Resolve into partial fractions.

1. $\frac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution: Let $\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

Multiplying both sides by $(x-1)^2(x-2)$, we get

$x^2 - 3x + 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$ ……… (i)

$x^2 - 3x + 1 = A(x^2 - 3x + 2) + B(x-2) + C(x^2 - 2x + 1)$

Put $x - 1 = 0$ i.e ; $x = 1$ in (i), we get

$(1)^2 - 3(1) + 1 = B(1-2)$

$1 - 3 + 1 = B(-1)$

$-1 = -B$

$B = 1$

Now, put $x - 2 = 0$ i.e ; $x = 2$ in (i), we get

$(2)^2 - 3(2) + 1 = C(2-1)^2$

$4 - 6 + 1 = C(1)^2$

$-1 = C(1)$

$C = -1$

by comparing co − efficients of x^2 on both sides, we get

$1 = A + C$ ……………………………… (ii)

Put $C = -1$ in (ii), we get

$1 = A + C$

$1 = A + (-1)$

$A = 2$

Hence, required partial fraction are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{x-2}$$

2. $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$

Solution:

Let $\dfrac{x^2+7x+11}{(x+2)^2(x+3)} = \dfrac{A}{(x+2)} + \dfrac{B}{(x+2)^2} + \dfrac{C}{(x+3)}$ ….. (k)

Multiplyingboth sides by $(x+2)^2(x+3)$, *we get*

$x^2 + 7x + 11 = A(x+2)(x+3) + B(x+3) + C(x+2)^2$ …………… (i)

$x^2 + 7x + 11 = A(x^2+5x+6) + B(x+3) + C(x^2+4x+4)$ ………. (ii)

Put $x + 2 = 0$ *i.e*; $x = -2$ *in* (i)

$(-2)^2 + 7(-2) + 11 = 0 + B(-2+3) + 0$

$4 - 14 + 11 = B(1)$

$1 = B$

$\boldsymbol{B = 1}$

Put $x + 3 = 0$ *or* $x = -3$ *in* (i), *we get*

$(-3)^2 + 79 - 3) + 11 = C(-3+2)^2$

$9 - 21 + 11 = C(-1)^2$

$-1 = C(1)$

$\boldsymbol{C = -1}$

by comparing co − *efficients of* x^2 *in* (ii), *we get*

$1 = A + C$

Put $C = -1$ *in it, we get*

$1 = A - 1$

$1 + 1 = A$

$A = 2$

Put values of A, B, C in (k), we get

Therefore, partial fractions are

$$\frac{2}{x+2} + \frac{1}{(x+2)^2} - \frac{1}{x+3}$$

Thus, $\dfrac{x^2+7x+11}{(x+2)^2(x+3)} = \dfrac{2}{x+2} + \dfrac{1}{(x+2)^2} - \dfrac{1}{x+3}$

3. $\dfrac{9}{(x-1)(x+2)^2}$

Solution: Let $\dfrac{9}{(x+2)^2(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{x-1}$ ………(K)

Multiplying both sides by $(x+2)^2(x-1)$, we get

$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$ ………(i)

$9 = A(x^2+x-2) + B(x-1) + C(x^2+4x+4)$ ….(ii)

Put $x + 2 = 0$ i.e ; $x = -2$ in (i), we get

$9 = B(-2-1)$

$9 = -3B$

$B = -3$

Put $x - 1 = 0$ i.e ; $x = 1$ in (i), we get

$9 = C(1+2)^2$

$9 = C(3)^2$

$9 = C(9)$

$C = 1$

by comparing co − efficient of x^2 in (ii), we get

$0 = A + C$

Put $C = 1$ in it, we get

$0 = A + 1$

$A = -1$

Putting values of A, B, C in (k), we get

Therefore, partial fractions are

$$= \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

Thus, $\dfrac{9}{(x+2)^2\,(x-1)} = \dfrac{1}{x-1} - \dfrac{1}{x+2} - \dfrac{3}{(x+2)^2}$

