Exercise 3.7

1. The surface area A of a cube varies directly as the square of the length l of an edge and A=27 square units when $l = 3$ units
Find (i) A when $l = 4$ units
(ii) l when $A = 12$ sq. units

Solution: Given that $A \propto l^2$

$A = kl^2$ ……………… (i)

Put $A = 27, l = 3$ in (i), we get

$27 = k(3)^2$

$27 = 9k$

$k = \frac{27}{9}$

$k = 3$

$A = 3l^2$

(i) Put $l = 4,\ k = 3$ in (i), we get

$A = 3 \times 4^2$

$A = 3 \times 16 = 48$ sq. units

(ii) Put $A = 12,\ k = 3$ in (i), we get

$12 = 3 \times l^2$

$l^2 = 4$ taking square root

$l = 2$ units

$A = 3 \times 16 = 48$ sq. units

$r^2 = 9$ taking square root on both sides

$r = 3$

2. The surface area S of the spherical varies directly as the square of radius r, and $S = 16\pi$ when $r = 2$. Find r when $S = 36\pi$.

Solution: $S \propto r^2$

$S = kr^2$ ……………… (i)

Put $S = 16\pi, r = 2$ in (i), we get

$16\pi = k(2)^2$

$16\pi = 4k$

$k = \frac{16\pi}{4}$

$k = 4\pi$
$S = 4\pi r^2$
Now, put $S = 36\pi$, $k = 4\pi$ in (i), we get
$36\pi = 4\pi r^2$

$r^2 = \frac{36\pi}{4\pi}$
$r^2 = 9$ taking square root on both sides
$r = 3$

3. **In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and $F = 32lb$ when $S = 1.6$ in. Find (i) S when $F = 50lb$. (ii) F when $S = 0.8\ in$.**

Solution: $F \propto S$
$F = kS$ …………….(i)
Put $F = 32, S = 1.6$ in (i), we get
$32 = k(1.6)$
$k = \frac{32}{1.6}$

$k = \frac{32 \times 10}{16}$

$k = 2 \times 10 = 20$
Put $k = 20$ in (i)
$F = 20S$ ……..(ii)

(i) Put $F = 50$ in (ii)
$50 = 20S$

$S = \frac{50}{20}$
$S = 2.5\ in$

(ii) Put $S = 0.8$ in (ii)
$F = (20)(0.8)$
$F = 16lb$

4. **The intensity I of light from a given source varies inversely as the depth d. If the pressure distance d from it. If the intensity is 20 candlepower at a distance of 12ft. From the source, find the intensity at a point 8ft. From the source.**

Solution: $I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2}$ ………..(i)

Put $I = 20$ *and* $d = 12$ *in* (i), we get

$$20 = \frac{k}{(12)^2}$$
$$k = 20 \times (12)^2$$
$$k = 20 \times 114$$
$$k = 2880$$

Put $k = 2880$ *in* (i), we get

$$I = \frac{2880}{d^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Now put $d = 8$ *in* (ii), we get

$$l = \frac{2880}{(8)^2}$$
$$I = \frac{2880}{64}$$
$$I = 45cp.$$

5. **The pressure P in a body of fluid varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft high is $2.25\frac{lb}{sq}$. in, how deep much the fluid be to exert a pressure of $9lb/sq$. in?**

Solution: $P \propto d$

$$P = kd \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Put $P = 2.25, d = 5$ *in* (i), *we get*

$$22.5 = k(5)$$
$$k = \frac{2.25}{5}$$
$$k = 0.45$$

Put $k = 0.45$ *in* (i)

$$P = 0.45d \ldots\ldots\ldots \text{(ii)}$$

Now put, $P = \frac{9lb}{sq}$. *in* (ii), we get

$$9 = 0.45d$$
$$d = \frac{9}{0.45}$$
$$d = 20ft$$

6. **Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days in $Rs.$ 286000, then find the labour cost of 600 workers for 18 days.**

Solution: $c \propto nd$

$c = knd$ ....(i)

Put $c = 286000, n = 800, \; d = 13$ *in* (i)

$286000 = k(800)(13)$

$$k = \frac{286000}{800 \times 13} = \frac{55}{2}$$

By putting the value of k in (i), *we get*

$$c = \frac{55}{2} nd \text{ .... (ii)}$$

Now, put $n = 600, d = 18$ *in* (ii), we get

$$c = \frac{55}{2} \times 600 \times 18$$

$= Rs. 297000$

7. **The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be a support a load of 28 tons?**

Solution: *Given that* $c \propto \frac{(d)^4}{l^2}$

$$c = k\frac{d^4}{l^2} \text{ .... (i)}$$

Put $c = 63, l = 30, d = 6$ *in* (i), *we get*

$$63 = k\frac{(6)^4}{30^2}$$

$$63 = k\frac{1296}{900}$$

$$k = 63 \times \frac{1296}{900}$$

$$k = \frac{175}{4}$$

Put $k = \frac{175}{4}$ *in* (i), *we get*

$$C = \frac{175(d)^4}{4l^2} \text{ .... (ii)}$$

Now, put $c = 28, d = 4$ *in* (ii), *we get*

$$28 = \frac{175 \times (4)^4}{4l^2}$$

$$l^2 = \frac{175 \times 256}{4 \times 28}$$

$$l^2 = 400$$

taking square root on both sides, we get

$$l = 20\, ft$$

8. **The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec are required for a 4- hp motor to lift 500lb through $40ft$, what power is required to lift 800 lb, through 120 ft in 40 sec.?**

Solution: *Given that* $T \propto \frac{wd}{p}$

$$T = k \times \frac{wd}{p} \ldots\ldots\ldots\ldots\ldots\ldots (\text{i})$$

Put $T = 25, p = 4, w = 500, d = 40$ *in* (i), *we get*

$$25 = \frac{k \times 500 \times 40}{4}$$

$$k = \frac{25 \times 4}{500 \times 40}$$

$$k = \frac{1}{200}$$

Put the value of k in (i), *we get*

$$T = \frac{wd}{200p} \ldots\ldots\ldots\ldots (\text{ii})$$

Now,

put $T = 40, W = 800, d = 120$ *in* (ii), *we get*

$$40 = \frac{800 \times 120}{200p}$$

$$p = \frac{800 \times 120}{200 \times 40}$$

$$P = 12hp$$

9. **The kinetic energy $(K.E)$ of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb [illegible]mobile travelling 4[illegible] ft/sec**

Solution: Given that $K.E \propto mv^2$

$K.E = kmv^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

Put $K.E = 4320, m = 45, v = 24$ in (i)

$4320 = k \times 45 \times (24)^2$

$4320 = k \times 45 \times 576$

$k = \frac{4320}{45 \times 576}$

$$k = \frac{1}{6}$$

Put this value of k in (i), we get

$K.E = \frac{1}{6} \times mv^2 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Put $m = 3000, v = 44$ in (ii), we get

$K.E = \frac{1}{6} \times 3000 \times 44^2$

$K.E = 500 \times 1936$

$K.E = 968000$ ft/lb