Exercise 4.3

Resolve into partial fractions.

1. $\dfrac{3x-11}{(x+3)(x^2+1)}$

Solution: Let $\dfrac{3x-11}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ ……… (i)

Multiplying both sides by $(x+3)(x^2+1)$, *we get*

$3x - 11 = A(x^2+1) + (Bx+C)(x+3)$ ……. (ii)

$3x - 11 = A(x^2+1) + Bx^2 + 3Bx + Cx + 3C$

$3x - 11 = Ax^2 + A + Bx^2 + 3Bx + Cx + 3C$

$3x - 11 = Ax^2 + Bx^2 + 3Bx + Cx + A + 3C$

$3x - 11 = (A+B)x^2 + (3B+C)x + A + 3C$ ..(iii)

$x + 3 = 0$ i.e ; $x = -3$ in (ii), we get

$3(-3) - 11 = A[(-3)^2 + 1]$
$-9 - 11 = A[9 + 1]$
$-20 = 10A$
$\boxed{A = -2}$
by comparing co – efficient of x^2*, we get*
$0 = A + B$
Put $A = -2$ *in it, we get*
$0 = -2 + B$
$B = 2$
by comparing co – efficients of x *in (iii), we get*
$0 = 3B + C$
Put $B = 2$ *in it*
$0 = 3(2) + C$
$0 = 6 + C$
$C = -6$
Putting values of A, B, C in (i), we get
Therefore, partial fraction are

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{2}{x+3} + \frac{2x-6}{x^2+1}$$

2. $\frac{3x+7}{(x^2+1)(x+3)}$

Solution: $= \frac{3x+7}{(x+3)(x^2+3)}$

$$= \frac{3x+7}{(x+3)(x^2+1)}$$

Let $\frac{3x+7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$ ……………………(i)

Multiplying by $(x+3)(x^2+1)$*, we get*
$3x + 7 = A(x^2 + 1) + (Bx + C)(x + 3)$ ………….(ii)
$3x + 7 = Ax^2 + A + Bx^2 + 3Bx + Cx + A + 3C$
$3x + 7 = Ax^2 + Bx^2 + 3Bx + Cx + A + 3C$
$3x + 7 = (A + B)x^2 + (3B + C)x + A + 3C$ ………(iii)
Put $x + 3 = 0$ *i.e;* $x = -3$ *in (ii), we get*
$3(-3) + 7 = A\{(-3)^2 + 1\}$
$-9 + 7 = A(9 + 1)$
$-2 = 10A$
$A = -\frac{2}{10}$
$A = -\frac{1}{5}$

by comparing co − efficients of x^2 in (iii), we get

$A + B = 0$

Put $A = -\frac{1}{5}$ in it, we get

$-\frac{1}{5} + B = 0$

$B = \frac{1}{5}$

by comparing co − efficeints of x in (iii), we get

$3 = 3B + C$

Put $B = \frac{1}{5}$ in it, we get

$3 = 3\left(\frac{1}{5}\right)$ in it, we get

$3 = 3\left(\frac{1}{5}\right) + C$

$3 = \frac{3}{5} + C$

$C = \frac{15-3}{5}$

$C = \frac{12}{5}$

Putting values of A, B, C in (i), we get

Therefore, partial fractions are

$$= -\frac{1}{5(x+3)} + \frac{\frac{1}{5}x + \frac{12}{5}}{x^2+1}$$

$$= -\frac{1}{5(x+3)} + \frac{x+12}{5(x^2+1)}$$

Hence, $\frac{3x+7}{(x+3)(x^2+1)} = \frac{1}{5(x+3)} + \frac{x+12}{5(x^2+1)}$

3. $\frac{1}{(x+1)(x^2+1)}$

Solution: Let $\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ ……………(i)

Multiplying both sides by $(x+1)(x^2+1)$, we get

$1 = A(x^2+1) + (Bx+C)(x+1)$

$1 = Ax^2 + A + Bx^2 + Bx + Cx + C$

$1 = Ax^2 + Bx^2 + Bx + Cx + A + C$

$1 = (A+B)x^2 + (B+C)x + A + C$ ……………(iii)