4. $\dfrac{x^4+1}{x^2(x-1)}$

$$= \frac{x^4+1}{x^3-x^2}$$

$$= x + 1 + \frac{x^2+1}{x^3-x^2}$$

By long division, we have

$$\boxed{\begin{array}{r} x+1 \\ x^3 - x^2 \overline{)x^4+1} \\ \pm x^4 \mp x^2 \\ \hline x^3 + 1 \\ \pm x^3 \mp x^2 \\ \hline x^2 + 1 \end{array}}$$

$$= x + 1 + \frac{x^2+1}{x^2(x-1)} \ldots\ldots\ldots\ldots (i)$$

Now consider $\frac{x^2+1}{x^2(x-1)}$

Now, Let $\frac{x^2+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Multiplying by $x^2(x-1)$, we get

$x^2 + 1 = A(x)(x-1) + B(x-1) + Cx^2 \ldots\ldots (iii)$

Put $x = 0$ in (iii)

$(1)^2 + 1 = B(0-1)$

$1 = B(-1)$

$1 = -B$

$\boldsymbol{B = -1}$

Put $x - 1 = 0$ i.e ; $x = 1$ in (iii), we get

$(1)^2 + 1 = C(1)^2$

$2 = C(1)$

$\boldsymbol{C = 2}$

by comparing co − efficient of x^2 in (iii), we get

$1 = A + C$

Put $C = 2$ in it

$1 = A + 2$

$A = 1 - 2 = -1$

Putting values of A, B, C in (ii) and from (i)

Therefore, partial fractions are

$$x + 1 - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-1}$$

Thus, $x + 1 + \frac{x^2+1}{x^2(x-1)} = x + 1 - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-1}$

5. $\frac{7x+4}{(3x+2)(x+1)^2}$

Solution:

Let $\frac{7x+4}{(x+1)^2(3x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{3x+2} \ldots\ldots\ldots\ldots (i)$

Multiplying both sides by $(x+1)^2(3x+2)$, we get

$7x+4 = A(x+1)(3x+2) + B(3x+2) + C(x+1)^2 \ldots\ldots (ii)$

$7x+4 = A(3x^2+5x+2) + B(3x+2) + C(x^2+2x+1) \ldots. (iii)$

Put $x+1=0$ i.e ; $x=-1$ in (ii), we get

$$7(-1)+4 = B\{3(-1)+2\}$$
$$-7+4 = B(-3+2)$$
$$-3 = -B$$
$$B = 3$$

Put $3x+2=0$ i.e; $x=-\frac{2}{3}$ in (ii), we get

$$7\left(-\frac{2}{3}\right)+4 = C\left(-\frac{2}{3}+1\right)^2$$
$$\frac{-14}{3}+4 = C\left(\frac{-2+3}{3}\right)^2$$
$$\frac{-14+12}{3} = C\left(\frac{1}{3}\right)^2$$
$$\frac{-2}{3} = \frac{1}{9}C$$
$$C = \left(-\frac{2}{3}\right)(9)$$
$$C = -6$$

by comparing co − efficients of x^2 in (iii), we get

$$0 = 3A + C$$

Putting $C=-6$ in it, we get

$$0 = 3A + (-6)$$
$$0 = 3A - 6$$
$$3A = 6$$
$$A = 2$$

Putting values of A, B, C in (i), we get

Therefore, partial fractions are

$$\frac{2}{x+1} + \frac{3}{(x+1)^2} - \frac{6}{3x+2}$$

Thus, $\frac{7x+4}{(x+1)^2(3x+2)} = \frac{2}{x+1} + \frac{3}{(x+1)^2} - \frac{6}{3x+2}$

6. $\frac{1}{(x-1)^2(x+1)}$

Solution: Let $\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \ldots\ldots\ldots(i)$

