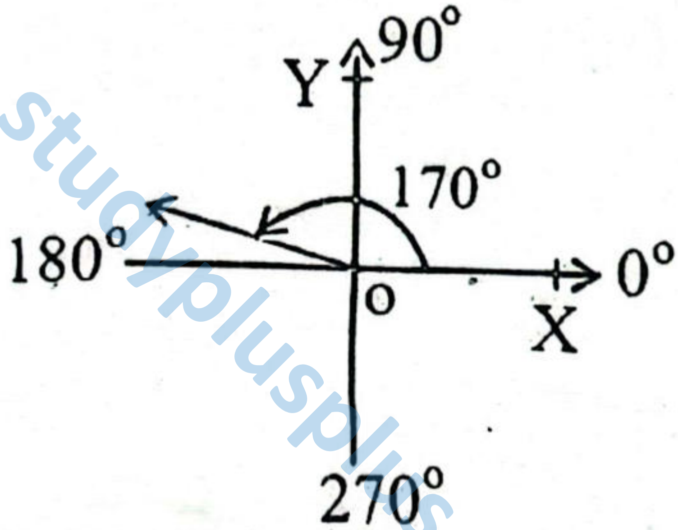
Exercise 7.3

1. **Locate each of the angles in standard position using a protractor or fair free hand guess. Also find a positive and a negative angle conterminal with each given angle.**

(i) **170°**

Sol.



Positve coterminal angle is

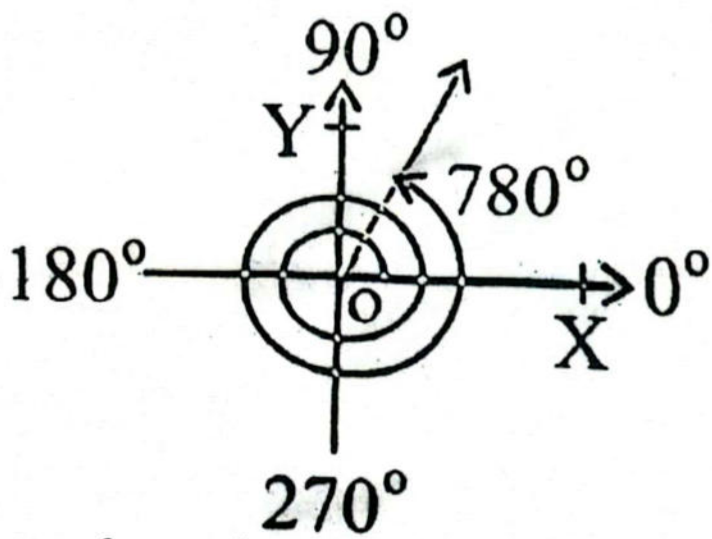
$360^\circ + 170^\circ = 530^\circ$

Negative Coterimal angle is

$-360^\circ + 170^\circ = -190^\circ$

(ii) **780°**

Sol.



$780^\circ = 360^\circ + 360^\circ + 60^\circ$

Positive coterminal angle

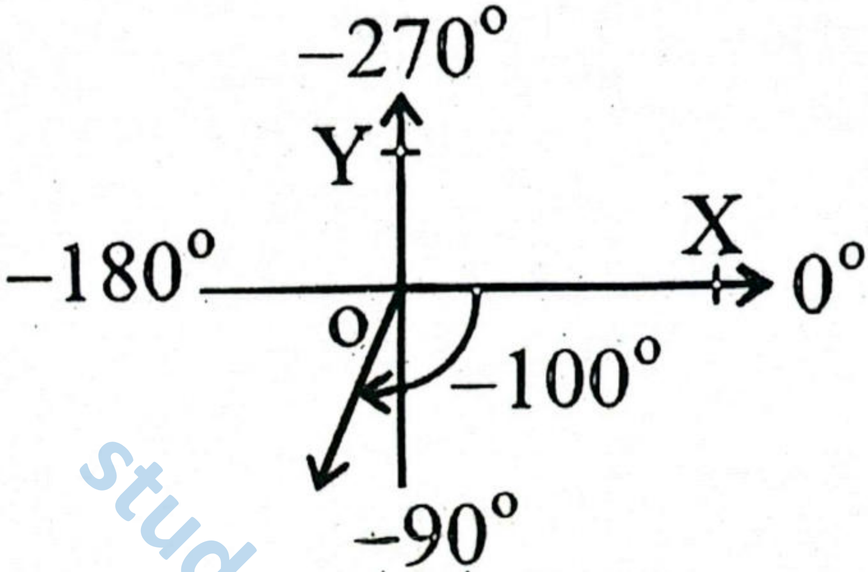
$780^\circ - 360^\circ - 360^\circ = 60^\circ$

Negative Coterminal angle is

$-360^\circ+60^\circ = -300^\circ$

(*iii*) $\quad$ **-100°**

Sol.



Positive Coterminal angle is

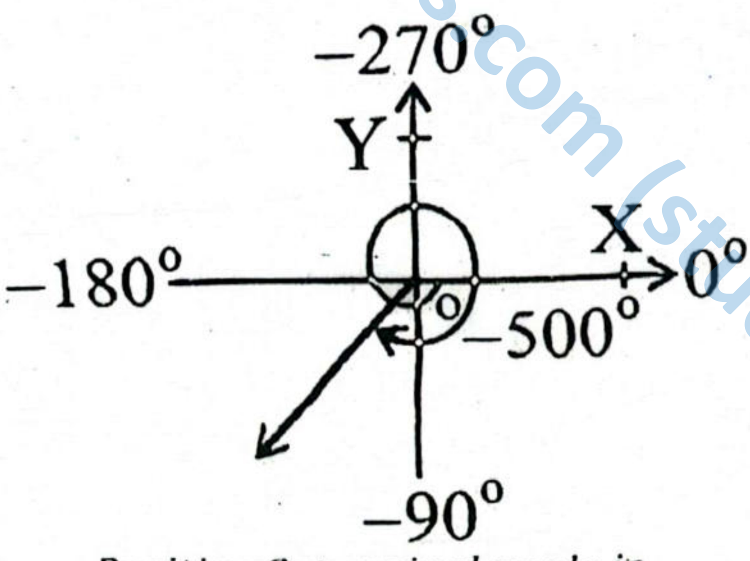
$360^\circ-100^\circ = 260^\circ$

Negative coterminal angle is

$-360^\circ-100^\circ = -460^\circ$

(*iv*) $\quad$ **-500°**

Sol.



