Exercise 4.1

Resolve into partial fractions.

1. $\frac{7x-9}{(x+1)(x-3)}$

Solution: Let $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{(x-3)}$ ……… (i)

Multiplying both sides by $(x+1)(x-3)$, we get

$7x - 9 = A(x-3) + B(x+1)$
$7x - 9 = Ax - 3A + Bx + B$
$7x - 9 = Ax + Bx - 3A + B$
$7x - 9 = (A+B)x - 3A + B$

by comparing co – efficients of x and constant terms

$A + B = 7$ ……………… (ii)
$-3A + B = -9$ ……………… (iii)

substracting (ii) from (iii), we get

$4A = 16$
$A = 4$

put $A = 4$ in (ii), we get

$4 + B = 7$
$B = 7 - 4 = 3$

Putting values of A, B in (i), we get

$$\frac{7x-9}{(x+1)(x-3)} = \frac{4}{x+1} + \frac{3}{x-3}$$

2. $\frac{x-11}{(x-4)(x+3)}$

Solution: Let $\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x-3}$ ……… (i)

Multiplying both sides by $(x-4)(x+3)$, we get

$x - 11 = A(x+3) + B(x-4)$
$x - 11 = Ax + 3A + Bx - 4B$
$x - 11 = Ax + Bx + 3A - 4B$
$x - 11 = (A+B)x + (3A - 4B)$

by comparing coefficients of x and constant terms

$A + B = 1$ ……………… (ii)
$3A - 4B = -11$ ……… (iii)

Multiplying (ii) by 3, on both sides, we get

$3A + 3B = 3$ ………………(iv)

Subtracting (iii) from (iv), we get

$7B = 14$

$B = 2$

Put $B = 2$ in (ii), we get

$A + 2 = 1$

$A = 1 - 2$

$A = -1$

Putting values of A, B in (i), we get

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

3. $\frac{3x-1}{x^2-1}$

Solution: Let $\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ ……… (i)

Multiplying both sides by $(x-1)(x+1)$, we get

$3x - 1 = A(x+1) + B(x-1)v$

$3x - 1 = Ax + A + Bx - B$

$3x - 1 = Ax + Bx + A - B$

$3x - 1 = (A + B)x + (A - B)$

by comparing coefficients of x and constant term.

$A + B = 3$ ………………(ii)

$A - B = -1$ ……………(iii)

by adding (ii), (iii), we get

$2A = 2$

$A = 1$

Put $A = 1$ in (ii), we get

$1 + B = 3$

$B = 3 - 1 = 2$

by putting values of A, B in (i), we get

$$\frac{3x-1}{x^2-1} = \frac{1}{x-1} + \frac{2}{x+1}$$

4. $\frac{x-5}{x^2+2x-3}$

Solution: $\frac{x-5}{x^2+2x-3}$

$$= \frac{x-5}{x^2-x+3x-3}$$

$$= \frac{x-5}{x(x-1)+3(x-1)}$$

$= \frac{x-5}{(x-1)(x+3)}$

Let $= \frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

Multiplying both sides by $(x-1)$, we get

$x - 5 = A(x+3) + B(x-1)$

$x - 5 = Ax + 3A + Bx - B$

$x - 5 = Ax + Bx + 3A - B$

$x - 5 = (A+B)x + (3A - B)$

by comparing co – efficients of x and constant terms

$A + B = 1$ ……… (ii)

$3A - B = -5$ ……. (iii)

$4A = -4$ (by adding ii and iii), we get

$A = -1$

Put $A = -1$ in (ii), we get

$-1 + B = 1$

$B = 1 + 1$

$B = 2$

Putting values of A, B in (i), we get

$\frac{x-5}{(x-1)(x+3)} = \frac{-1}{x-1} + \frac{2}{x+3}$

5. $\frac{3x+3}{(x-1)(x+2)}$

Solution: Let $\frac{3x+3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$ …… (i)

Multiplying both sides by $(x-1)(x+2)$, we get

$3x + 3 = A(x+2) + B(x-1)$

$3x + 3 = Ax + 2A + Bx - B$

$3x + 3 = Ax + Bx + 2A - B$

$3x + 3 = (A+B)x + (2A - B)$

by comparing coefficients of x and constant terms

$A + B = 3$ ………… (ii)

$2A - B = 3$ ……… (iii)

$3A = 6$ (*by adding* ii and iii), we get

$A = 2$

Put $A = 2$ in (ii), we get

$2 + B = 3$

$B = 3 - 2 = 1$

by puttting values of A, B in (i), we get

$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{1}{x+2}$

6. $\frac{7x-25}{(x-4)(x-3)}$

Solution: Let $\frac{7x-25}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$ ……… (i)

Multiplying both sides by $(x-4)(x-3)$, we get

$7x - 25 = A(x-3) + B(x-4)$

$7x - 25 = Ax - 3A + Bx - 4B$

$7x - 25 = (A+B)x - 3A - 4B$

by comparing coefficients of x and constant terms.

