



Exercise 6.2



1. What do you understand by measure of central tendency.

Ans. The specific value of the variable around which the majority of the observation tend to concentrate is called tendency.

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2. Define (i) Arithmetic mean (ii) Geometric mean, (iii) Harmonic mean (iv) mode (v) median.

Ans. Mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number of observation.

(i) **Geometric Mean:**

Geometric mean of a variable x is not positive root of the product of the $x_1, x_2, x_3, \dots, x_n$ observation.

$$G.M = (x_1 \times x_2 \times x_3 \dots x_n)^{1/n}$$

(iii) **Harmonic Mean:**

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \dots, x_n$ observations.

$$H.M = \frac{n}{\sum \frac{1}{x}}$$

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(iv) **Mode:**

The most repeated value in an observation is called its mode.

(v) **Median:**

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.

3. Find arithmetic mean by direct method for the following set of data:

(i) 12, 14, 17, 20, 24, 29, 35, 45.

Solution:

$$\begin{aligned} A.M = \bar{X} &= \frac{\sum x}{n} = \frac{12+14+17+20+24+29+35+45}{8} \\ &= \frac{196}{8} \\ &= 24.5 \end{aligned}$$

(ii) 200, 225, 350, 375, 320, 290.

$$\begin{aligned} A.M = \bar{X} &= \frac{\sum x}{n} = \frac{200+225+350+375+320+290}{7} \\ &= \frac{2030}{7} \\ &= 290 \end{aligned}$$

4. For each of the data in Q.no3., compute arithmetic mean using indirect method.

(i) 12, 14, 17, 20, 24, 29, 35, 45

Solution:

Take any constant say 24 and take deviations from it (24).

X	$D = X - A$
12	$12 - 24 = -12$
14	$14 - 24 = -10$
17	$17 - 24 = -7$
20	$20 - 24 = -4$
24	$24 - 24 = 0$
29	$29 - 24 = 5$
35	$35 - 24 = 11$
45	$45 - 24 = 21$
$n = 8$	$\Sigma D = 4$

$$\begin{aligned}\bar{X} &= A + \frac{\Sigma D}{n} \\ &= 24 + \frac{4}{8} = 24 + \frac{1}{2} = 24\frac{1}{2} = 24.5\end{aligned}$$

(ii) 200, 225, 350, 270, 320, 290

Solution:

Let We Take $A = 270$

X	$D = X - A$
200	$200 - 270 = -70$
225	$225 - 270 = -45$
350	$350 - 270 = 80$
375	$375 - 270 = 105$
270	$270 - 270 = 0$
320	$320 - 270 = 50$
290	$290 - 270 = 20$
$n = 7$	$\Sigma D = 140$

$$\begin{aligned}\Sigma D &= 140 \\ n &= 7\end{aligned}$$

$$\bar{X} = A + \frac{\Sigma D}{n} = 270 + \frac{140}{7}$$

$$\bar{X} = 270 + 20 = 290$$

5. The marks obtained by students of class of XI in mathematics are given below. Compute arithmetic mean by direct and indirect methods.

Classes/Groups	Frequency
0 - 9	2
10 - 19	10
20 - 29	5
30 - 39	9
40 - 49	6
50 - 59	7
60 - 69	1

Direct Method:

Class/groups	mid points (x)	f	f(x)
0 - 9	4.5	2	$4.5 \times 2 = 9.0$
10 - 19	14.5	10	$14.5 \times 10 = 145.0$
20 - 29	24.5	5	$24.5 \times 5 = 122.5$
30 - 39	34.5	9	$34.5 \times 9 = 310.5$
40 - 49	44.5	6	$44.5 \times 6 = 267.0$
50 - 59	54.5	7	$54.5 \times 7 = 381.5$
60 - 69	64.5	1	$64.5 \times 1 = 64.5$
$n = \sum f = 40$			1300