Positive Coterminal angle is

$720^\circ-500^\circ = 220^\circ$

Negative Coterminal angle is

$360^\circ-500^\circ = -140^\circ$

2. **Identify the closest quadrantal angles between which the following angles lies.**

(i) 156° **(ii)** 318° **(iii)** 572° **(iv)** -330°

Solution:

(*i*)	156°	(*ii*)	318°
	$90^\circ, 180^\circ$		$270^\circ, 360^\circ$
(*iii*)	572°	(*iv*)	-330°
	$540^\circ, 630^\circ$		$0^\circ, 90^\circ$

3. Write the closest quadrantal angles between which the angle lies. Write your answers in radian measure.

(i) $\frac{\pi}{3}$ **(ii)** $\frac{3\pi}{4}$ **(iii)** $\frac{-\pi}{4}$ **(iv)** $\frac{-3\pi}{4}$

Solution:

(*i*) $\frac{\pi}{3}$

The closest quadrantal angles between which $\frac{\pi}{3}$ lies are 0 *and* $\frac{\pi}{2}$

$0 < \frac{\pi}{3} < \frac{\pi}{2}$

(*ii*) $\frac{3\pi}{4}$

The closest quadrantal angles between which $\frac{3\pi}{4}$ lies are $\frac{\pi}{2}$ *and* π

$\frac{\pi}{2} < \frac{3\pi}{4} < \pi$

(*iii*) $-\frac{\pi}{4}$

The closest quadrantal angles between which $-\frac{\pi}{4}$ lies are $-\frac{\pi}{2}$ *and* 0

$-\frac{\pi}{2} < -\frac{\pi}{4} < 0$

(*iv*) $-\frac{3\pi}{4}$

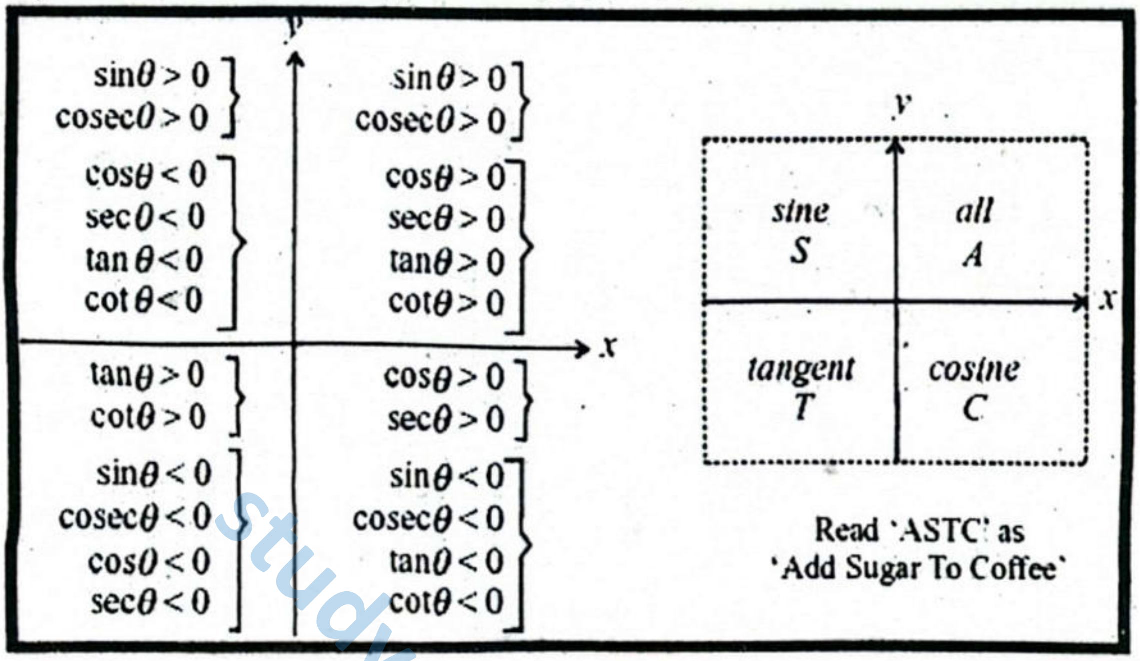
The closest quadrantal angles between which $-\frac{3\pi}{4}$ lies are $-\pi$ *and* $-\frac{\pi}{2}$

$-\pi < -\frac{3\pi}{4} < -\frac{\pi}{2}$

4. In which quadrant θ lie when

(*i*)	$\sin\theta > 0$,	$\tan\theta < 0$
(*ii*)	$\cos\theta < 0$,	$\sin\theta < 0$
(*iii*)	$\sec\theta > 0$,	$\sin\theta < 0$

(iv)	$\cos\theta < 0$,	$\tan\theta < 0$
(v)	$\operatorname{cosec}\theta > 0$,	$\cos\theta > 0$
(vi)	$\sin\theta < 0$,	$\sec\theta < 0$



Solution:

(i) $\sin\theta > 0$

$\tan\theta < 0$

"θ" lies in *Quadrant: II*

as $\sin\theta$ *is + ve*

$\tan\theta$ *is − ve in Quadrant II*

(ii) $\cos\theta < 0$

$\sin\theta < 0$

"θ" lies in *Quadrant: III*

$\cos\theta$ *is − ve*

$\sin\theta$ *is − ve in Quadrant III*

(iii) $\sec\theta > 0$

$\sin\theta < 0$

"θ" lies in *Quadrant: IV*

$\sec\theta$ *is + ve*

$\sin\theta$ *is − ve in Quadrant IV*

(iv) $\cos\theta < 0$

$\tan\theta < 0$

"θ" lies in *Quadrant: II*

$\cos\theta$ *is − ve*

$\tan\theta$ *is − ve in Quadrant II*

(v) $\operatorname{Cosec}\theta > 0$

$\cos\theta > 0$

Quadrant: I

$cosec\theta + ve$

$\text{Cos}\,\theta + ve$ *in Quadrant I*

(*vi*) $\boldsymbol{\sin\theta} < 0$

$\boldsymbol{\sec\theta} < 0$

"θ" lies in *Quadrant: III*

$\text{Sin}\theta$ *is* $-ve$

$\sec\theta$ *is* $-ve$ *in Quadrant III*

5. Fill in the blanks.

(i) $\cos(-150^\circ) = \ldots\ldots\ldots\ldots \cos 150^\circ$

(ii) $\sin(-310^\circ) = \ldots\ldots\ldots\ldots \sin 310^\circ$

(iii) $\tan(-210^\circ) = \ldots\ldots\ldots\ldots \tan 210^\circ$

(iv) $\cot(-45^\circ) = \ldots\ldots\ldots\ldots \cot 45^\circ$

(v) $\sec(-60^\circ) = \ldots\ldots\ldots\ldots \sec 60^\circ$

(vi) $cosec(-137^\circ) = \ldots\ldots\ldots\ldots cosec\ 137^\circ$

Solution:

(i) $\cos(-150^\circ) = +\cos 150^\circ$

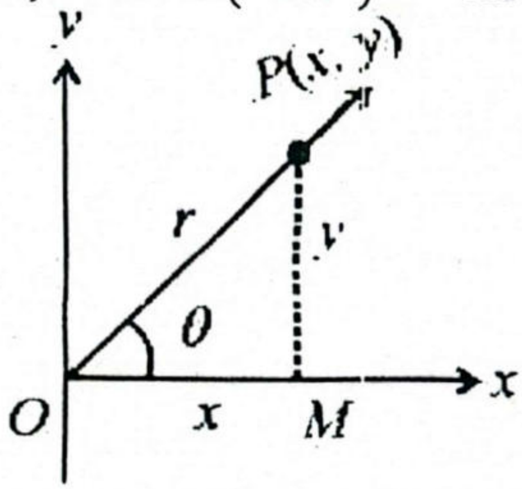
(ii) $\sin(-310^\circ) = -\sin 310^\circ$

(iii) $\tan(-210^\circ) = -\tan 210^\circ$

(iv) $\cot(-45^\circ) = -\cot 45^\circ$

(v) $\sec(-60^\circ) = +\sec 60^\circ$

(vi) $cosec\,(-137^\circ) = -cosec\ 137^\circ$



$$\left(\because \sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}\right)$$

6. The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

(i) $(-2, 3)$ (ii) $(-3, -4)$ (iii) $(\sqrt{2}, 1)$

Solution:

(i) $(-2, 3)$

Here $x = -2$ and $y = 3$

So, the quadrant of θ is II.
Now; by Pythagorean theorem,
We have

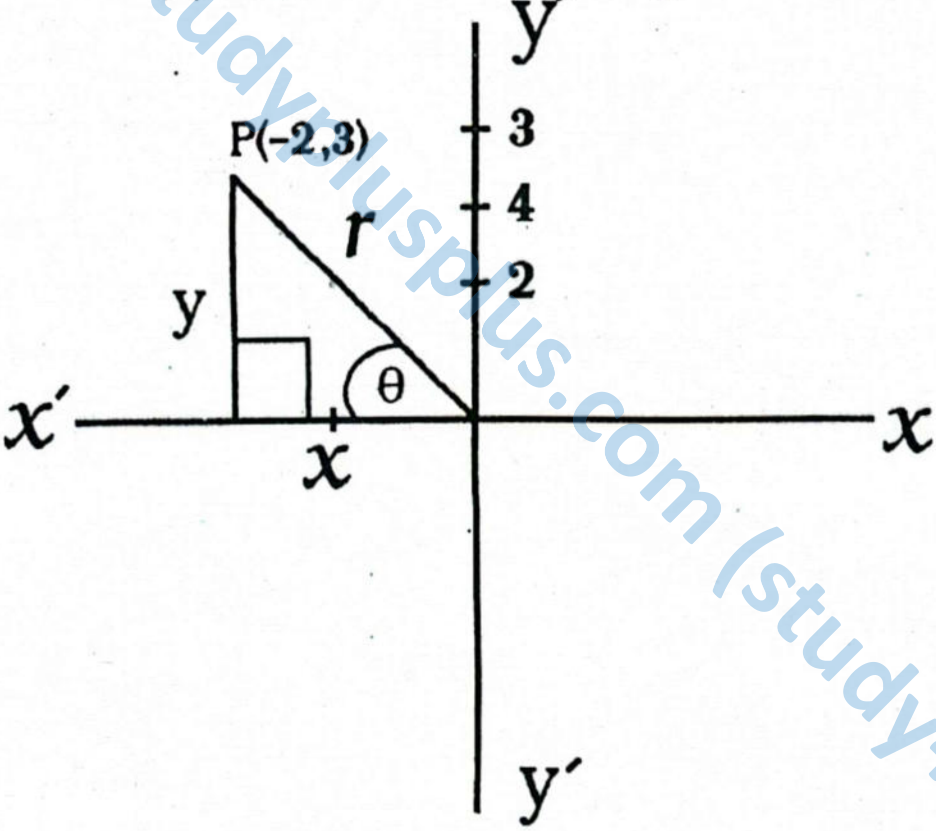
$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(3-2)^2 + (3)^2}$$
$$= \sqrt{4+9}$$
$$= \sqrt{13}$$