$A + B = 7$ ……… (ii)

$-3A - 4B = -25$ ……… (iii)

Multiply (ii) by 3, we get

$3A + 3B = 21$ ……… (iv)

$-B = -4$ (by adding iii and iv), we get

$B = 4$

Put $B = 4$ in (ii), we get

$A + 4 = 7$

$A = 7 - 4$

$A = 3$

by putting values of A, B in (i), we get

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4} + \frac{4}{x-3}$$

7. $\frac{x^2+2x+1}{(x-2)(x+3)}$

Solution: $\frac{x^2+2x+1}{(x-2)(x+3)}$

by long division, we have

$= \frac{x^2+2x+1}{x^2+x-6}$

$$x^2+x-6\ \overline{\smash{)}\,x^2+2x+1}\quad 1$$
$$\underline{\pm x^2 \pm x \mp 6}$$
$$x+7$$

$= 1 + \frac{x+7}{x^2+x-6}$

$= 1 + \frac{x+7}{(x-2)(x+3)}$

Let $\frac{x+7}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$ ……… (i)

Multiplying both sides by $(x-2)(x+3)$, we get

$x + 7 = A(x+3) + B(x-2)$

$x + 7 = Ax + 3A + Bx - 2B$

$x + 7 = Ax + Bx + 3A - 2B$

$x + 7 = (A + B)x + 3A - 2B$

by comparing coefficient of x and constant terms.

$A + B = 1 \ldots\ldots\ldots\ldots\ldots (ii)$

$3A - 2B = 7 \ldots\ldots\ldots\ldots (iii)$

Multiplying (ii) by 2, we get

$2A + 2B = 2 \ldots\ldots\ldots\ldots (iv)$

$5A = 9$ (by addding iii and iv), *we get*

$A = \frac{9}{5}$

Put $A = \frac{9}{5}$ in (ii), we get

$\frac{9}{5} + B = 1$

$B = 1 - \frac{9}{5}$

$B = \frac{5-9}{5} = -\frac{4}{5}$

Thus required partial fractions are $\frac{\frac{9}{5}}{x-2} + \frac{\frac{-4}{5}}{x+3}$

Hence, $\frac{x^2 + 2x + 1}{(x-2)(x+3)} = 1 + \frac{9}{5(x-2)} - \frac{4}{5(x+3)}$

8. $\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

Solution: $\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

by long division, we have

$$\begin{array}{r|l} 3x^2 - 2x - 1 & 2x + 3 \\ & \overline{)6x^3 - 5x^2 - 7} \\ & \underline{\pm 6x^3 \mp 4x^2 \mp 2x} \\ & 4x^2 + 2x - 7 \\ & \underline{\pm 9x^2 \mp 6x \mp 3} \\ & 8x - 4 \end{array}$$

$= 2x + 3 + \frac{8x-4}{3x^2-2x-1}$

$= 2x + 3 + \left(\frac{8x-4}{3x^2-3x+x-1}\right)$

$3x^2 - 2x - 1$

$= 2x + 3 + \frac{8x-4}{3x(x-1)+1(x-1)}$

$= 2x + 3 + \frac{8x-4}{(x-1)(3x+1)}$ ……….(i)

Let $\frac{8x-4}{(x-1)(3x+1)} = \frac{A}{x-1} + \frac{B}{3x+1}$ ……….(K)

Multiplying both sides by $(x-1)(3x+1)$, we get

$8x - 4 = A(3x+1) + B(x-1)$

$8x - 4 = 3Ax + A + Bx - B$

$8x - 4 = 3Ax + Bx + A - B$

$8x - 4 = (3A + B)x + (A - B)$

by comparing coefficients of x and constant terms

$3A + B = 8$ ……….(ii)

$A - B = -4$ ……….(iii)

$4A = 4$ (by adding ii and iii), we get

$A = 1$

Put $A = 1$ in (ii)

$3(1) + B = 8$

$B = 8 - 3$

$B = 5$

Putting values of A, B in , we get

$\frac{8x-4}{(x-1)(3x+1)} = \frac{1}{x-1} + \frac{5}{3x+1}$

Hence, $2x + 3 + \frac{8x-4}{(x-1)(3x+1)} = 2x + 3 + \frac{1}{x-1} + \frac{5}{3x+1}$