1. if $X = \{1, 3, 5, 7, \dots \dots 19\}$

$Y = \{0, 2, 4, 6, 8, \dots \dots 20\}$

$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$,

then find the following:

(i) $X \cup (Y \cup Z)$

Solution: $X \cup (Y \cup Z)$

$= \{1,3,5,7, \dots ,19\} \cup [\{0,2,4,6,8 \dots .20\} \cup \{2,3,5,7,11,13,17,19,23\}]$

$= \{1,3,5,7 \dots ,19\} \cup \{0,2,3,4, \dots ,8,10,11,12,13,14,16,17,18,19,20,23\}$

$= \{0,1,2,3, \dots \dots .20,23\}$

(ii) $(X \cup Y) \cup Z$

Solution: $(X \cup Z) \cup Z$

$= [\{1,3,5,7, \dots 19\} \cup \{0,2,4,6,8 \dots 20\}] \cup \{2,3,5,7,11,13,17,19,23\}$

$= \{0,1,2,3 \dots 20\} \cup \{2,3,5,7,11,13,17,19,23\}$

$= \{0,1,2,3 \dots \dots ,20,23\}$

(iii) $X \cap (Y \cap Z)$

Solution: $X \cap (Y \cap Z)$

$= \{1,3,5,7, \dots ,19\} \cap [\{0,2,4,6 \dots ,20\} \cap \{2,3,5,7,11,13,17,19,23\}]$

$= \{1,3,5,7 \dots ,19\} \cap \{2\}$

$= \{ \ \}$

(iv) $(X \cap Y) \cap Z$

Solution: $(X \cap Y) \cap Z$

$= [\{1,3,5,7, \dots ,19\}] \cap \{0,2,4,6 \dots ,20\} \cap \{2,3,5,7,11,13,17,19,23\}$

$= \{1,3,5,7, \dots .19\} \cap \{2\}$

$= \{ \ \}$

(v) $\quad X \cup (Y \cap Z)$

Solution: $\quad X \cup (Y \cap Z)$

$= \{1,3,5,7, \ldots ,19\} \cup [\{0,2,4,6,8, \ldots ,20\} \cap \{2,3,5,7,11,13,17,19,23\}]$

$= \{1,3,5,7, \ldots .19\} \cup \{2\}$

$= \{1,2,3,5,7 \ldots .,19\}$

(vi) $\quad (X \cup Y) \cap (X \cup Z)$

Solution: $\quad (X \cup Y) = \{1,3,5,7 \ldots .,19\} \cup \{0,2,4,6,8, \ldots ..20\}$

$\qquad\qquad\qquad = \{0,1,2,3,4, \ldots .,20\}$

$\qquad (X \cup Z) = \{1,3,5,7, \ldots 19\} \cup \{2,3,5,7,11,13,17,19,23\}$

$\qquad\qquad\qquad = \{1,2,3,5,7,11,13,17,19,23\}$

$(X \cup Y) \cap (X \cup Z) = \{0,1,2,3,4 .. 20\} \cap \{1,2,3,5,7,11,13,17,19,23\}$

$\qquad\qquad\qquad = \{1,2,3,5,7 \ldots .,19\}$

(vii) $\quad X \cap (Y \cup Z)$

$\qquad Now, Y \cup Z = \{0,2,4,6,8, \ldots .20\} \cap \{2,3,5,7,11,13,17,19,23\}$

$\qquad\qquad\qquad = \{0,2,3,4, \ldots .8,10,11,12,13,14,16,17,18,19,20,23\}$

$\qquad X \cap (Y \cup Z) = \{1,3,5,7, \ldots .19\} \cap \{0,2,3,4, \ldots 8,10,11,12,13,$

$\qquad\qquad\qquad = 14,16,17,18,9,20,23\}$

$\qquad\qquad\qquad = \{3,5,7,11,13,17,19\}$

(viii) $\quad (X \cap Y) \cup (X \cap Z)$

Solution: $\quad X \cap Y$

$\qquad = \{1,3,5,7, \ldots .19\}\{0,2,4,6,8 \ldots ..,20\}$

$\qquad = \{ \quad \}$

$\qquad X \cap Z$

$\qquad = \{1,3,5,7,11, \ldots .,19\} \cap \{2,3,5,7,11,13,17,19,23\}$

$\qquad = \{3,5,7,11,13,17,19\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (ii)$

$\qquad Now, \quad (X \cap Y) \cup (X \cap Z)$

$\qquad = \{ \quad \} \cup \{3,5,7,11,13,17,19\}$

$\qquad = \{3,5,7,11,13,17,19\}$

2. $\quad$ **If A= \{1, 2, 3, 4, 5, 6\}, B= \{2, 4, 6, 8\}, C= \{1, 4, 8\}.**

$\qquad$ **Prove the following identities:**

Solution:

(i) $\quad A \cap B = B \cap A$

$\qquad L.H.S. = A \cap B$

$\qquad = \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$

$\qquad = \{2,4,6\}$

$\qquad R.H.S. = B \cap A$

$\qquad = \{2,4,6,8\} \cap \{1,2,3, \ldots .,6\}$

$\qquad = \{2,4,6\}$

$\qquad Hence \ Proved, \quad A \cap B = B \cap A$

(ii) $A \cup B = B \cup A$

$L.H.S = A \cup B$

$= \{1,2,3, \ldots ,6\} \cup \{2,4,6,8\}$

$= \{1,2,3, \ldots 6,8\}$

$R.H.S = B \cup A$

$= \{2,4,6,8\} \cup \{1,2,3, \ldots ,6\}$

$= \{1,2,3, \ldots ,6,8\}$

Hence Proved, $A \cup B = B \cup A$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C)$

$= \{1,2,3, \ldots ,6\} \cap [\{2,4,6,8\} \cup \{1,4,8\}]$

$= \{1,2,3 \ldots ,6\} \cap \{1,2,4,6,8\}$

$= \{1,2,4,6\}$

Hence Proved, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$= \{1,2,3, \ldots ,6\} \cup [\{2,4,6,8\} \cap \{1,4,8\}]$

$= \{1,2,3, \ldots ,6\} \cup \{4,8\}$

$= \{1,2,3, \ldots 6,8\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$= [\{1,2,3, \ldots ,6\} \cup \{2,4,6,8\}] \cap \{1,2,3, \ldots ,6\} \cup \{1,4,8\}$

$= \{1,2,3, \ldots ,6,8\} \cap \{1,2,3, \ldots ,6,8\}$

$= \{1,2,3, \ldots ,6,8\}$

Hence Proved, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

3. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$.

Then verify the De–Morgan's laws

i.e., $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Solution:

(i) $(A \cap B)' = A' \cup B'$

$(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)'$

$A \cap B = \{1,3,5,7,9\} \cap \{2,3,5,7\}$

$= \{3,5,7\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,2,3, \ldots ,10\} - \{3,5,7\}$

$= \{1,2,4,6,8,9,10\} \ldots \ldots \ldots \ldots (i)$

$R.H.S = A' \cup B'$

$A' = (U - A) = \{1,2,3, \ldots ,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$$B' = (U - B) = \{1,2,3,\ldots,10\} - \{2,3,5,7\}$$
$$= \{1,4,6,8,9,10\}$$

Now, $A' \cup B'$

$$A' \cup B' = \{2,4,6,8,9,10\} \cup \{1,4,6,8,9,10\}$$
$$= \{1,2,4,6,8,9,10\} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

From (i), (ii), we get

Hence Proved, $(A \cap B)' = A' \cup B'$

(ii) $(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)'$

$A \cup B = \{1,3,5,7,9\} \cup \{2,3,5,7\}$
$$= \{1,2,3,5,7,9\}$$

$(A \cup B)' = \{1,2,3,\ldots.10\} - \{1,2,3,5,7,9\}$
$$= \{4,6,8,10\} \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$R.H.S. = A' \cap B'$

$A' = (U - A) = \{1,2,3,\ldots,10\} - \{1,3,5,7,9\}$
$$= \{2,4,6,8,10\}$$

$B' = (U - B) = \{1,2,3,\ldots,10\} - \{2,3,5,7\}$
$$= \{1,4,6,8,9,10\}$$

$A' \cap B' = \{2,4,6,8,10\} \cap \{1,4,6,8,9,10\}$
$$= \{4,6,8,10\} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Hence from (i) and (ii)

It is Proved that, $(A \cup B)' = A' \cap B'$

4. If $U = \{1, 2, 3, \ldots, 20\}$.

$X = \{1, 3, 5, 7, 9, 15, 18, 20\}$

and $Y = \{1, 3, 5, \ldots, 17\}$, then show that

Solution:

(i) $X - Y = X \cap Y'$

$L.H.S. = X - Y$

$X - Y = \{1,3,5,7,9,15,18,20\} - \{1,3,5,\ldots,17\}$
$$= \{18,20\} \ldots\ldots\ldots(i)$$

$R.H.S = X \cap Y'$

$Y' = U - Y$
$$= \{1,2,3,\ldots.20\} - \{1,3,5,7,9,11,13,15,17\}$$
$$= \{2,4,6,8,10,12,14,16,18,19,20\}$$

$X \cap Y' = \{1,3,5,7,9,15,18,20\} \cap \{2,4,6,8,10,12,14,16,18,19,20\}$
$$= \{18,20\} \ldots\ldots\ldots(ii)$$

Hence from (i) and (ii)

It is Proved that, $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

$L.H.S = Y - X$

$$Y - X = \{1,3,5,7,9,11,13,15,17\} - \{1,3,5,7,9,15,18,20\}$$
$$= \{5,11,13,17\} \quad \dots\dots\dots(i)$$

$R.H.S = Y \cap X'$

$X' = U - X$
$$= \{1,2,3,\dots,20\} - \{1,3,7,9,15,18,20\}$$
$$= \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$$

$Y \cap X' = \{1,3,5,7,9,11,13,15,17\} \cap \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$
$$= \{5,11,13,17\} \quad \dots\dots\dots(i)$$

Hence from (i) and (ii)

It is Proved that, $\quad Y - X = Y \cap X'$