1. If $L = \{a, b, c\}$, $M = \{3, 4\}$, then find two binary relations of $L \times M$ and $M \times L$.

Solution: $\quad L = \{a, b, c\}, \quad M = \{3, 4\}$

$$L \times M = \{a, b, c\} \times \{3, 4\}$$
$$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$$

Then $R_1 = \{(a, 3), (b, 4), (c, 3)\}$

$\quad\quad R_2 = \{(a, 4), (b, 3), (c, 4)\}$

Now $M \times L = \{3, 4\} \times \{a, b, c\}$

$$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$$

Here, $\quad R_1 = \{(3, a), (4, a), (4, c)\}$

$\quad\quad \underline{R_2 = \{(3, b), (4, c)\}}$

2. If $Y = \{-2, 1, 2\}$, then make two binary relations for $Y \times Y$. Also find their domain and range.

Solution: $\quad Y = \{-2, 1, 2\}$

$$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$$

$$= \{(-2,-2),(-2,1),(-2,2),(1,-2),(1,1),(1,2)$$
$$(2,-2),(2,1),(2,2)\}$$
$$R_1 = \{(-2,-2),(-2,1),(1,2)(2,2)\}$$
Dom $R_1 = \{-2,1,2\}$

Range $R_1 = \{-2,1,2\}$

and $R_2 = \{(-2,1),(1,1),(-2,2)\}$

Dom $R_2 = \{1,2\}$

Range $R_2 = \{1,2\}$

3. **If $L = \{a,b,c\}$, and $M = \{d,e,f,g\}$, then find two binary relations in each:**

Solution:

(i) $L \times L$

$$L = \{a,b,c\}$$
Now $L \times L = \{a,b,c\} \times \{a,b,c\}$
$$= \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),$$
$$(c,a),(c,b),(c,c)\}$$
$$R_1 = \{(a,a)\}$$
$$R_2 = \{(a,b),(b,b),(c,b)\}$$

(ii) $L \times M$

$$L = \{a,b,c\}$$
$$M = \{d,e,f,g\}$$
Now $L \times M = \{a,b,c\} \times \{d,e,f,g\}$
$$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),$$
$$(b,f),(b,g),(c,d),(c,e),(c,f),(c,g)\}$$
Here, $R_1 = \{(a,d),(b,f)\}$
$$R_2 = \{(b,e),(b,g),(c,d),(c,e)\}$$

(iii) $M \times M$

$$M = \{d,e,f,g\}$$
Now $M \times M = \{d,e,f,g\} \times \{d,e,f,g\}$
$$= \{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),$$
$$(e,f),(e,g),(f,d),(f,e),(f,f),(f,g),$$
$$(g,d),(g,e),(g,f),(g,g)\}$$
Here, $R_1 = \{(d,e),(d,f),(f,f)\}$
$$R_2 = \{(d,f),(e,d),(e,e),(g,g)\}$$

4. **If set M has 5 elements, then find the number of binary relations in M.**

Solution: Numbers of elements in M = 5

Numbers of elements in M = 5

Numbers of binary relations in $M \times M$
$$= 2^{5 \times 5} = 2^{25}$$

5. If $L = \{x | x \in N \wedge x \leq 5\}$, $M = \{y | y \in P \wedge y < 10\}$, then make the following relations from L to M

Solution:

(i) $R_1 = \{(x, y) | y < x\}$

$L = \{x | x \in N \wedge x \leq 5\}$

Thus, $L = \{1, 2, 3, 4, 5\}$

and $M = \{y | y \in P \wedge y < 10\}$ Prime = 2

Thus, $M = \{2, 3, 5, 7\}$

Now, $L \times M = \{1, 2, 3, 4, 5\}, \{2, 3, 5, 7\}$

$= \{1, 2, (1,3), (1,5), (1,7), (2,2), (2,3), (2,5),$
$(2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3),$
$(4,5), (4,7), (5,2), (5,3), (5,5), (5,7)\}$

$R_1 = \{(x, y) | y < x\}$

$= \{(3,2), (4,2), (4,3), (5,2), (5,3)\}$

Dom $R_1 = \{3, 4, 5\}$

Range $R_1 = \{2, 3\}$

(ii) $R_2 = \{(x, y) | y = x\}$

$R_2 = \{(2,2), (3,3), (5,5)\}$

Dom $R_2 = \{2, 3, 5\}$ $y = x$.

