Exercise 5.3

1. *If* $U = \{1, 2, 3, 4, \ldots\ldots 10\}$,

   $A = \{1, 3, 5, 7, 9\}$

   $B = \{1, 4, 7, 10\}$.

   then verify the following questions.

Solution:

(i) $A - B = A \cap B'$

$L.H.S = A - B$

$= \{1,3,5,7,9\} - \{1,4,7,10\}$

$= \{3,5,9\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$R.H.S = A \cap B'$

$Now\ B' = (U - B) = \{1,2,3,4, \ldots\ldots 10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$A \cap B' = \{1,3,5,7,9\} \cap \{2,3,5,6,8,9\}$

$= \{3,5,9\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Hence from (i) and (ii)

It is Proved that, $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

$L.H.S = B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$

$= \{4,10\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$R.H.S = B \cap A'$

$Now\ A' = (U - A) = \{1,2,3, \ldots 10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B \cap A' = \{1,4,7,10\} \cap \{2,4,6,8,10\}$

$= \{4,10\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Hence from (i) and (ii)

It is Proved that, $B - A = B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)'$

$A \cup B = \{1,3,5,7,9\} \cup \{1,4,7,10\}$

$= \{1,3,4,5,7,9,10\}$

$(A \cup B)' = U - (A \cup B) = \{1,2,3, \ldots\ldots 10\} - \{1,3,4,5,7,9,10\}$

$= \{2,6,8\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$R.H.S. = A' \cap B'$

$Now\ A' = (U - A) = \{1,2,3, \ldots\ldots, 10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$and\ B' = (U - B) = \{1,2,3, \ldots\ldots, 10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$Now\quad A' \cap B' = \{2,4,6,8,10\} \cap \{2,3,5,6,8,9\}$

$= \{2,6,8\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Hence from (i) and (ii)

It is Proved that, $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

$L.H.S. = (A \cap B)'$

$A \cap B = \{1,3,5,7,9\} \cap \{1,4,7,10\}$

$= \{1,7\}$

$(A \cap B)' = U - (A \cap B) = \{1,2,3, \ldots\ldots, 10\} - \{1,7,\}$

$= \{2,3,4,5,6,8,9,10\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$R.H.S. = A' \cup B'$

$A' = (U - A) = \{1,2,3, \ldots\ldots 10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = (U - B) = \{1,2,3, \ldots\ldots, 10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$A' \cup B' = \{2,4,6,8,10\} \cup \{2,3,5,6,8,9\}$

$= \{2,3,4,5,6,8,9,10\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

Hence from (i) and (ii)

It is Proved that, $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B'$

$L.H.S. = (A - B)'$

$A - B = \{1,3,5,7,9\} - \{1,4,7,9\}$

$= \{3,5,9\}$

$(A - B)' = U - (A - B)$

$= \{1,2,3, \ldots\ldots, 10\} - \{3,5,9\}$

$= \{1,2,4,6,7,8,10\} \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$R.H.S. = A' \cup B'$

$A' = (U - A) \quad = \{1,2,3, \ldots\ldots, 10\} - \{1,4,7,10\}$

$= \{2,4,6,8,10\}$

$B' = (U - B) = \{1,2,3, \dots \dots ,10\} - \{1,4,7,9\} = \{1,4,7,10\}$

Now $A' \cup B' = \{2,4,6,8,10\} \cup \{1,4,7,10\}$

$= \{1,2,4,6,7,8,10\} \dots \dots \dots \dots \dots \dots (ii)$

Hence from (i) and (ii)

It is Proved that, $(A - B)' = A' \cup B$

(vi) $(B - A)' = B' \cup A$

$L.H.S. = (B - A)'$

$B - A = \{1,4,7,10\} - \{1,3,5,7,9\}$

$= \{4,10\}$

$(B - A)' = U - (B - A) = \{1,2,3, \dots \dots ,10\} - \{4,10\}$

$= \{1,2,3,5,6,7,8,9\} \dots \dots \dots \dots \dots \dots \dots \dots \dots (i)$

$R.H.S = B' \cup A$

$A' = (U - A) = \{1,2,3, \dots .10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = (U - B) = \{1,2,3, \dots ,10\} - \{1,4,7,10\}$

$= \{2,3,4,5,6,8,9,10\} \dots \dots \dots \dots \dots \dots (ii)$

Hence from (i) and (ii)

