1. Find a third proportional to
(i) 6, 12

Solution: *Let x be the third proportional,*

Then $\quad 6 : 12 :: 12 : x$

Product of extremes = Product of means.

$$6x = 12 \times 12$$

$$x = \frac{12 \times 12}{6}$$

$$x = 24$$

**(ii)** $\quad a^3, 3a^2$

**Solution:** Let $x$ be the third proportional.

Then $\quad a^3 : 3a^2 :: 3a^2 : x$

$$a^3 x = 3a^2 \times 3a^2$$

$$x = \frac{9a^4}{a^3}$$

$$x = 9a$$

**(iii)** $\quad a^2 - b^2, a - b$

**Solution:** Let $x$ be the third proportional, then

$$(a^2 - b^2) : (a - b) :: (a - b) : x$$

Product of extremes = Product of means.

$$(a^2 - b^2)x : (a - b) :: (a - b) : x$$

$$x = \frac{(a-b)(a-b)}{a^2 - b^2}$$

$$= \frac{(a-b)(a-b)}{(a-b)(a+b)}$$

$$x = \frac{a-b}{a+b}$$

**(iv)** $\quad (x - y)^2 . x^3 - y^3$

**Solution:** Let $z$ be the third proportional,

Then $\quad (x - y)^2 : (x^3 - y^3) :: (x^3 - y^3) : z$

$$(x - y)^2 z = (x^3 - y^3)(x^3 - y^3)$$

$$z = \frac{(x^3 - y^3)(x^3 - y^3)}{(x - y)^2}$$

$$= \frac{(x-y)(x^2 + xy + y^2)(x-y)(x^2 + xy + y^2)}{(x-y)(x-y)}$$

$$z = (x^2 + xy + y^2)^2$$

**(v)** $\quad (x + y)^2, x^2 - xy - 2y^2$

**Solution:** Let $z$ be the third proportinal, then

$$(x + y)^2 : (x^2 - xy - 2y^2) :: (x^2 - xy - 2y^2) : z$$

Product of extremes = Product of means.

$$z(x + y)^2 = (x^2 - xy - 2y^2)(x^2 - xy - 2y^2)$$

$$z = \frac{\left[x^2 - 2xy + xy - 2y^2\right]\left[x^2 - 2xy + xy - 2y^2\right]}{(x + y)^2}$$

$$= \frac{[x(x-2y) + y(x-2y)][x(x-2y) + y(x-2y)]}{(x + y)^2}$$

$$= \frac{[(x - 2y)(x + y)][(x - 2y)(x + y)]}{(x + y)^2}$$

$$(x - 2y)(x - 2y)$$

$$z = (x - 2y)^2$$

**(vi)** $\quad \dfrac{p^2 - q^2}{p^3 + q^3} : \dfrac{p - q}{p^2 - pq + q^2}$

**Solution:** Let $x$ be the third proportional, then

$$= \frac{p^2 - q^2}{p^3 + q^3} : \frac{p - q}{p^2 - pq + q^2} :: \frac{p - q}{p^2 - pq + q^2} : x$$

Product of extremes = Product of means.

$$\left(\frac{p^2 - q^2}{p^3 + q^3}\right)(x) = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2}$$

$$x = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2} \times \frac{p^3 \times q^3}{p^2 - q^2}$$

$$x = \frac{(p-q)(p-q)(p + q)(p^2 - pq + q^2)}{(p^2 - pq + q^2)(p^2 - pq + q^2)(p - q)(p + q)}$$

$$x = \frac{p - q}{p^2 - pq + q^2}$$

**2. Find a fourth proportional to**

**(i)** $\quad \underline{5, 8, 15}$

**Solution:** Let $x$ be the fourth proportional, then

$$= 5 : 8 :: 15 : x$$

Product of extremes = Product od means.

$$5x = 8 \times 15$$

$$x = \frac{8 \times 15}{5}$$

$$x = 8 \times 3$$

$$x = 24$$

**(ii)** $\quad 4x^4, 2x^3, 18x^5$

**Solution:** Let $y$ be the fourth proportional, then

$$4x^4 : 2x^3 :: 18x^5 : y$$

Product of extremes = Product of means.

$$(4x^4)y = (2x^3)(18x^5)$$

$$y = \frac{(2x^3)(18x^5)}{4x^4}$$

$$y = 9x^4$$

**(iii)** $15a^5b^6, 10a^2b^5, 21a^3b^3$

**Solution:** Let $x$ be the fourth proportional, then

$$= 15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : x$$

Product of extremes = Product of means.

$$(15a^5b^6)x = (10a^2b^5)(21a^3b^3)$$

$$x = \frac{210a^5b^8}{15a^5b^6}$$

$$x = 14b^2$$

**(iv)** $x^2 - 11x + 24, (x-3), 5x^4 - 40x^3$

**Solution:** Let $y$ be the fourth proportional, then

$$= (x^2 - 11x + 24) : x - 2 :: (5x^4 - 40x^3) : y$$

Product of extremes = Product of means.

$$(x^2 - 11x + 24)y = (x-3)(5x^4 - 40x^3)$$

$$y = \frac{(x-3)(5x^4 - 40x^3)}{x^2 - 11x + 24}$$

$$= \frac{(x-3)(5x^3)(x-8)}{x^2 - 8x - 3x + 24}$$

$$= \frac{5x^3(x-3)(x-8)}{x(x-8) - 3(x-8)}$$

$$= \frac{5x^3(x-3)(x-8)}{(x-3)(x-8)}$$

$$y = 5x^3$$

**(v)** $p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$

**Solution:** Let $x$ be the fourth proportional, then

$$= (p^3 + q^3) : p^2 - q^2 :: (p^2 - pq + q^2) : x$$

Product of extremes = Product of means.