$$\bar{X} = A + \frac{\sum D}{n} = \frac{1300}{40}$$

$$= 32.5$$

Indirect, short cut method

Classes limits	f	mid-point s(x)	$D = X - A$	$U = \frac{D}{10}$	fD	$\frac{f(U)}{3} = \frac{f(D)}{3}$
0 - 9	2	4.5	$4.5 - 34.5 = -30$	-3	-60	-6
10 - 19	10	14.5	$14.5 - 34.5 = -20$	-2	-200	-20
20 - 29	5	24.5	$24.5 - 34.5 = -10$	-1	-50	-5
30 - 39	9	34.5	$34.5 - 34.5 = 0$	0	0	0
40 - 49	6	44.5	$44.5 - 34.5 = 10$	1	60	6
50 - 59	7	54.5	$54.5 - 34.5 = 20$	2	140	14
60 - 69	1	64.5	$64.5 - 34.5 = 30$	3	30	3
Total	40				-80	-8

$$\bar{X} = A + \frac{\sum D}{\sum f} \text{ or using: } \bar{X} = A + \frac{\sum f(U)}{\sum f} \times h$$

$$= 34.5 + \frac{(-80)}{40}$$

$$= 34.5 + \frac{-8}{40} \times h$$

$$= 34.5 - 2$$

$$= 32.5$$

$$= 34.5 + \frac{-8}{40} \times 10$$

$$= 34.5 + \frac{-8}{40} \times 10$$

$$= 34.5 - 2$$

$$= 32.5$$

6. The following data relates to the age of children in a school. Compute the mean age by direct and short-cut method taking any provisional mean. (Hint. Take $A=8$)

Classes/Groups	Frequency
4 - 6	10
7 - 9	20
10 - 12	13
13 - 15	7
Total	50

Also compute Geometric mean and Harmonic mean.

Solution: Direct Method.

Class Limits	mid points (x)	f	fx
4 - 6	5	10	$5 \times 10 = 50$
7 - 9	8	20	$8 \times 20 = 160$
10 - 12	11	13	$11 \times 13 = 143$
13 - 15	14	7	$14 \times 7 = 98$
Total	$\Sigma f = 50$		$\Sigma f(x) = 451$

$$A.M = \frac{\Sigma f(x)}{\Sigma f} = \frac{451}{50} = 9.02$$

Indirect/short method

Class limits	f	Mid-points (x)	$D = X - A$	$U = \frac{D}{3}$	fD	$\frac{f(U)}{f(D)}$ = $\frac{3}{3}$
4 - 6	10	5	$5 - 11 = -6$	$= -2$	-60	$= -20$
7 - 9	20	8	$8 - 11 = -3$	$= -1$	-60	$= -20$
10 - 12	13	11	$11 - 11 = 0$	$= 0$	-0	$= 0$
13 - 15	7	14	$14 - 11 = 3$	$= 1$	21	$= 7$
	50				-99	-33

$$\bar{X} = A + \frac{\Sigma D}{\Sigma f} \text{ or using: } \bar{X} = A + \frac{\Sigma f(U)}{\Sigma f} \times h$$

$$= 11 - \frac{99}{50}$$

$$= 11 - \frac{33}{50} \times 3$$

$$\begin{aligned}
 &= 11 - 1.98 &= 11 - \frac{99}{50} \\
 &= 11 - 1.98 &= 11 - 1.98 \\
 &= 9.02 &= 9.02
 \end{aligned}$$

Geometric Mean:

We proceed as follows:

Class limits	f	Mid-points x	log x	f log x
4 - 6	10	5	0.69897	6.9897
7 - 9	20	8	0.90309	18.0618
10 - 12	13	11	1.04139	13.53807
13 - 15	7	14	1.14613	8.02291
$\Sigma f = 50$				$\Sigma f \log x = 46.61248$

$$\begin{aligned}
 G.M &= \text{Antilog} \left(\frac{\Sigma f \log x}{\Sigma f} \right) \sqrt{\quad} \\
 &= \text{Anti log} \frac{46.61248}{50} \\
 &= \text{Antilog } 9322496 \\
 &= 8.553
 \end{aligned}$$

$$\begin{aligned}
 G.M &= \frac{\Sigma f \log x}{\Sigma f} \\
 A.M &= \frac{\Sigma x}{n} = \frac{\Sigma f x}{\Sigma f} \\
 \text{Direct} &= A + \frac{\Sigma D}{n} \\
 &= A + \frac{\Sigma D}{\Sigma f}
 \end{aligned}$$

Harmonic Mean:

Class Limits	f	mid-points (x)	$\frac{f}{x}$
4 - 6	10	5	$\frac{10}{5} = 2.0$
7 - 9	20	8	$\frac{20}{8} = 2.5$
10 - 12	13	11	$\frac{13}{11} = 1.18$
13 - 15	7	14	$\frac{7}{14} = 0.50$
	$\Sigma f = 50$		$\Sigma \frac{f}{x} = 6.18$

$$H.M = \frac{\Sigma f}{\Sigma \frac{f}{x}} = \frac{50}{6.18} = 8.09$$

7. The following data show the number of children in various families. Find mode and median.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5.

Solution: Writing the observation in ascending order.
 2, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 11, 11, 12.

Mode: The most frequent observation = 9, 4 ✓
 Number of observations = 38 ✓
 Therefore, median is the mean of 19th and 20th observation = $\frac{7+7}{2} = 7$

8. Find Mode number of heads for the following distribution showing the number of heads when 5 coins are tossed. Also determine median.

Meelba
 $(\frac{n}{2})$

X (number of heads)	Frequency (number of times)
1	3
2	8
3	5
4	3
5	1

how much the value has come.

$n = \sum f = 20$

Solution: Mode: The most frequent observation = 2
 For median, we make cumulative frequency column.

x	Frequency	Cumulative frequency
1	3	3
2	8	3 + 8 = 11
3	5	11 + 5 = 16
4	3	16 + 3 = 19
5	1	19 + 1 = 20

Median = the class containing $(\frac{n}{2})^{\text{th}}$ observation.
 = the class containing $(\frac{20}{2})^{\text{th}}$ observation.
 = the class containing (10)th observations.

$$\text{Median} = 2$$

9. The following frequency distribution the weights of boys in kilogram. Compute mean, median, mode.

Class Intervals	Frequency
1 - 3	2
4 - 6	3
7 - 9	5
10 - 12	4
13 - 15	6
16 - 18	2
19 - 21	1

Solution:

Classes limits	f	mid-points (x)	fx	Class boundaries	Cumulative frequency
1 - 3	2	2	4	0.5 - 3.5	2
4 - 6	3	5	15	3.5 - 6.5	2 + 3 = 5
7 - 9	5	8	40	6.5 - 9.5	5 + 5 = 10
10 - 12	4	11	44	9.5 - 12.5	10 + 4 = 14
13 - 15	6	14	84	12.5 - 15.5	14 + 6 = 20
16 - 18	2	17	34	15.5 - 18.5	20 + 2 = 22
19 - 21	1	20	20	18.5 - 21.5	22 + 1 = 23
	23		241		

$$\text{Mean} = \bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{241}{23}$$

$$= 10.78$$

Median:

Median class = class containing $\left(\frac{n}{2}\right)^{\text{th}}$ observation.

$$= \left(\frac{23}{2}\right)^{\text{th}} = (11.5)^{\text{th}} \text{ observation.}$$

Median class is 9.5 - 12.5

Here $l = 9.5$

$$c = 10$$

$$f_1 = 4$$

$$h = 3$$



$$\bar{X} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$= 9.5 + \frac{3}{4} \left(\frac{23}{2} - 10 \right)$$

$$= 9.5 + \frac{3}{4} \left(\frac{3}{2} \right)$$

$$= 9.5 + \frac{9}{8}$$

$$= 9.5 + 1.125$$

$$\approx 10.625$$

$$\therefore \text{Mode} = 12.5 + \frac{6-4}{2(6)-4-2} \times 3$$

$$= 12.5 + \frac{2}{6} \times 3$$

$$= 12.5 + 1$$

$$= 13.5$$

10. A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

(i) If the weight accorded these marks are 4, 3, 3, 2 and 2, respectively, what is an appropriate average mark?