The six trigonometric ratios are

$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$ $\quad$ $Cosec\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$

$Cos\,\theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$ $\quad$ $sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-2}$

$Tan\,\theta = \frac{y}{x} = \frac{3}{-2}$ $\quad$ $Cot\theta = \frac{x}{y} = \frac{-2}{3}$

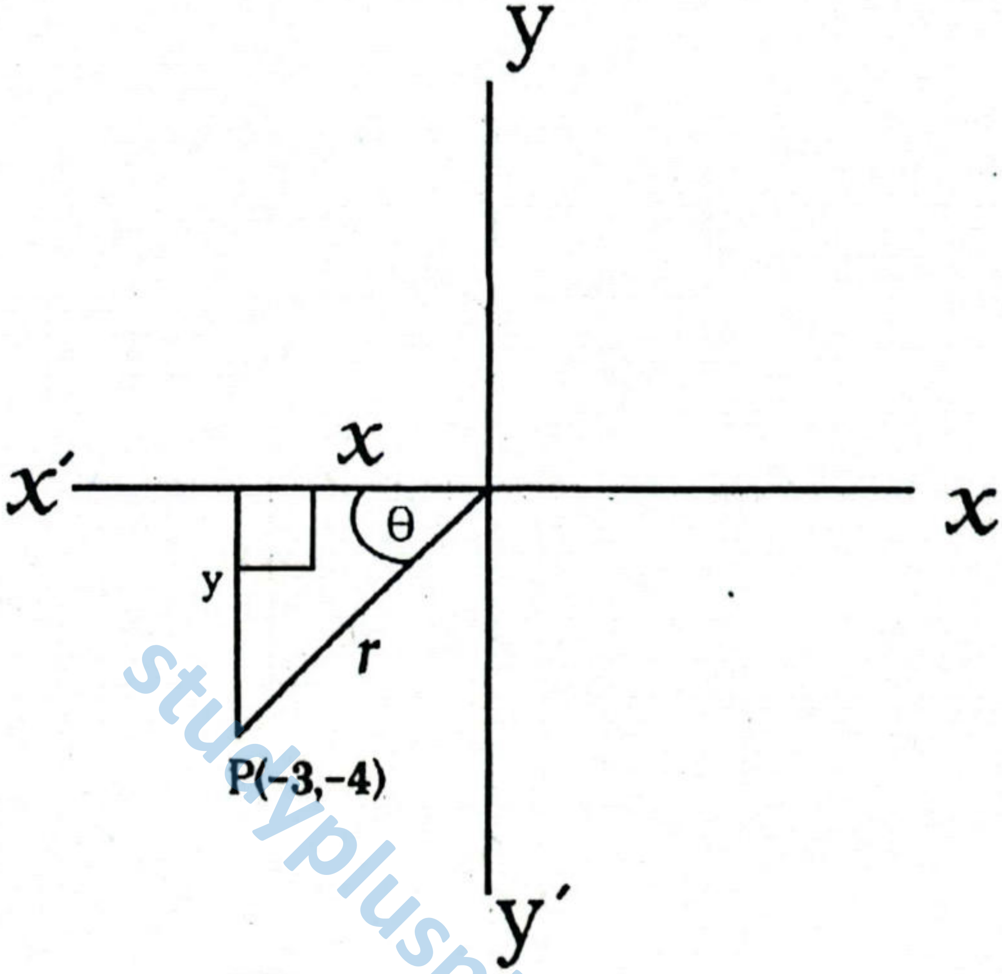


Solution:

(*i*) $\quad (-3, -4)$

Here $x = 3$ and $y = -4$
So, the quadrant of θ is III.
Now; by Pythagorean theorem,
We have

$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$



$= \sqrt{25} \qquad = 5$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = -\frac{4}{5} \qquad \text{Cosec}\,\theta = \frac{r}{y} = \frac{5}{-4}$

$\text{Cos}\,\theta = \frac{x}{r} = -\frac{3}{4} \qquad \text{Sec}\,\theta = \frac{r}{x} = \frac{5}{-3}$

$\text{Tan}\,\theta = \frac{y}{r} = \frac{-4}{-3} = \frac{4}{3} \qquad \text{Cot}\,\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$

Solution:

(*iii*) $P(\sqrt{2}, 1)$

Here $x = \sqrt{2}$ *and* $y = 1$

So, the quadrant of θ *is I*

Now by Pythagorean theorem,

We have

$r = \sqrt{x^2 + y^2}$

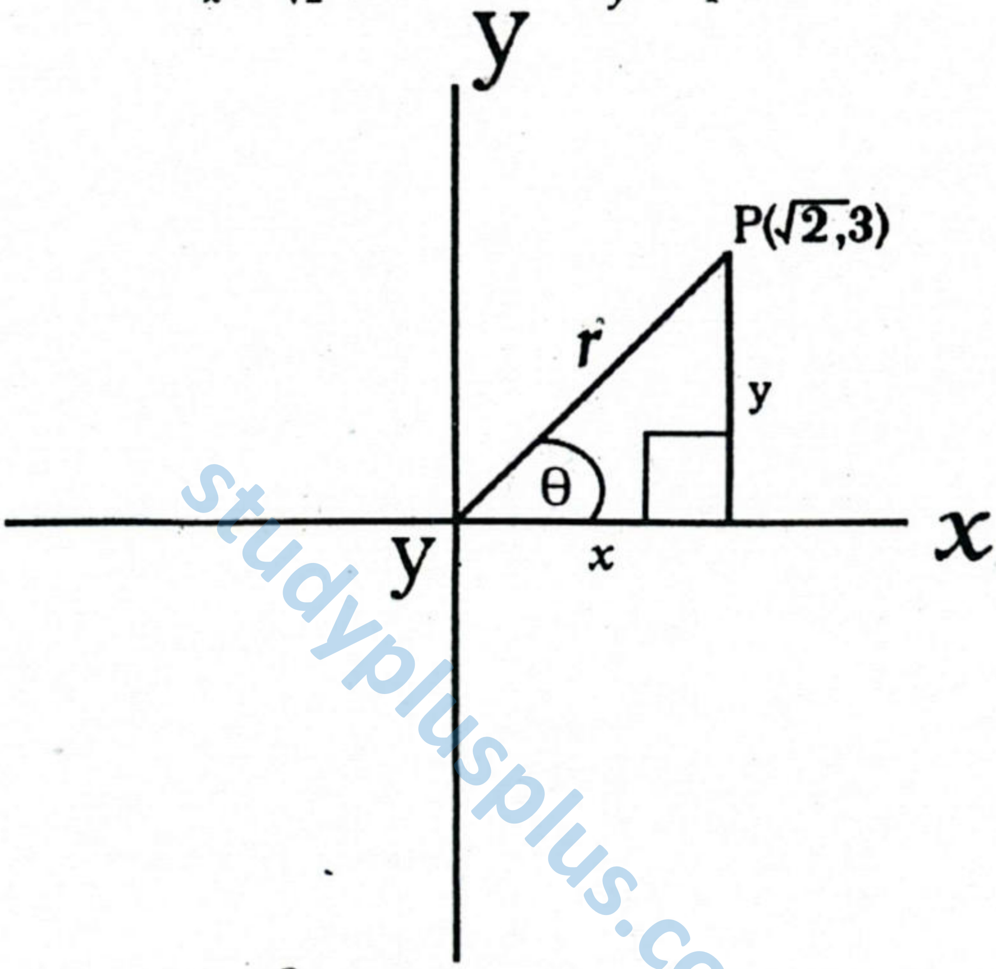
$r = \sqrt{(\sqrt{2})^2 + (1)^2} \quad = \sqrt{2+1} \quad = \sqrt{3}$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = \frac{1}{\sqrt{3}} \qquad \text{Cosec}\,\theta = \frac{r}{y} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$ $\quad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{2}{3}}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$ $\quad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$



7. If $\cos\theta = \frac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

Solution: *Since* $\text{Cos}\theta = \frac{-2}{3}$

and teminal arm of the angle θ *is in quadrant II.*

$$\text{So } x = -2 \text{ and } r = 3$$

By Pythagorean theorem, we have

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(3)^2 - (-2)^2}$$

$$y = \sqrt{9 - 4}$$

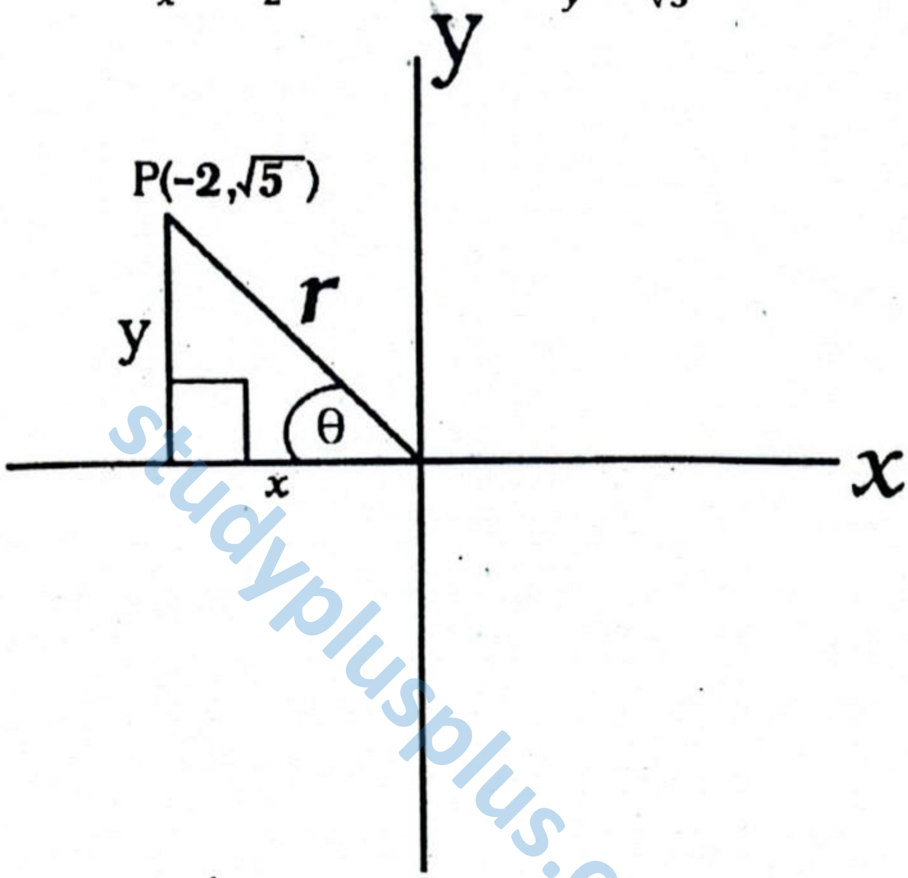
$$y = \sqrt{5}$$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$ $\text{Cosec}\,\theta = \frac{r}{y} = \frac{3}{\sqrt{5}}$ *or* $\frac{3\sqrt{5}}{5}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{-2}{3}$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{-3}{2}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{-2}{\sqrt{5}}$