$$(p^3 + q^3)x = (p^2 - q^2)(p^2 - pq + q^2)$$

$$x = \frac{(p^2 - q^2)(p^2 - pq + q^2)}{p^3 + q^3}$$

$$= \frac{(p+q)(p-q)(p^2-pq+q^2)}{(p+q)(p^2-pq+q^2)}$$

$$x = p - q$$

**(vi)** $(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$

**Solution:** Let $x$ be the fourth proportional, then

$$= (p^2 - q^2)(p^2 + pq + q^2):(p^3 + q^3) :: (p^3 - q^3) : x$$

Product of extremes = Product of means.

$$(p^2 - q^2)(p^2 + pq + q^2)x = (p^3 + q^3)(p^3 - q^3)$$

$$x = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2 + pq + q^2)}$$

$$= \frac{(p+q)(p^2-pq+q^2)(p-q)(p^2 + pq + q^2)}{(p+q)(p-q)(p^2 + pq + q^2)}$$

$$x = p^2 - pq + q^2$$

**3.** **Find a mean proportional between.**

**(i)** 20, 45

**Solution:** Let $x$ be the mean proportional, then

$$= 20 : x :: x : 45$$

Product of means = Product of extremes

$$x^2 = 20 \times 45$$

$$x^2 = 900$$

Taking square root on both sides, we get

$$x = \pm 30$$

**(ii)** $20x^3y^5, 5x^7y$

**Solution:** Let $z$ be the mean proportional, then

$$= 20x^3y^5 : z :: z : 5x^7y$$

Product of means = Product of extremes

$$z^2 = 20x^3y^5 \times 5x^7y$$

$$= 100x^{10}y^6$$

$$z^2 = (10x^5y^3)^2$$

Taking square root on both sides, we get

$$z = \pm 10x^5y^3$$

**(iii)** $15p^4qr^3, 135q^5r^7$

**Solution:** Let $x$ be the mean proportional, then

$$= 15p^4qr^3 : x :: x : 135q^5r^7$$

$$x \times x = 15p^4qr^3 \times 135q^5r^7$$

$$= 2025p^4q^6r^{10}$$
$$x^2 = (45)^2(p^2q^3r^5)^2$$

*Taking square root on both sides, we get*
$$x = \pm 45p^2q^3r^5$$

**(iv)** $\quad x^2 - y^2, \dfrac{x-y}{x+y}$

**Solution:** *Let z be the mean proportional, then*
$$= (x^2 - y^2) : z :: z : \dfrac{x-y}{x+y}$$

*Product of means = Product of extremes*
$$z^2 = (x^2 - y^2)\left(\dfrac{x-y}{x+y}\right)$$
$$= \dfrac{(x+y)(x-y)(x-y)}{(x+y)}$$
$$z^2 = (x-y)^2$$

*Taking square root on both sides, we get*
$$z = \pm(x - y)$$

**4.** **Find the values of the letter involved in the following continued proportions.** 2nd value e m

**(i)** $\quad 5, \textcircled{p}, 45$

**Solution:** *Since 5, p and 45 are in continued propotion*

*Therefore* $\quad 5 : p :: p : 45$

*Product of means = Product of extremes*
$$p \times p = 5 \times 45$$
$$p^2 = 225$$
$$p^2 = (15)^2$$

*Taking square root on both sides, we get*
$$p = \pm 15$$

**(ii)** $\quad 8, x, 18$

**Solution:** $\quad$ *Since 8, x and 18 are in continued propotion*

*Here,* $\quad 8 : x :: x : 18$

*Product of means = Product of extremes*
$$x \times x = 8 \times 18$$
$$x^2 = 144$$

*Taking square root on both sides, we get*
$$x = \pm 12$$

**(iii)** $\quad 12, 3p - 6, 27$

**Solution:** *Since 12, 3p − 6 and 27 are in continued propotion*

Therefore    $12 : (3p - 6) :: (3p - 6) : 27$

Product of means = Product of extremes

$$(3p - 6)(3p - 6) = 12 \times 27$$

$$(3p - 6)^2 = 324$$

$$\sqrt{(3p - 6)^2} = \pm\sqrt{324}$$

$$3p - 6 = \pm 18$$

| | | |
|---|---|---|
| $3p - 6 = 18$ | or | $3p - 6 = -18$ |
| $3p = 18 + 6$ | or | $3p = -18 + 6$ |
| $3p = 24$ | or | $3p = -12$ |
| $p = 8$ | or | $p = -4$ |

**(iv)**    $7, m - 3, 28$

**Solution:** *Since* $7, m - 3$ *and* $28$ *are in continued propotion*

Therefore    $7 : (m - 3) :: (m - 3) : 28$

*Product of means = Product of extremes*

$$(m - 3)(m - 3) = 7 \times 28$$

$$(m - 3)^2 = 196$$

$$\sqrt{(m - 3)^2} = \pm\sqrt{196}$$

$$m - 3 = \pm 14$$

| | | |
|---|---|---|
| $m - 3 = 14$ | or | $m - 3 = -14$ |
| $m = 14 + 3$ | or | $m = -14 + 3$ |
| $m = 17$ | or | $m = -11$ |