It is Proved that, $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

$L.H.S = (A - B)'$

$A - B = \{1,3,5,7,9\} - \{1,4,7,10\}$

$= \{3,5,9\}$

$(A - B)' = U - (A - B) = \{1,2,3, \dots .10\} - \{3,5,9\}$

$= \{1,2,4,6,7,8,10\} \dots \dots \dots \dots \dots \dots \dots (i)$

$R.H.S = A' \cup B$

$A' = U - A = \{1,2,3, \dots ,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

Now $A' \cup B = \{2,4,6,8,10\} \cup \{1,4,7,10\}$

$= \{1,2,4,6,7,8,10\} \dots \dots \dots \dots \dots \dots \dots \dots (ii)$

Hence from (i) and (ii)

It is Proved that, $(A - B)' = A' \cup B$

(vi) $(B - A)' = B' \cup A$

$L.H.S. = (B - A)'$

$B - A = \{1,4,7,10\} - \{1,3,5,7,9\}$

$= \{4,10\}$

$(B - A)' = U - (B - A)$

$= \{1,2,3, \dots \dots ,10\} - \{4,10\}$

$= \{1,2,3,4,5,6,7,8,9\} \dots \dots \dots \dots \dots \dots \dots (i)$

$R.H.S = B' \cup A$

$B' = U - B$

$= \{1,2,3, \ldots\ldots, 10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$B' \cup A = \{2,3,5,6,8,9\} \cup \{1,3,5,7,9\}$

$= \{1,2,3,5,6,7,8,9\}$

Hence from (i) and (ii)

It is Proved that, $(B - A)' = B' \cup A$

2. ***If*** $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$

$A = \{1, 3, 5, 7, 9\}; B = \{1, 4, 7, 10\};$

$C = \{1, 5, 8, 10\}$

then verify the following:

Solution:

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

$L.H.S = (A \cup B) \cup C$

$= [\{1,3,5,7,9\} \cup \{1,4,7,10\}] \cup \{1,5,8,10\}$

$= \{1,3,4,5,7,9,10\} \cup \{1,5,8,10\}$

$= \{1,3,4,5,7,8,9,10\}$ ……………………… (i)

$R.H.S = A \cup (B \cup C)$

$= \{1,3,5,7,9\} \cup [\{1,4,7,10\} \cup \{1,5,8,10\}]$

$= \{1,3,4,5,7,9\} \cup \{1,4,5,7,8,10\}$

$= \{1,3,4,5,7,8,9,10\}$ ……………………… (ii)

From (i), (ii), we get

$(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S. = (A \cap B) \cap C$

$= [\{1,3,5,7,9\} \cap \{1,4,7,10\}] \cap \{1,5,8,10\}$

$= \{1,7\} \cap \{1,5,8,10\}$

$= \{1\}$ ……………………………………… (i)

$R.H.S. = A \cap (B \cap C)$

$= \{1,3,5,7,9\} \cap [\{1,4,7,10\} \cap \{1,5,8,10\}]$

$= \{1,3,5,7,9\} \cap \{1,10\}$

$= \{1\}$ ……………………………………… (ii)

From (i), (ii), we get

$(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution: $L.H.S = A \cup (B \cap C)$

$= \{1,3,5,7,9\} \cup [\{1,4,7,10,\} \cap \{1,5,8,10\}]$

$= \{1,3,5,7,9\} \cup \{1,10\}$

$= \{1,3,5,7,9,10\}$ ……………………………… (i)

$R.H.S = (A \cup B) \cap (A \cup C)$

$= [\{1,3,5,7,9\} \cup \{1,4,7,10\}] \cap [\{1,3,5,7,9\} \cup \{1,5,8,10\}]$

$= \{1,3,4,5,7,9,10\} \cap \{1,3,5,7,8,9,10\}$
$= \{1,3,5,7,9,10\} = \ldots\ldots\ldots (ii)$

From (i), (ii), we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution: $L.H.S. = A \cap (B \cup C)$
$= \{1,3,5,7,9\} \cap [\{1,4,7,10\} \cup \{1,5,8,10\}]$
$= \{1,3,5,7,9\} \cap \{1,4,5,7,8,10\}$
$= \{1,5,7\} \ldots\ldots\ldots (i)$