8. **If tan $\theta = \frac{4}{3}$ and sin $\theta < 0$, find the values of other trigonometric functions at θ.**

Solution: *Since* $\tan\theta = \frac{4}{3}$ *and* $\sin\theta < 0$

(*teminal arm of the angle* $\boldsymbol{\theta}$ ***is in quadrant III***)

$$\text{So } x = 3 \text{ and } y = 4$$

By Pythagorean theorem, we have

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(3)^2 + (4)^2}$$

$$r = \sqrt{9 + 16}$$

$$r = \sqrt{25}$$

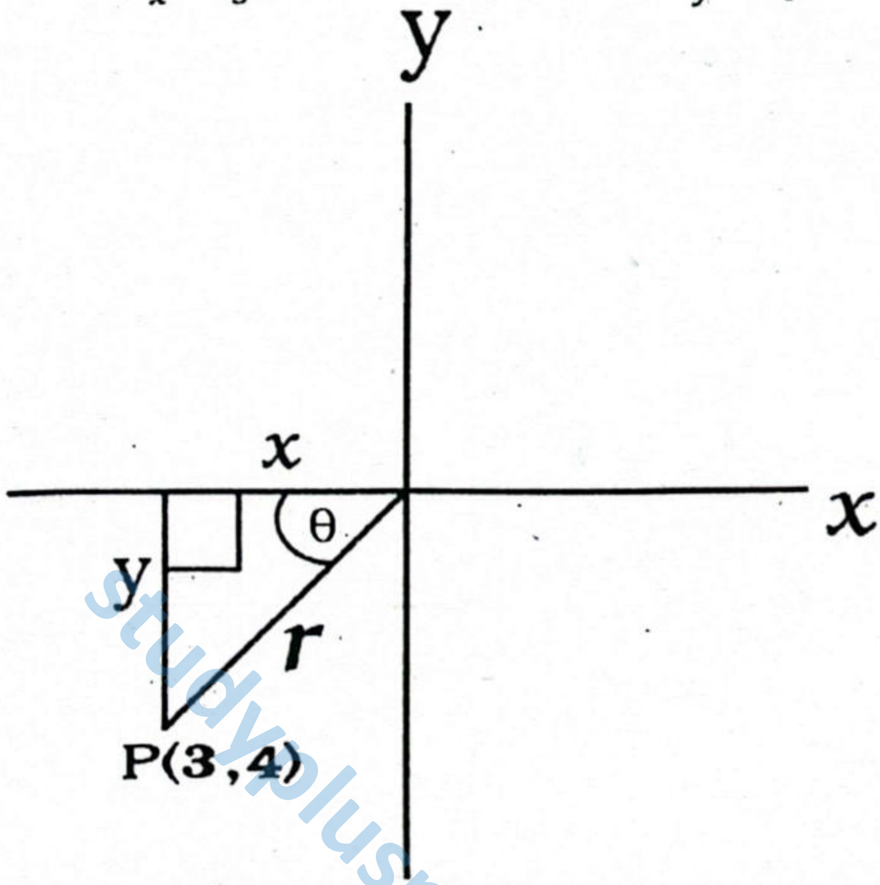
$$r = 5$$

The six trigonometric ratios are

$\text{Sin}\,\theta = -\frac{y}{r} = -\frac{4}{5}$ $\text{Cosec}\,\theta = \frac{r}{y} = -\frac{5}{4}$

$\text{Cos}\,\theta = -\frac{x}{r} = -\frac{3}{5}$ $\text{Sec}\,\theta = -\frac{r}{x} = -\frac{5}{3}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{4}{3}$ $\quad\quad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{3}{4}$



9. **If $\sin\theta = \frac{-1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant III, find the values of $\tan\theta$, $\sec\theta$, and $\operatorname{cosec}\theta$.**

Solution:

$$Since\ Sin\theta = \frac{-1}{\sqrt{2}}$$

*As Sin θ is negative in III and IV quadrant, therefore, in this case θ lies in **IV quadrant** because θ does not lies in III quadrant.*

