Exercise 3.6

1. If $a : b = c : d, (a, b, c, d \neq 0)$, then show that

(i) $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

Solution: $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

$\because a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk, \quad c = dk$

L.H.S	R.H.S
$\frac{4a-9b}{4a+9b}$ .... (i)	$\frac{4c-9d}{4c+9c}$ .... (ii)
Put $a = bk$ in (i)	Put $c = dk$ in (ii)
$= \frac{4bk-9b}{4bl+9b}$	$= \frac{4dk-9b}{4dk+9b}$
$= \frac{b(4k-9)}{b(4k+9)}$	$= \frac{d(4k-9)}{d(4k+9)}$
$= \frac{4k-9}{4k+9}$	$= \frac{4k-9}{4k+9}$

L.H.S $=$ R.H.S

(ii) **If $\frac{a}{b} = \frac{c}{d}$ then show that $\frac{6a-5b}{6a+5b} = \frac{4c-5d}{4c+5d}$**

Solution: $\frac{6a-5b}{6a+5b} = \frac{4c-5d}{4c+5d}$

Let $\frac{a}{b} = \frac{c}{d} = k$

Hence, $a = bk, \ c = dk$

L.H.S	R.H.S
$= \frac{6a-5b}{6a+5b}$	$= \frac{6c-5d}{6c+5d}$
Put $a = bk$	Put $a = dk$
$= \frac{6bk-5b}{6bk+5b}$	$= \frac{6dk-5d}{6dk+5d}$
$= \frac{b(6k-5)}{b(6k+5)}$	$= \frac{d(6k-5)}{d(6k+5)}$
$= \frac{6k-5}{6k+5}$	$= \frac{6k-5}{6k+5}$

L.H.S $=$ R.H.S

(iii) If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Let $\frac{a}{b}=\frac{c}{d}=k$

Then $a=bk,\quad c=dk$

L.H.S	R.H.S
$=\frac{a}{b}\ldots\ldots\ldots$ (i)	$=\sqrt{\frac{a^2+c^2}{b^2+a^2}}\ldots\ldots$ (ii)
Put $a=bk$ in (i)	Put $a=bk$, $c=dk$ in (ii)
$=\frac{bk}{b}$	$=\sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$
$=k$	$=\sqrt{\frac{b^2k^2+d^2k^2}{(b^2+d^2)}}$
	$=\sqrt{\frac{k^2(b^2+d^2)}{(b^2+d^2)}}$
	$=\sqrt{k^2}$
	$=k$

L.H.S = R.H.S

(iv) If $\frac{a}{b}=\frac{c}{d}$ then $a^6+c^6 : b^6+a^6 = a^3c^3 : b^3d^3$

Let $\frac{a}{b}=\frac{c}{d}=k$

$a=bk, c=dk$

L.H.S	R.H.S
$\frac{a^6+c^6}{b^6+d^6}$ ……… (i)	$\frac{a^3c^3}{b^3d^3}$ …… (ii)
Put $a = bk$	Put $c = dk$
Put $c = dk$, in (i)	$c = dk$ in (ii)
$\frac{(bk)^6+(dk)^6}{b^6+d^6}$	$\frac{(bk)^3(dk)^3}{b^3d^3}$
Common $= \frac{b^6k^6+d^6k^6}{b^6+d^6}$	$\frac{b^3k^3d^3k^3}{b^3d^3}$ + of powers.
$= \frac{k^6(b^6+d^6)}{b^6+d^6}$	$= k^{3+3}$
$= k^6$	$= k^6$

L.H.S = R.H.S

(v) If $\frac{a}{b} = \frac{c}{d}$ then $p(a+b)+qb : p(c+d)+qd = a : c$

Solution: Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk, c = dk$

L.H.S	R.H.S
$= \frac{p(a+b)+qb}{p(c+d)+qd}$ …. (i)	$= \frac{a}{c}$ …… (ii)
Put $a = bk$	Put $c = dk$
$c = dk$, in (i)	$a = bk$ in (ii)
$= \frac{p(bk+b)+qb}{p(dk+d)+qd}$	$= \frac{bk}{dk}$
$= \frac{pbk+pb+qb}{pdk+pd+qd}$	$= \frac{b}{a}$
$= \frac{b(pk+p+q)}{d(pk+p+q)}$	
$= \frac{b}{a}$	

L.H.S = R.H.S

(vi) **If** $\frac{a}{b}=\frac{c}{d}$ **then** $a^2+b^2 : \frac{a^2}{a+b} = c^2+d^2 : \frac{c^2}{c+d}$