$R.H.S = (A \cap B) \cup (A \cap C)$
$= [\{1,3,5,7,9\} \cap \{1,4,7,10\}] \cap [\{1,3,5,7,9\} \cap \{1,5,8,10\}]$
$= \{1,7\} \cup \{1,5\}$
$= \{1,5,7\} \ldots\ldots\ldots (ii)$

From (i), (ii), we get

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. ***If $U = N$; then verify De – Morgan's Law by using $A = \phi$ and $B = P$.***

Solution: *De – morgan's Law*

(i) $(A \cup B)' = A' \cap B'$

Here $U = N = \{1,2,3, \ldots\}$
$A = \phi$
$B = P = \{2,3,5,7,11,13,17, \ldots\}$

Let us prove that:
$(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)'$
$A \cup B = \{\ \} \cup \{2,3,5,7,11,13 \ldots\}$
$= \{2,3,5,7,13,13 \ldots\}$
$(A \cup B)' = U - (A \cup B)$
$= \{1,2,3, \ldots\} - \{2,3,5,7,11,,13,17, \ldots\}$
$= \{1,4,6,8,9,10,12,14,15,6,18 \ldots\} \ldots (i)$

$R.H.S = A' \cap B'$
$A' = U - A$
$= \{1,2,3, \ldots\} - \{\ \}$
$= \{1,2,3, \ldots\}$
$B' = U - B$
$= \{1,2,3, \ldots\} - \{2,3,5,7,11,13,17, \ldots\}$
$= \{1,4,6,8,9,10,12,14,15,16, \ldots\}$

Now,
$A' \cap B' = \{1,2,3, \ldots\} \cap \{1,4,6,8,9,10,12,14,15,16, \ldots\}$
$= \{1,4,6,8,9,10,12,14,15,16, \ldots\} \ldots\ldots (ii)$

From $(i), (ii)$, we get

$(A \cup B)' = A' \cap B'$

(ii) $(A' \cap B') = A' \cup B'$

$L.H.S = (A' \cap B')$

$A \cap B = \{\ \} \cap \{2,3,5,7,11,13,17, \ldots \ldots\}$

$= \{\ \}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,2,3, \ldots \ldots\} - \{\ \}$

$= \{1,2,3,4, \ldots .\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i)$

$R.H.S = A' \cup B'$

Now $A' = U - A$

$= \{1,2,3, \ldots .\} - \{\ \}$

$= \{1,2,3, \ldots\}$

and $B' = U - B$

$= \{1,2,3, \ldots \ldots\} - \{2,3,5,7,11,13,17, \ldots \ldots .\}$

$= \{1,4,6,8,9,10,12, \ldots \ldots\}$

Now $A' \cup B' = \{1,2,3, \ldots .\} \cup \{1,4,6,8,9,10,12\}$

$= \{1,2,3,4, \ldots \ldots\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (ii)$