$$So, \quad y = -1 \text{ and } r = \sqrt{2}$$

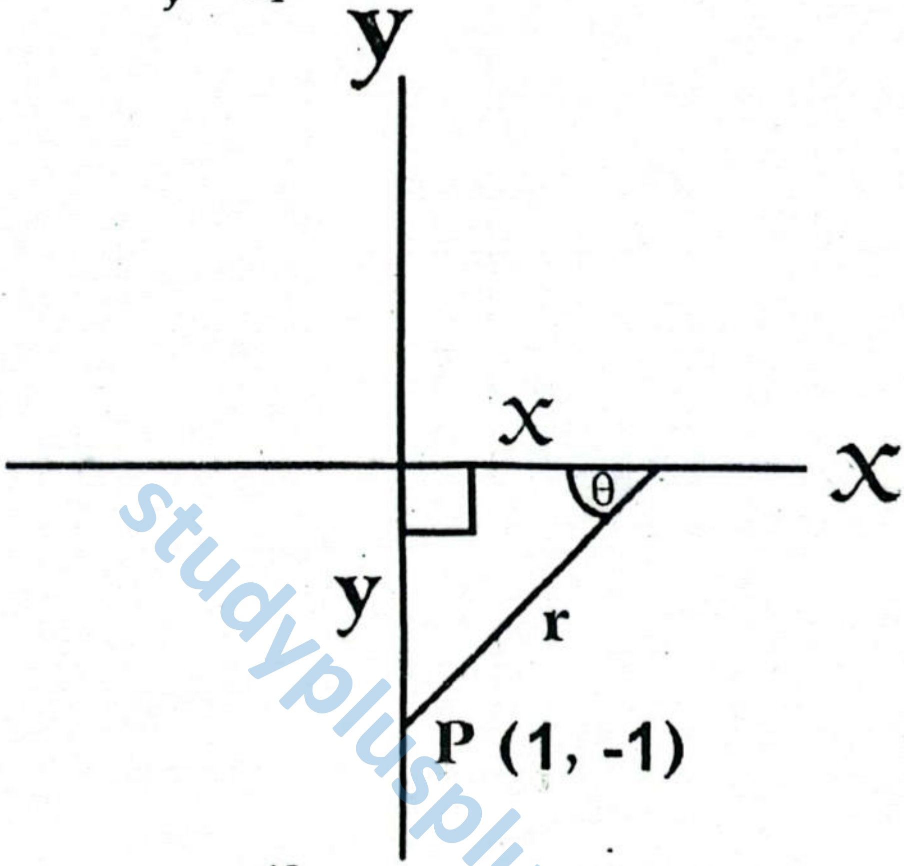
By Pythagorean theorem, we have

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$x = \sqrt{\left(\sqrt{2}\right)^2 - (-1)^2}$$
$$x = \sqrt{2 - 1}$$
$$x = \sqrt{1}$$
$$x = 1$$

The six trigonometric ratios are

$$\text{Tan}\,\theta = \frac{y}{x} = \frac{-1}{1} = -1 \qquad \text{Sec}\,\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\text{Cosec}\theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$



10. **If cosec $\theta = \frac{13}{12}$ and sec > 0, find the remaining trigonometric functions.**

Solution:

$$\text{Since cosec}\,\theta = \frac{13}{12} \text{ and sec } > 0$$

$$(\text{teminal arm of the angle } \theta \text{ is in quadrant } I)$$

$$So\ y = 12 \text{ and } r = 13$$

By Pythagorean theorem, we have

$$r^2 = x^2 + y^2$$

$$x^2 = r^2 - y^2$$

$$x = \sqrt{r^2 - y^2}$$

$$x = \sqrt{(13)^2 - (12)^2}$$

$$x = \sqrt{169 - 144}$$

$$x = \sqrt{25}$$

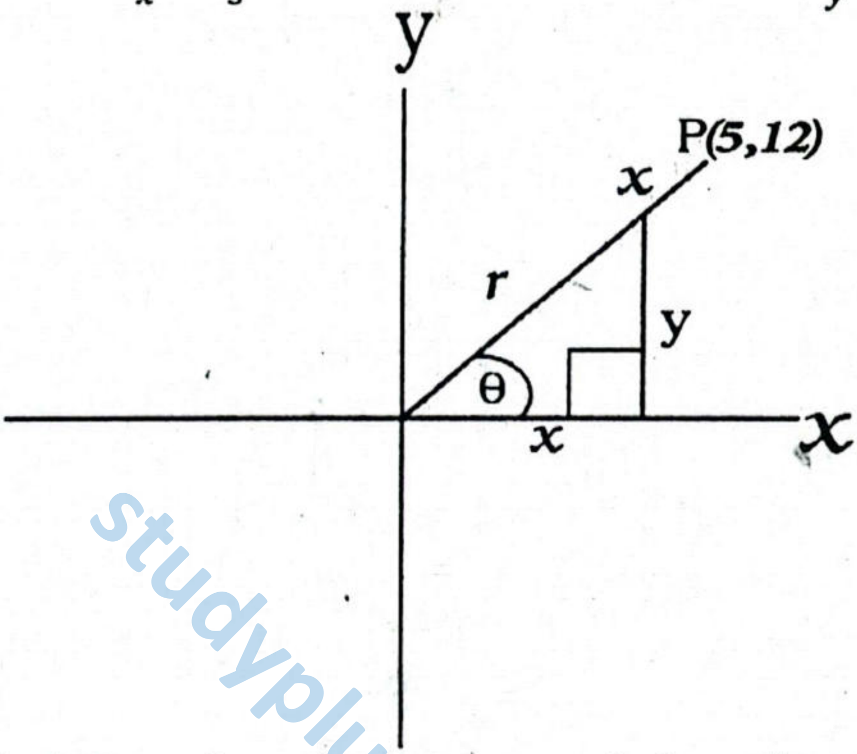
$$x = 5$$

The six trigonometric ratios are

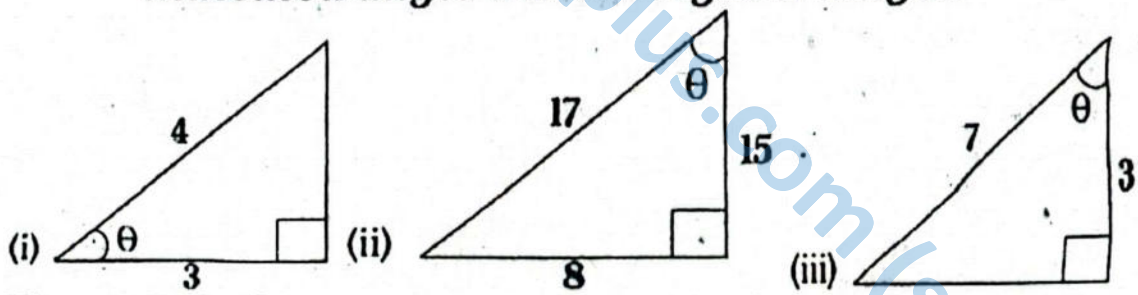
$$\text{Sin}\,\theta = -\frac{y}{r} = \frac{12}{13} \qquad \text{Cosec}\,\theta = \frac{r}{y} = \frac{13}{12}$$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{5}{13}$ $\qquad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{13}{5}$