Solution: $\frac{a}{b}=\frac{c}{d}=k$

then $a = bk,\ c = dk$

$L.H.S = R.H.S$

L.H.S	R.H.S
$= \dfrac{a^2+b^2}{\frac{a^2}{a+b}}$	$= \dfrac{c^2+d^2}{\frac{c^3}{c+d}}$
$= (a^2+b^2)\times\frac{(a+b)}{a^2}$ ... (i)	$(c^2+d^2)\times\frac{(c+d)}{c^3}$ .... (ii)
Put $a = bk$ in (i)	Put $c = dk$ in (ii)
$[(bk)^2+b^2]\times\left[\frac{bk+b}{b^3k^3}\right]$	$= (d^2k^2+d^2)\times\left(\frac{dk+d}{b^3k^3}\right)$
$= (b^2k^2+b^2)\left(\frac{bk+b}{b^3k^3}\right)$	$= \frac{d^2(k^2+1)(d)(k+1)}{d^3k^3}$
$\frac{b^2(k^2+1)(b)(k+1)}{b^3k^3}$	$= \frac{(k^2+1)(k+1)}{k^3}$
$\frac{(k^2+1)(k+1)}{k^3}$	

$L.H.S = R.H.S$

(vii) **If** $\frac{a}{b}=\frac{c}{d}$ **then** $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Solution: $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

$\Rightarrow \frac{a}{a-b}\times\frac{b}{a+b} = \frac{c}{c-d}\times\frac{d}{c+d}$

Let $\frac{a}{b}=\frac{c}{d}=k$

then $a = bk,\ \ c = dk$

L.H.S	R.H.S
	$\frac{c}{c-d}\times\frac{d}{c+d}$
Put $a=bk$	Put $c=dk$
$=\frac{bk}{bk-b}\times\frac{b}{bk+b}$	$=\frac{dk}{dk-b}\times\frac{d}{dk+b}$
$=\frac{bk}{b(k-1)}\times\frac{b}{bk+b}$	$=\frac{dk}{d(k-1)}\times\frac{d}{d(k+1)}$
$=\frac{k}{k-1}\times\frac{1}{k+1}$	$=\frac{k}{k-1}\times\frac{d}{d(k+1)}$
$=\frac{k}{(k-1)(k+1)}$	$=\frac{k}{(k-1)(k+1)}$
$=\frac{k}{k^2-1}$	$=\frac{k}{k^2-1}$

L.H.S = R.H.S

2. *If* $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ $(a,b,c,d,e,f\neq 0)$, *then show that*

(i) $\frac{a}{b}=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

Solution: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$$\frac{a}{b}=k, \quad \frac{c}{d}=k \text{ and } \frac{e}{f}=k$$

$$a=bk, \quad c=dk, \quad e=fk$$

L.H.S	R.H.S
$=\frac{a}{b}$	$=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$
$=\frac{bk}{b}$	$=\sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$
$=k$	$=\sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$
	$=\sqrt{k^2}$
	$=k$

$$L.H.S = R.H.S$$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

Solution: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k \text{ and } \frac{c}{d} = k \text{ and } \frac{e}{f} = k$$

$$a = bk \quad c = dk \quad e = fk$$

L.H.S

$$\frac{ac+ce+ea}{bd+df+fb}$$

$$= \frac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb}$$

$$= \frac{bdk^2 + dfk^2 + fbk^2}{bd + df + bf}$$

$$= k^2$$

R.H.S

$$= \left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$$

$$= \left[\frac{(bk)(dk)(fk)}{bdf}\right]^{\frac{2}{3}}$$

$$= \left[\frac{bdfk^3}{bdf}\right]^{\frac{2}{3}}$$

$$= [k^3]^{\frac{2}{3}}$$

$$= k^2$$

$$L.H.S = R.H.S$$

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ca}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$$\frac{a}{b} = k \text{ and } \frac{c}{d} = k \text{ and } \frac{e}{f} = k$$

$$a = bk \quad c = dk \quad e = fk$$

L.H.S

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$$

$$= \frac{(bk)(dk)}{bd} + \frac{(dk)(fk)}{df} + \frac{(fk)(bk)}{bf}$$

$$= \frac{bdk^2}{bk} + \frac{dfk^2}{df} + \frac{fbk^2}{fb}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2$$

R.H.S

$$= \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$= \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2$$