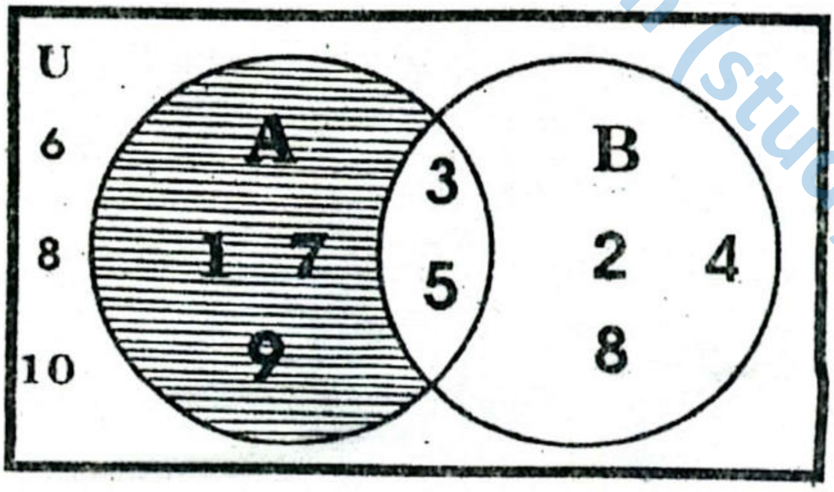
From $(i), (ii)$, we get

$(A \cap B)' = A' \cup B'$

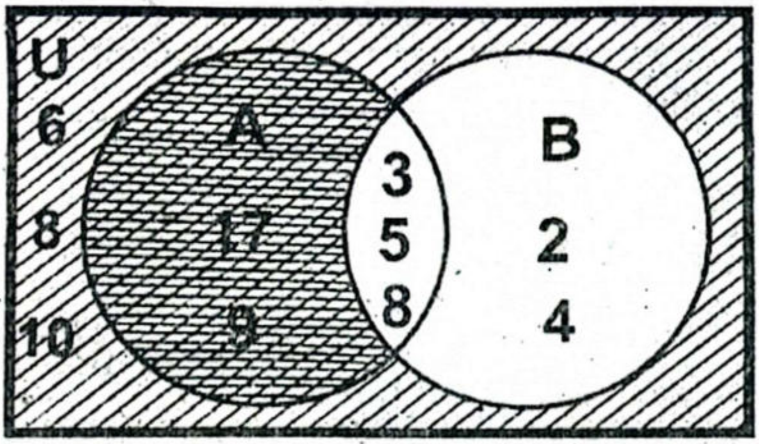
4. *If* $U = \{1, 2, 3, 4, \ldots \ldots, 10\}, A = \{1, 3, 5, 7, 9\}$ *and* $B = \{2, 3, 4, 5, 8\}$, ***then prove the following questions by Venn diagram:***

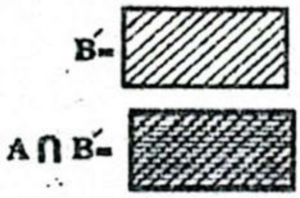
(i) $A - B = A \cap B'$

Solution:



A-B=





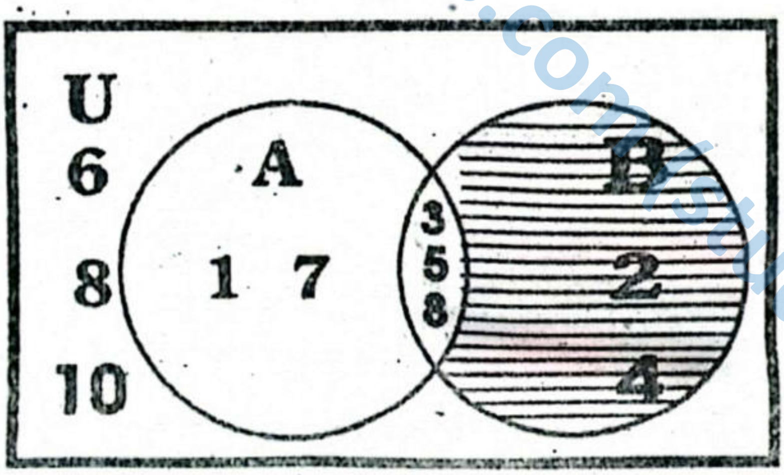
Note: *In fig(i) horizontal lines area and in fig (ii) double shaded area are the same therefore*

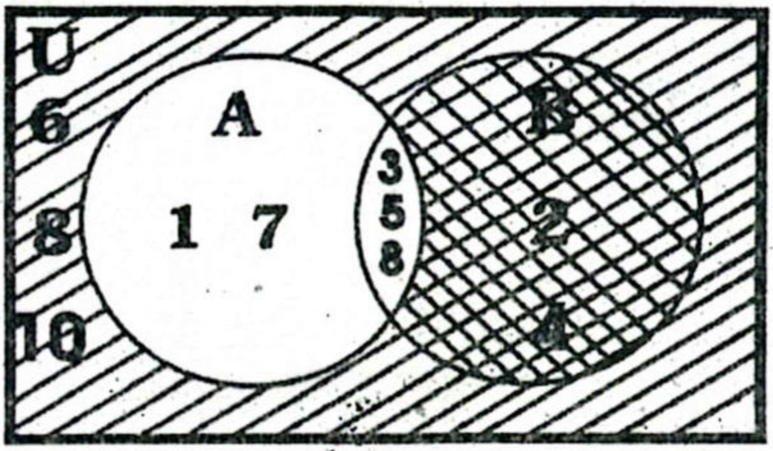
$$A - B = A \cap B'$$

(ii) $B - A = B \cap A'$

Solution:

$U = \{1, 2, 3, 4, \ldots\ldots, 10\}$
$A = \{1,3,5,7,9\}$
$B = \{2,3,4,5,8\}$





In fig (i) horizontal lines area and in fig (ii)double shaded area are the same.

