3 = 5^3 \times 3^3$$

$$R^3 = (15)^3$$

By taking cube root on both sides, we get

$$R = 15$$

6. If w varies directly as u^3 and $w = 81$ when $u = 3$. Find w, when $u = 5$.

Solution: *Given that* $w \propto u^3$

$w = ku^3$ ……………………… (i)

Put $w = 81$ *and* $u = 3$ *in* (i)

$$81 = k(3)^3$$

$$81 = 27k$$

$$k = \frac{81}{27} = 3$$

Put $k = 3$ *in* (i), *we get*

$$w = 3u^3$$

Now put $u = 5$ *in* $w = 3u^3$

$$w = 3(5)^3$$

$$w = 3 \times 125$$

$$w = 375$$

7. If y varies inversely as x and $y = 7$ when $x = 2$, find y when $x = 126$

Solution: *Given that* $y \propto \frac{1}{x}$

$y = \frac{k}{x}$ ……………………… (i)

Put $y = 7$ *and* $x = 2$ *in* (i)

$$7 = \frac{k}{2}$$

$$(7)(2) = k$$

$$k = 14$$

Put $k = 14$ *in* $y = \frac{k}{x}$

$$y = \frac{14}{x}$$

Now put $x = 126$ *in* $y = \frac{14}{x}$

$$y = \frac{14}{126}$$

$$y = \frac{1}{9}$$

8. If $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$.

Solution: Given that $y \propto \frac{1}{x}$

$$y = \frac{k}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Put $y = 4, x = 3$ in (i), we get

$$4 = \frac{k}{3}$$

$$(4)(3) = k$$

$$k = 12$$

$$y = \frac{12}{x}$$

Now put $y = 24, k = 12$ in (i), we get

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$x = \frac{1}{2}$$

9. If $w \propto \frac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \frac{175}{4}$.

Solution: Given that $w \propto \frac{1}{z}$

$$w = \frac{k}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Put $w = 5, z = 7$ in (i), we get

$$5 = \frac{k}{7}$$

$$(5)(7) = k$$

$$k = 35$$

$$w = \frac{35}{z}$$

Now put $z = \frac{175}{4}, k = 35$ in (i), we get

$$w = \frac{35}{\frac{175}{4}}$$

$$w = \frac{4 \times 35}{175}$$

$w = \frac{4}{5}$

10. $A \propto \frac{1}{r^2}$ **and** $A = 2$ **when** $r = 3$**, find** r **when** $A = 72$.

Solution: *Given that* $A \propto \frac{1}{r^2}$

$A = \frac{k}{r^2}$ ………………………(i)

Put $A = 2, and\ r = 3$ *in* (i), *we get*

$2 = \frac{k}{3^2}$

$2 = \frac{k}{9}$

$k = 18$

$A = \frac{18}{r^2}$

Now, put $A = 72,\ k = 18$ *in* (i), *we get*

$72 = \frac{18}{r^2}$

$72r^2 = 18$

$r^2 = \frac{18}{72}$

$r^2 = \frac{1}{4}$

$r = \pm\frac{1}{2}$

11. $a \propto \frac{1}{b^2}$ **and** $a = 3$ **when** $b = 4$**, find** a**, when** $b = 8$**.**

Solution: *Given that* $a \propto \frac{1}{b^2}$

$a = \frac{k}{b^2}$ ………………………(i)

Put $a = 3, b = 4\ in(i), we\ get$

$3 = \frac{k}{4^2}$

$3(4)^2 = k$

$3 \times 16 = k$

$k = 48$

$a = \frac{48}{b^2}$

Now put $b = 8,\ k = 48$ *in* (i)

$a = \frac{48}{8^2}$

$$a = \frac{48}{64}$$

$$a = \frac{3}{4}$$

12. **$V \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$, find V when $r = 6$ and r when $V = 320$.**

Solution: *Given that* $V \propto \frac{1}{r^3}$

$$V = \frac{k}{r^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Put $V = 5, r = 3$ in (i), we get

$$5 = \frac{k}{3^3}$$

$$5(3)^3 = k$$

$$5 \times 27 = k$$

$$k = 135$$

$$V = \frac{135}{r^3}$$

(i) *Put* $r = 6, k = 135$ *in* (i), *we get*

$$V = \frac{135}{6^3}$$

$$V(6)^3 = 135$$

$$216V = 135$$

$$V = \frac{135}{216} = \frac{5}{8}$$

(ii) *Also put* $V = 320, k = 135$ *in* (i), *we get*

$$320 = \frac{135}{r^3}$$

$$320r^3 = 135$$

$$r^3 = \frac{135}{320} = \frac{27}{64}$$

$$r^3 = \left(\frac{3}{4}\right)^2$$

Taking cube root on both sides, we get

$$r = \frac{3}{4}$$

13. **$m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$, find m when $n = 6$ and n when $m = 432$.**

Solution: *Given that* $m \propto \frac{1}{n^3}$

$$m = \frac{k}{n^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Put $m = 2, n = 4$ in (i), we get

$$2 = \frac{k}{4^3}$$

$$(2)(4)^3 = k$$

$$k = 2 \times 64$$

$$k = 128$$

$$m = \frac{128}{n^3}$$

(i) Now, put $n = 6, k = 128$ in (i), we get

$$m = \frac{128}{6^3}$$

$$m(6)^3 = 128$$

$$216m = 128$$

$$m = \frac{128}{216} = \frac{16}{27}$$

(ii) Put $m = 432, k = 128$ in (i), we get

$$432 = \frac{128}{n^3}$$

$$432n^3 = 128$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

Taking cube root on both sides, we get

$$\therefore \quad n = \frac{2}{3}$$

In $a : b :: b : c$

c is called the third proportional and b is called mean proportional.

In $a : b :: c : d$

d is called the fourth proportional.