(ii) What is the average mark if equal weights are used?

Solution:

Marks (x)	Weights	xw
73	4	73 × 4 = 292
82	3	82 × 3 = 246
80	3	80 × 3 = 240
67	2	67 × 2 = 134
62	2	62 × 2 = 124
Σx = 364	Σw = 14	Σxw = 1036

$$\bar{X}_w = \frac{\sum xw}{\sum w} = \frac{1036}{14} = 74$$

$$\bar{X} = \frac{\sum x}{n} = \frac{364}{5}$$

= Some values
no. of values.

$$\bar{X} = 72.8$$

11. On a vacation trip a family bought 21.3 liter of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liter at 40.90 rupees per liter.

Solution:

Litter X	Price per liter. W	Amount XW
21.3	36.90	$(21.3)(39.90) = 849.87$
18.7	42.90	$(18.7)(42.90) = 802.23$
23.5	40.90	$(23.5)(40.90) = 961.15$
$\Sigma X = 63.5$		$\Sigma XW = 2613.25$

$$\text{Mean price} = \frac{\Sigma XW}{\Sigma X} = \frac{2613.25}{63.5} = 41.15 \text{ rupees per liter.}$$

12. Calculate simple moving average of 3 years from the following data:

Years	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Values	102	108	130	140	158	180	196	210	210	230

Solution:

Years	Values	3-years moving Total	3-years moving Average
2001	102	—	—
2002	108	340	$340/3 = 113.33$
2003	130	378	$378/3 = 126.00$
2004	140	428	$428/3 = 159.33$
2005	158	478	$478/3 = 159.33$
2006	180	534	$534/3 = 178.00$
2007	196	586	$586/3 = 195.33$
2008	210	626	$626/3 = 208.67$
2009	220	660	$660/3 = 220.00$
2010	230	—	

13. Determine graphically for the following data and check your answer by using formulae.
- Median and Quartiles by using cumulative frequency polygon.
 - Mode using Histogram.

Class Boundaries	Frequency
10 - 20	2
20 - 30	5
30 - 40	9
40 - 50	6
50 - 60	4
60 - 70	1

Solution: (i)

Class Boundaries	f	Cumulative frequency
10 - 20	2	2
20 - 30	5	7
30 - 40	9	16
40 - 50	6	22
50 - 60	4	26
60 - 70	1	27
	$\Sigma f = 27$	

Median class = $\left(\frac{n}{2}\right)^{\text{th}}$ observation = $\left(\frac{27}{2}\right)^{\text{th}}$
 $= (13.5)^{\text{th}}$ observation

Median = $\tilde{x} = l + \frac{h}{f} \left(\frac{n}{2} - c\right)$

Here $l = 30, h = 10, f = 9$
 $n = 27, c = 7$

Median = $\tilde{X} = 30 + \frac{10}{9} \left(\frac{27}{2} - 7\right)$

$= 30 + \frac{10}{9} \left(\frac{13}{2}\right)$

$= 30 + \frac{10}{9} \left(\frac{13}{2}\right)$

$= 30 + \frac{65}{9} = 30 + 7.22 = 37.22$

Now we will find Q_3 as:

We find $3 \left(\frac{n}{4}\right)^{\text{th}}$ observations.

$Q_3 = 3 \left(\frac{n}{4}\right)^{\text{th}}$ observations.

$= 3 \left(\frac{27}{4}\right)^{\text{th}}$ observations.