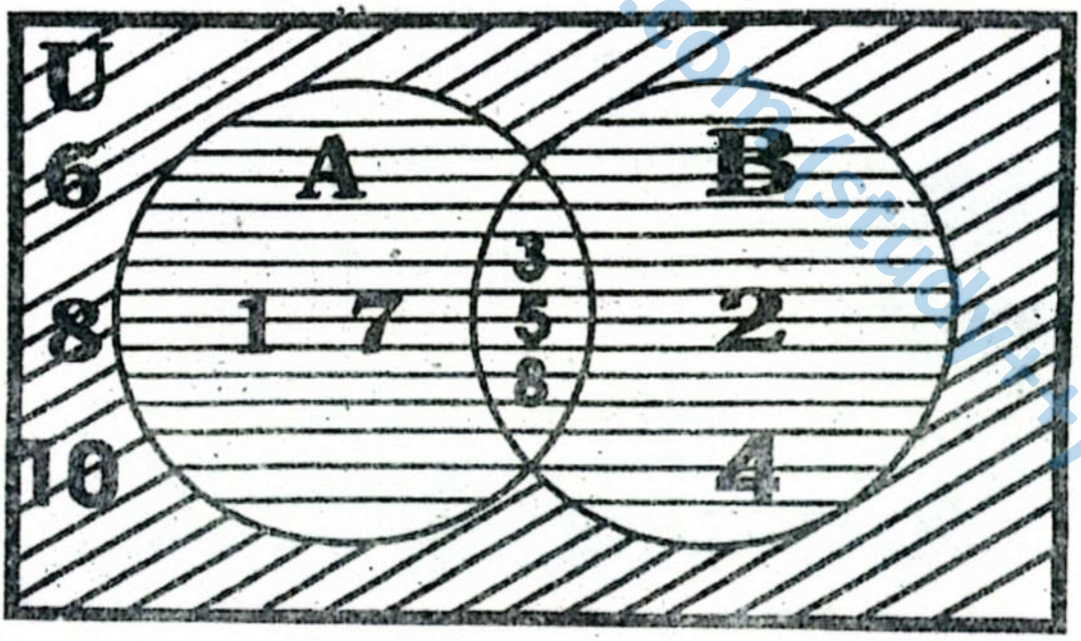
Therefore $B - A = B \cap A'$

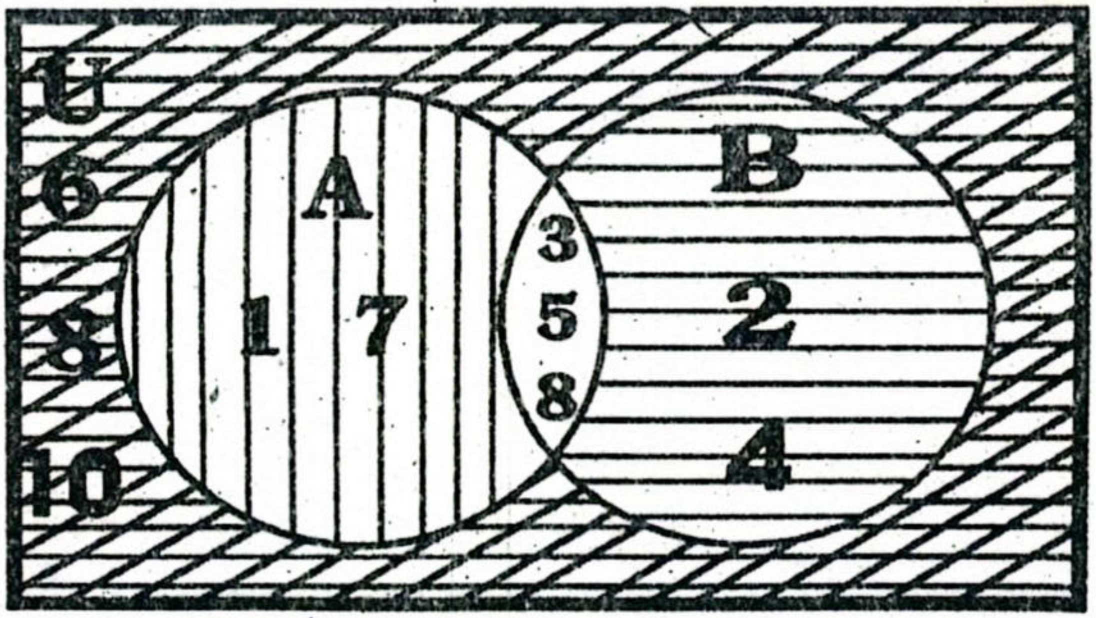
(iii) $(B - A)' = A' \cap B'$

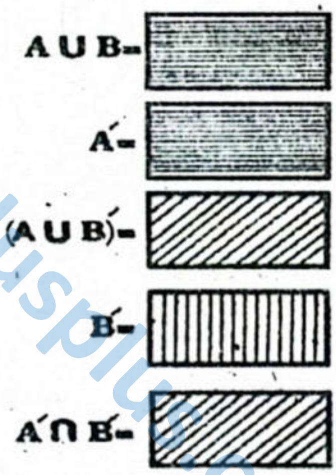
$U = \{1, 2, 3, \ldots\ldots, 10\}$

$A = \{1,3,5,7,9\}$