Put $x + 1 = 0$ i.e ; $x = -1$ in (ii), we get

$$1 = A(-1)^2 + 1$$
$$1 = A(1+1)$$
$$1 = 2A$$
$$A = \frac{1}{2}$$

by comparing co – efficient of x^2 *in (iii), we get*

$$0 = A + B$$

Put $A = \frac{1}{2}$ *in it, we get*

$$0 = \frac{1}{2} + B$$
$$B = -\frac{1}{2}$$

by comparing co – efficient of x *in (iii), we get*

$$0 = B + C$$

Put $B = -\frac{1}{2}$ *in it, we get*

$$0 = -\frac{1}{2} + C$$
$$C = \frac{1}{2}$$

Putting values of A, B, C in (i), we get
Partial fractions are

$$= \frac{1}{2(x+1)} + \frac{-\frac{1}{2}x + \frac{1}{2}}{(x^2+1)}$$
$$= \frac{1}{2(x+1)} - \frac{x-1}{2(x^2+1)}$$

Thus, $$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{x-1}{2(x^2+1)}$$

4. $\frac{9x-7}{(x+3)(x^2+1)}$

Solution: *Let* $\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$ ……………(i)

Multiplying both sides by $(x+3)(x^2+1)$*, we get*

$$9x - 7 = A(x^2+1) + (Bx+C)(x+3) \ldots\ldots (ii)$$
$$9x - 7 = Ax^2 + A + Bx^2 + 3Bx + Cx + 3C$$
$$9x - 7 = (A+B)x^2 + (3B+C)x + (A+3C) = 0$$

Put $x + 3 = 0$ *i.e*; $x = -3$ *in (ii), we get*

$$9(-3) - 7 = A((-3)^2 + 1)$$

$$-27 - 7 = A(9 + 1)$$

$$-34 = 10A$$

$$A = -\frac{34}{10}$$

$$A = -\frac{17}{5}$$

by comparing co − efficients of x^2 *in (iii), we get*

$$0 = A + B$$

Put $A = -\frac{17}{5}$ *in it, we get*

$$0 = -\frac{17}{5} + B$$

$$B = \frac{17}{5}$$

by comparing co − effficients of x in (iii), we get

$$9 = 3B + C$$

Putting $B = \frac{17}{5}$ *in it, we get*

$$9 = 3\left(\frac{17}{5}\right) + C$$

$$9 = \frac{51}{5} + C$$

$$C = 9 - \frac{51}{5}$$

$$= \frac{45-51}{5}$$

$$C = -\frac{6}{5}$$

Putting values of A, B, C in (i), we get

Therefore, partial fraction are

$$= -\frac{17}{5(x+3)} + \frac{\frac{17}{5}x - \frac{6}{5}}{x^2+1}$$

Thus, $\frac{9x-7}{(x+3)(x^2+1)} - \frac{17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$

5. $\frac{3x+7}{(x+3)(x^2+4)}$

Solution: *Let* $\frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+c}{x^2+4}$ ……………(i)

Multiplying both sides by $(x+3)(x^2+4)$*, we get*

$$3x + 7 = A(x^2 + 4) + (Bx + C)(x + 3) \;\ldots\ldots\;\text{(ii)}$$

$3x + 7 = Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C$

$3x + 7 = Ax^2 + Bx^2 + 3Bx + Cx + 4A + 3C$

$3x + 7 = (A + B)x^2 + (3B + C)x + 4A + 3C$ ……….(iii)

Put $x = -3$ *in (ii), we get*

$3(-3) + 7 = A[(-3)^2 + 4]$

$-9 + 7 = A(9 + 4)$

$-2 = 13A$

$A = -\frac{2}{13}$

by comparing co − efficients of x^2 *in (iii), we get*

$0 = A + B$

Put $A = -\frac{2}{13}$ *in it, we get*

$0 = -\frac{2}{13} + B$

$B = \frac{2}{13}$

by comparing co − efficients os x in (iii), we get

$3 = 3B + C$

Put $B = \frac{2}{13}$ *in it, we get*

$3 = 3\left(\frac{2}{13}\right) + C$

$C = 3 - \frac{6}{13}$

$C = \frac{39-6}{13}$

$C = \frac{33}{13}$

Putting values of A, B, C in (i), we get
Therefore, partial fractions are

$= -\frac{2}{13(x+3)} + \frac{\frac{2}{13}x + \frac{33}{13}}{(x^2+4)}$

$= -\frac{2}{13(x+3)} + \frac{2x + 33}{13(x^2+4)}$

Thus, $\frac{3x+7}{(x+3)(x^2+4)} = -\frac{2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$