Range $R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y) | y + x = 6\}$

$R_3 = \{(1,5), (3,3), (4,2)\}$

Dom $R_3 = \{1, 3, 4\}$

Range $R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y | y - x = 2)\}$

$R_4 = \{(1,3), (3,5), (5,7)\}$

Dom $R_4 = \{1, 3, 5\}$

Range $R_4 = \{3, 5, 7\}$

Also write the domain and range of each relation.

6. **Indicate relations, into functions, one-one function, on to function, and bijective function from the following. Also find their domain and the range.**

Solution:

(i) $R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Bijective

Dom $R_1 = \{1, 2, 3, 4\}$

Range $R_1 = \{1, 2, 34\}$

(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

Rel

Dom R_2 = {1,2,3}

Range R_2 = {1,2,4,5}

(iii) $R_3 = \{(b,a),(c,a),(d,a)\}$

Function

Dom R_3 = {a, b, c}

Range R_3 = {a}

(iv) $R_4 = \{(a,b),(b,a),(c,d),(d,e)\}$

Onto function

Dom R_4 = {1,2,3,4,5}

Range R_4 = {1,3,4}

(v) $R_5 = \{(a,b),(b,a),(c,d),(d,e)\}$

One – one function

Dom R_5 = {a, b, c, d}

Range R_5 = {a, b, d, e}

(vi) $R_6 = \{(1,2),(2,3),(1,3),(3,4)\}$

Relation

Dom R_6 = {1,2,3}

Range R_6 = {2,3,4}

(vii) $R_7 =$
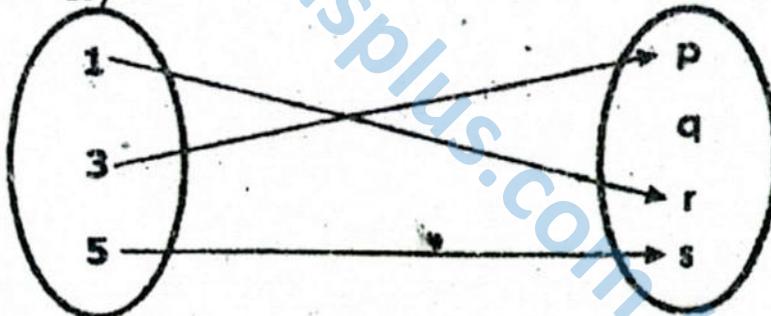
Dom R_6 = {1,2,3}

Range R_7 = {r, p, s}

(viii) $R_8 =$
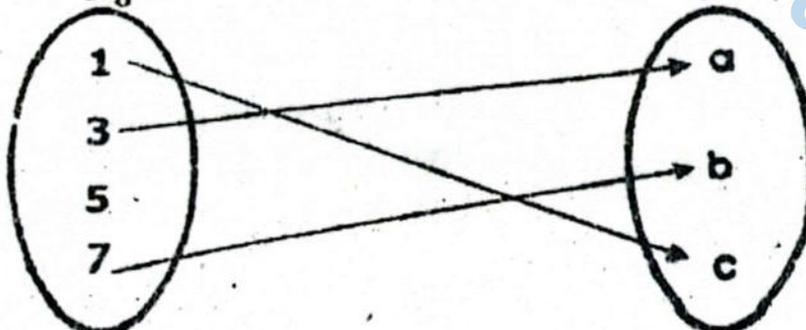
Relation

Dom R_8 = {1,3,7}

Range R_8 = {a, b, c}