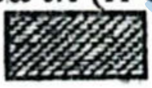
$B = \{2,3,4,5,8\}$







In fig(i) slanting line area in $(A \cup B)' = \{6,8,10\}$

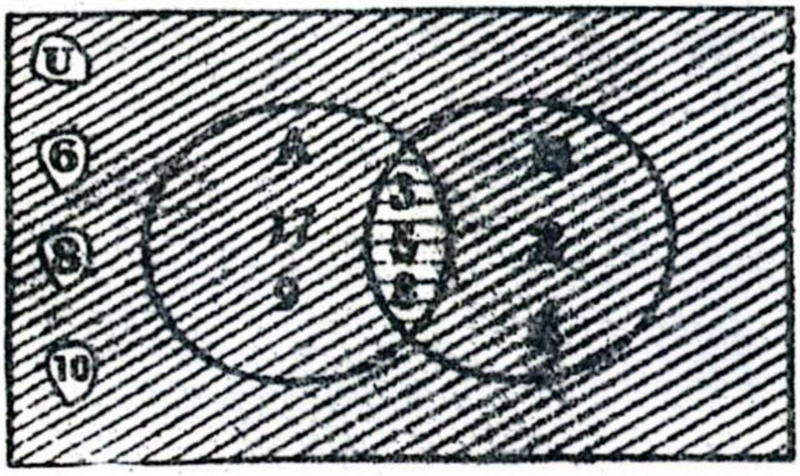
In fig (ii) area shown by  *is* $A' \cap B' = \{6,8,10\}$