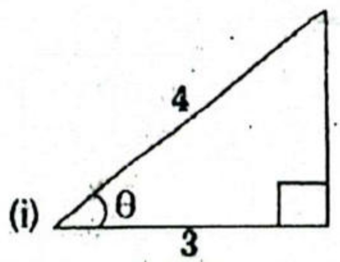
$\text{Tan}\,\theta = \frac{y}{x} = \frac{12}{5}$ $\qquad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{5}{12}$



11. ***Find the value of trigonometric functions at the. indicated angle θ in the right triangle.***



(i) Solution:



As $x = 3$ and $r = 4$

By Pythagorean theorem, we have

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$y = \sqrt{(4)^2 - (3)^2}$$

$$y = \sqrt{16 - 9}$$

$$y = \sqrt{7}$$

The six trigonometric ratios are

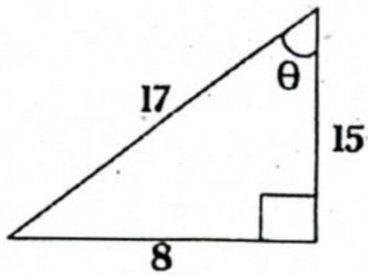
$\text{Sin}\,\theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$ $\quad$ $\text{Cosec}\,\theta = \frac{r}{y} = \frac{4}{\sqrt{7}}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{3}{4}$ $\quad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{4}{3}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{\sqrt{7}}{3}$ $\quad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{3}{\sqrt{7}}$

(ii) Solution:

(ii)



As $r = 17$, $x = 15$ *and* $y = 8$

The six trigonometric ratios are

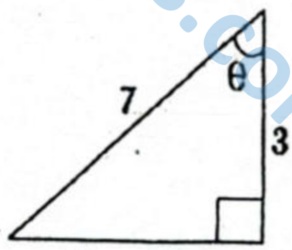
$\text{Sin}\,\theta = \frac{y}{r} = \frac{8}{17}$ $\quad$ $\text{Cosec}\,\theta = \frac{r}{y} = \frac{17}{8}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{15}{17}$ $\quad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{15}{17}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{8}{15}$ $\quad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{8}{15}$

(iii) Solution:

(iii)



As $x = 3$ *and* $r = 7$

By Pythagorean theorem, we have

$$r^2 = x^2 + y^2$$
$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$
$$y = \sqrt{(7)^2 - (3)^2}$$
$$y = \sqrt{49 - 9}$$
$$y = \sqrt{40}$$
$$y = 2\sqrt{10}$$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = \frac{2\sqrt{10}}{7}$ $\text{Cosec}\,\theta = \frac{r}{y} = \frac{7}{2\sqrt{10}}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{3}{7}$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{7}{3}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{2\sqrt{10}}{3}$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{3}{2\sqrt{10}}$

12. Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

(i) **tan 30°**

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$tan30^\circ = tan\left(2\pi + \frac{\pi}{6}\right) = tan\frac{\pi}{6}\,tan30^\circ = \frac{1}{\sqrt{3}}$

(ii) **tan 330°**

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= tan(360^\circ - 30^\circ)$

$= tan\left(2\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -tan30^\circ = -\frac{1}{\sqrt{3}}$

(iii) **sec 330°**

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \sec\,(360^\circ - 30^\circ)$

$= sec\left(2\pi - \frac{\pi}{4}\right) = sec\frac{\pi}{6} = sec30^\circ = \frac{2}{\sqrt{3}}$

(iv) **cot** $\frac{\pi}{4}$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= cot\left(2\pi + \frac{\pi}{4}\right) = cot\frac{\pi}{4} = cot\,45^\circ = \frac{1}{1} = 1$

(v) **cos** $\frac{2\pi}{3}$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= cos\left(\pi - \frac{\pi}{3}\right) = -cos\frac{\pi}{3} = cos\,60^\circ = -\frac{1}{2}$

(vi) **cosec** $\frac{2\pi}{3}$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= cosec\left(\pi - \frac{\pi}{3}\right) = cosec\frac{\pi}{3} = cosec\,60^\circ = \frac{2}{\sqrt{3}}$

(vii) **cos** (-450°)

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \cos(-360^\circ - 90^\circ)$

$= \cos\left[-2\pi - \frac{\pi}{2}\right]$

$= \cos\left(-\frac{\pi}{2}\right) = cos\,90^\circ = 0$

(viii) **tan** (-9π)

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \tan(-9\pi)$

$= \tan(-8\pi - \pi)$

$= \tan(-\pi) = -\tan 180^\circ = 0$

(ix) $\cos\left(\frac{-5\pi}{6}\right)$

Ans *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \cos\left(-\pi + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6} = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

(x) $\sin\frac{7\pi}{6}$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \sin\left(\pi + \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{6} = -\sin 30^\circ = -\frac{1}{2}$

(xi) $\cot\frac{7\pi}{6}$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \cot\left(\pi + \frac{\pi}{6}\right)$

$= \cot\frac{\pi}{6} = -\cot 30^\circ = \frac{1}{\sqrt{3}}$

(xii) $\cos 225^\circ$

Ans. *Since,* $2k\pi + \theta = \theta$ *where* $k \in Z$

$= \cos\left(\frac{5\pi}{4}\right)$

$= \cos\left(\pi + \frac{5\pi}{4}\right)$

$-\cos\frac{\pi}{4} = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$