$= 3(6.75)^{\text{th}}$ observations.

$= (20.25)^{\text{th}}$ observations.

Q_3 . Class is 40 - 50

$$\text{Now } Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - c \right)$$

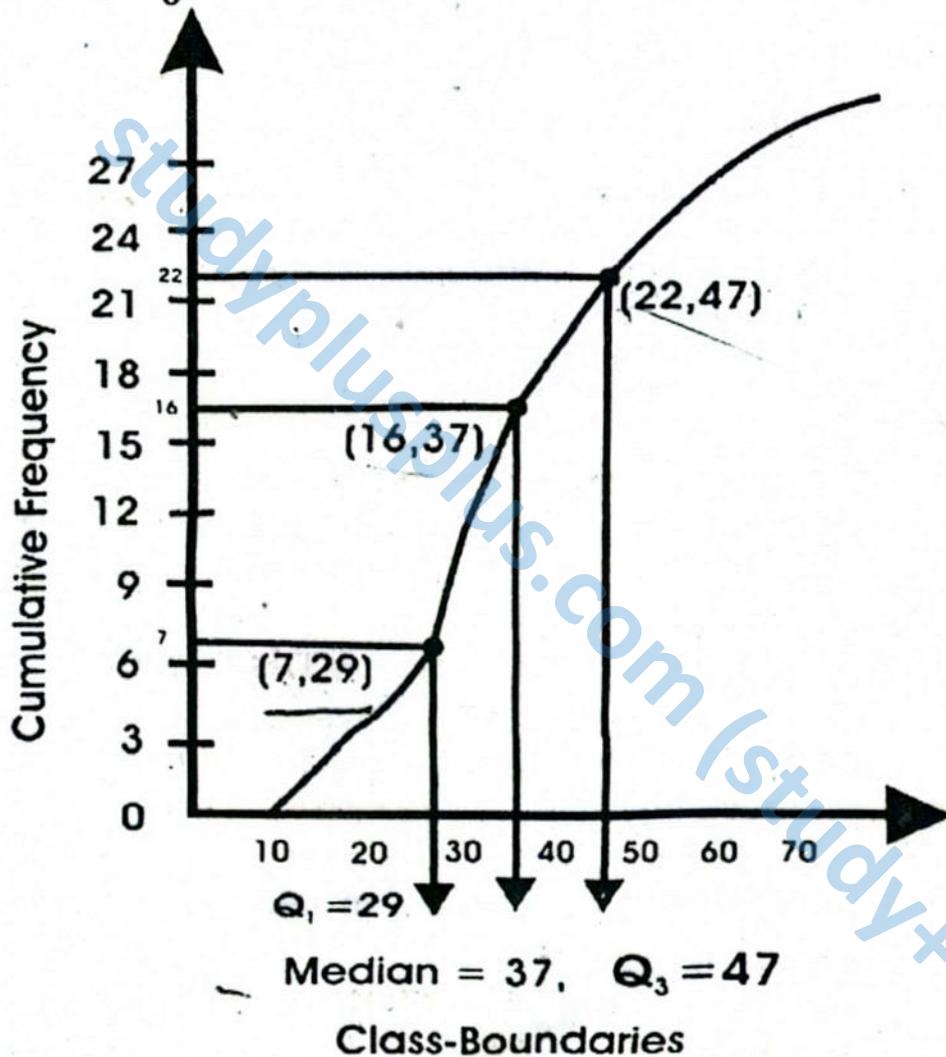
Here, $l = 40$, $h = 10$, $f = 6$

$n = 27$; $c = 16$

$$Q_3 = 40 + \frac{10}{6} \left(\frac{3 \times 27}{4} - 16 \right)$$

$$= 40 + \frac{10}{6} (20.25 - 16)$$

$$= 40 + \frac{10}{6} (4.25) = 40 + 7.08 = 47.08$$



Median = 37

$Q_3 = 47$

Solution: (ii) Mode using Histogram.