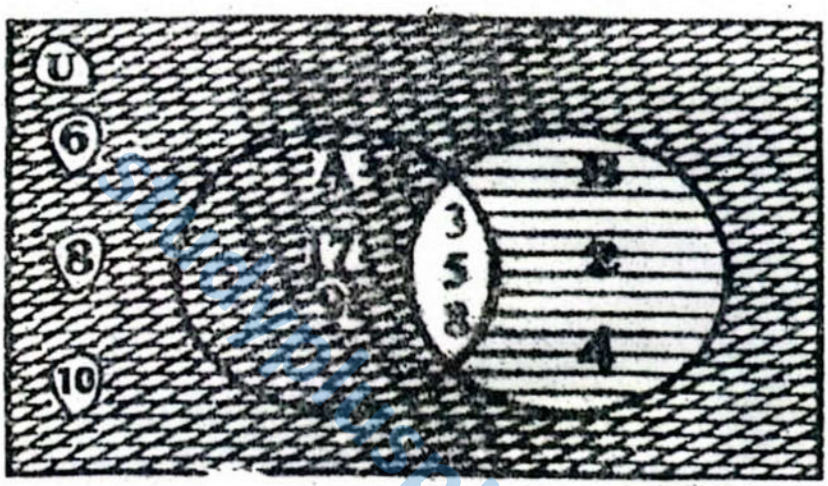
(iv) $(A \cap B)' = A' \cup B'$

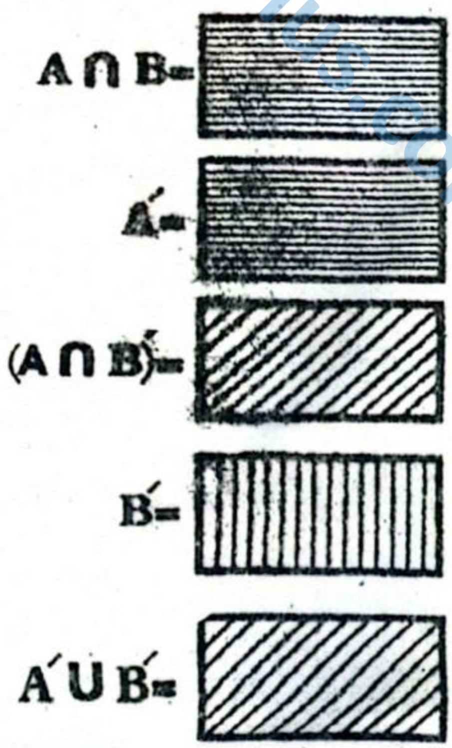
$U = \{1,2,3,\ldots\ldots,10\}$

$A = \{1,3,5,7,9\}$

$B = \{2,3,4,5,8\}$





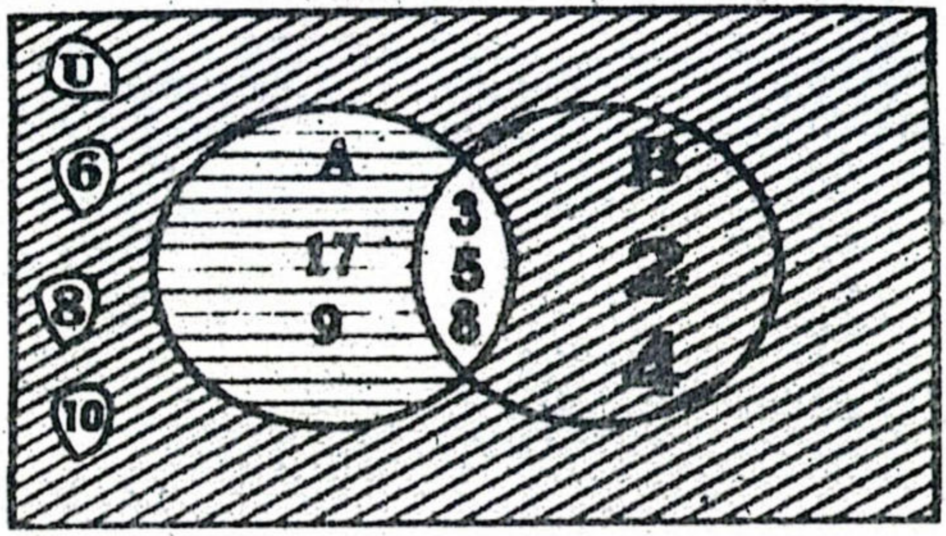


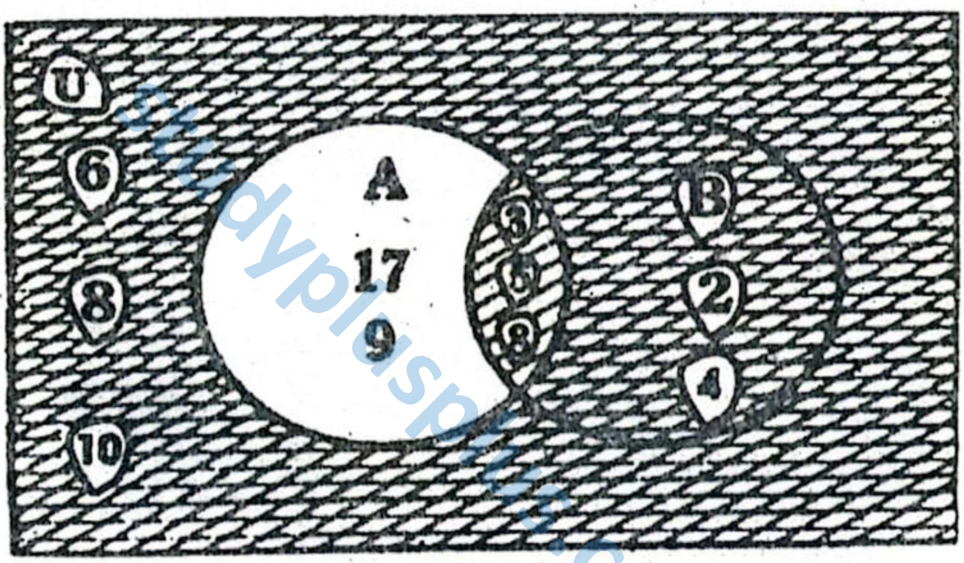
$(A-B)' = A' \cup B'$

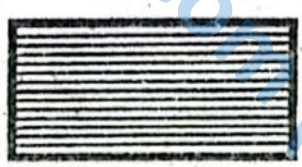
$U = \{1,2,3,\ldots\ldots,10\}$

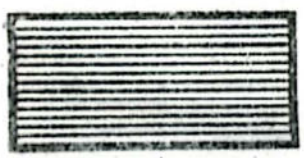
$A = \{1,3,5,7,9\}$

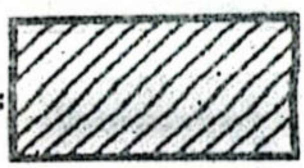