Multiplying both sides by $(x-1)^2(x+1)$, *we get*

$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \ldots\ldots\ldots(ii)$

$1 = A(x^2-1) + B(x+1) + C(x^2-2x+1) \ldots\ldots(iii)$

Put $x - 1 = 0$ *i.e* ; $x = -1$ *in (ii), we get*

$1 = B(1+1)$

$1 = 2B$

$B = \frac{1}{2}$

Put $x + 1 = 0$ *i.e* ; $x = -1$ *in* (ii), *we get*

$1 = C(-1-1)^2$

$1 = C(-2)^2$

$1 = 4C$

$C = \frac{1}{4}$

by comparing co − efficients of x^2 *in (iii), we get*

$0 = A + C$

Put $C = \frac{1}{4}$ *in it, we get*

$0 = A + \frac{1}{4}$

$A = -\frac{1}{4}$

Putting values of A, B, C in (i), we get

Therefore, partial fractions are

$\frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$

$= \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$

Thus, $\frac{1}{(x-1)^2(x+1)} = \frac{1}{4(x+1)} = \frac{1}{4(x-1)} = \frac{1}{2(x-1)}$

7. $\frac{3x^2+15x+16}{(x+2)^2}$

Solution: $= \frac{3x^2+15x+16}{(x+2)^2} = \frac{3x^2+15x+16}{x^2+4x+4}$

$x^2 + 4x + 4 \; \overset{3}{\overline{)3x^2+15x+16}}$

$\underline{\pm 3x^2 \pm 12x \pm 12}$

$3x + 4$

$$= \frac{3x^2+15x+16}{x^2+4x+4} = 3 + \frac{3x+4}{(x+2)^2}$$

$$= 3 + \frac{3x+4}{(x+2)^2} \ldots\ldots\ldots\ldots\ldots (i)$$

Now consider $\frac{3x+4}{x^2+4x+4}$

$$\frac{3x+4}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} \ldots\ldots\ldots (ii)$$

Multiplying by $(x+2)^2$, we get

$3x+4 = A(x+2) + B \ldots\ldots\ldots\ldots (iii)$

Put $x+2=0$ i.e ; $x=-2$ in (iii), we get

$3(-2)+4 = A(-2+2) = B$

$-6+4 = B$

$B = -2$

by comparing co – efficients of x in (iii), we get

$3 = A$

Putting values of A, B in (ii)joining with (i)

Therefore, partial fractions are

$$3 + \frac{3}{x+2} - \frac{2}{(x+2)^2}$$

$$\text{Thus, } 3 + \frac{3x+4}{(x+2)^2} = 3 + \frac{3}{x+2} - \frac{2}{(x+2)^2}$$

8. $\frac{1}{(x^2-1)(x+1)}$

Solution:

$$= \frac{1}{(x^2-1)(x+1)}$$

$$= \frac{1}{(x-1)(x+1)(x+1)}$$

$$= \frac{1}{(x-1)(x+1)^2}$$

$$= \frac{1}{(x+1)^2(x-1)}$$

Let $\frac{1}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} \ldots\ldots\ldots\ldots\ldots (i)$

Multiplying both sides by $(x+1)^2(x-1)$, we get

$1 = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \ldots\ldots (ii)$

$1 = A(x^2-1) + B(x-1) + C(x^2+2x+1) \ldots\ldots (iii)$

Put $x+1=0$ i.e ; $x=-1$ in (ii), we get

$1 = B(-1-1)$

$1 = -2B$

$B = -\frac{1}{2}$

Put $x - 1 = 0$ *i.e* ; $x = 1$ *in* (*iii*), *we get*

$1 = C(2)^2$

$1 = 4C$

$C = \frac{1}{4}$

by comparing co − *efficients of* x^2 *in* (*iii*), *we get*

$0 = A + C$

Put $C = \frac{1}{4}$ *in it, we get*

$0 = A + \frac{1}{4}$

$A = -\frac{1}{4}$

Putting values of A, B, C *in* (*i*), *we get*

Therefore, partial fraction are

$$= -\frac{1}{4(x+1)} - \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)}$$

$$= \frac{1}{4(x-1)} - \frac{1}{4(x+1)} \frac{1}{2(x+1)^2}$$

Thus, $$\frac{1}{(x+1)^2(x-1)} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$