6. $\frac{x^2}{(x+2)(x^2+4)}$

Solution: Let $\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$ ………… (i)

Mutiplying both sides by $(x+2)(x^2+4)$, we get

$x^2 = A(x^2+4) + (Bx+C)(x+2)$ ………… (ii)

$x^2 = Ax^2 + 4A + Bx^2 + 2Bx + Cx + 2C$

$x^2 = (A+B)x^2 + (2B+C)x + (4A+2C)$ ….. (iii)

Put $x+2=0$ i.e ; $x=-2$ in (ii), we get

$(-2)^2 = A[(-2)^2+4]$

$4 = A(4+4)$

$4 = 8A$

$A = \frac{1}{2}$

by comparing co − efficients of x^2 in (iii), we get

$1 = A + B$

Put $A = \frac{1}{2}$ in it, we get

$1 = \frac{1}{2} + B$

$B = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

$B = \frac{1}{2}$

by comparing co − efficients of in (iii), we get

$0 = 2B + C$

Put $B = \frac{1}{2}$ in it, we get

$0 = 2\left(\frac{1}{2}\right) + C$

$0 = 1 + C$

$C = -1$

Putting values of A, B, C in (i), we get

$\frac{1}{2(x+2)} + \frac{\frac{1}{2}x-1}{x^2+4}$

$= \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$

Thus, $\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{1}{2(x+2)} + \dfrac{x-2}{2(x^2+4)}$

7. $\dfrac{1}{x^3+1}\left[\text{Hint } \dfrac{1}{x^3+1} = \dfrac{1}{(x+1)(x^2-x+1)}\right]$

Solution: $= \dfrac{1}{x^3+1}$

$= \dfrac{1}{(x+1)(x^2-x+1)}$

Let $\dfrac{1}{(x+1)(x^2-x+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-x+1}$ ……… (i)

Multiplying both side by $(x+1)(x^2-x+1)$, *we get*

$1 = A(x^2 - x + 1) + (Bx + C + 1)$ ……………… (ii)

$1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$

$1 = Ax^2 + Bx^2 + Bx + Cx - Ax + A + C = 0$

$1 = (A + B)x^2 + (B + C - A)x + A + C = 0$ ……. (iii)

Put $x + 1 = 0$ i.e ; $x = -1$ in (ii), we get

$1 = A[(-1)^2 - (-1) + 1]$

$1 = A[1 + 1 + 1]$

$1 = 3A$

$A = \dfrac{1}{3}$

by comparing co – efficients of x^2 *in* (iii), *we get*

$0 = A + B$

Put $A = \dfrac{1}{3}$ in it, we get

$1 = \dfrac{1}{3} + C$

$C = 1 - \dfrac{1}{3}$

$= \dfrac{3-1}{3}$

$C = \dfrac{2}{3}$

Putting values of A, B, C in (i), we get

Therefore, partial fractions are

$= \dfrac{1}{3(x+1)} = \dfrac{x-2}{3(x^2-x+1)}$

Thus, $\dfrac{1}{(x+1)(x^2-x+1)} = \dfrac{1}{3(x+1)} - \dfrac{x-2}{3(x^2-x+1)}$

8. $\frac{x^2+1}{x^3+1}$

Solution: $= \frac{x^2+1}{x^3+1}$

$= \frac{x^2+1}{(x+1)(x^2-x+1)}$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ ……………. (i)

Multiplying by $(x+1)(x^2-x+1)$, *we get*

$x^2+1 = A(x^2-x+1)+(Bx+C)(x+1)$ …… (ii)

$x^2+1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$

$x^2+1 = Ax^2 + Bx^2 + Bx + Cx - Ax + A$

Put $x+1=0$ *i.e* ; $x=-1$ *in (ii)*

$(-1)^2+1 = A[(-1)^2 - 1) + 1$

$1+1 = A(1+1+1)$

$2 = 3A$

$A = \frac{2}{3}$

by comparing co − efficients of x^2 *in (iii), we get*

$1 = A + B$

Put $A = \frac{2}{3}$ *in it, we get*

$1 = \frac{2}{3} + C$

$C = 1 - \frac{2}{3}$

$C = \frac{1}{3}$

Putting $A = \frac{2}{3}$, $B = \frac{1}{3}$, $C = \frac{1}{3}$ in (i), we get

Therefore, partial fraction are

$\frac{2}{3(x+1)} + \frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)}$

$\frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$

Thus, $\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$