$B = \{2,3,4,5,8\}$

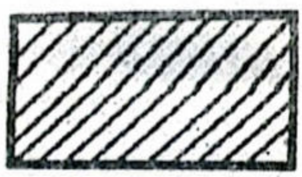




A-B= 

A′= 

(A − B)′= 

A ∪ B′= 

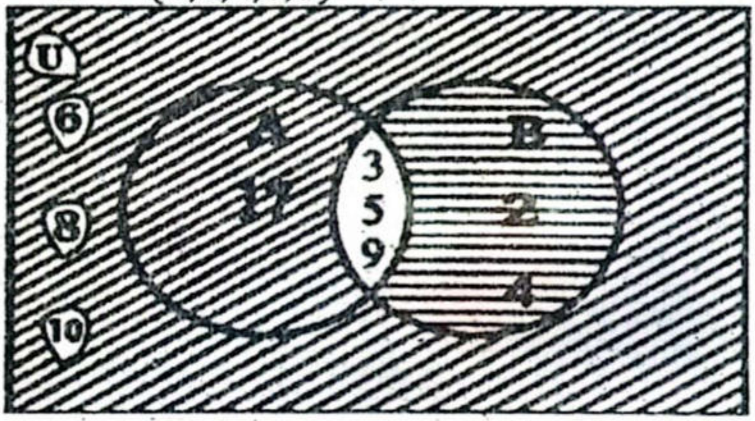
Area in both the cases is the same.
Thus, $(A - B) = A' \cup B$

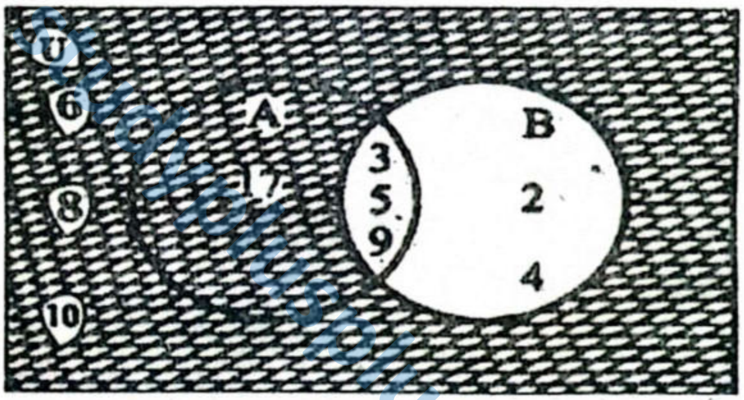
(vi) $(B-A)' = B' \cap A$

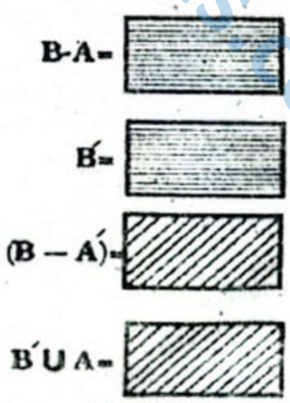
$U = \{1, 2, 3, \ldots\ldots, 10\}$

$A = \{1,3,5,7,9\}$

$B = \{2,3,4,5,8\}$







Fig(i), (ii) show that $(B-A)' = B' \cup A$.