Exercise 7.4

In Problem 1 – 6, simplify each expression to a single trigonometric function.

Important trigonometric relations

$\text{Sin}^2\theta + \cos^2\theta = 1$

$1 + \tan^2 = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$

(i) $\dfrac{\sin^2\theta}{\cos^2\theta}$

Solution:

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta \times \tan\theta$

$= \tan^2\theta$

2. $\tan x \sin x \sec x$

Solution:

$= \dfrac{\sin x}{\cos x} \times \sin x \times \dfrac{1}{\cos x}$

$= \dfrac{\sin x}{\cos x} \times \dfrac{\sin x}{\cos x}$

$= \tan x \times \tan x$

$= \tan^2 x$

3. $\dfrac{\tan x}{\sec x}$

Solution:

$= \tan x \times \cos x$ $\quad \{\because \cos x = \dfrac{1}{\sec x}\}$

$= \dfrac{\sin x}{\cos x} \times \cos x$

$= \sin x$

4. $1 - \cos^2 x$

Solution:

$= \sin^2 x + \cos^2 x - \cos^2 x$ $\quad \{\because \text{Sin}^2\theta + \cos^2\theta = 1\}$

$= \sin^2 x$

5. $\sec^2 x - 1$

Solution:

$$= 1 + \tan^2 - 1 \qquad \{\because 1 + \tan^2 = \sec^2\theta\}$$
$$= \tan^2 x$$

6. $\sin^2 x.\cot^2 x.$

Solution:

$$= \sin^2 x \frac{\cos^2 x}{\sin^2 x}$$
$$= \cos^2 x$$

In problems 7 – 24, verify the identities.

7. $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$

Solution:

$$L.H.S = (1 - \sin\theta)(1 + \sin\theta)$$
$$= (1)^2 - \sin^2\theta$$
$$= 1 - \sin^2\theta$$
$$= \cos^2\theta = R.H.S$$
$$\{\because 1 - \sin^2\theta = \cos^2\theta\}$$

8. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution:

$$L.H.S. = \frac{\sin\theta + \cos\theta}{\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$$
$$= \tan\theta + 1$$
$$\tan\theta + 1 = R.H.S.$$

9. $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$

Solution:

$$L.H.S. = (\tan\theta + \cot\theta)(\tan\theta)$$
$$= \tan\theta\tan\theta + \cot\theta\tan\theta$$
$$= \tan^2\theta + \cot\theta \times \frac{1}{\cot\theta}$$
$$= \tan^2\theta + 1 = \sec^2\theta = R.H.S$$
$$\{\because 1 + \tan^2 = \sec^2\theta\}$$

10. $(\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$

Solution:

$$L.H.S. = (\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta)$$
$$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta} - \sin\theta)\right)$$

$$= \left(\frac{\cos\theta+1}{\sin\theta}\right)\left(\frac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}\right)$$

$$= \frac{\sin\theta\cos\theta-\sin\theta\cos^2\theta+\sin\theta-\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta-\sin\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta\,(1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\cos^2}{\cos\theta}$$

$$= \sec\theta - \cos\theta = R.H.S.$$

11. $\frac{\sin\theta+\cos\theta}{\tan^2\theta-1} = \frac{\cos^2\theta}{\sin\theta-\cos\theta}$

Solution:

$$L.H.S. = \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta}{\cos^2\theta}-1}$$

$$= \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta-\cos^2\theta}{\cos^2\theta}}$$

$$= \frac{(\sin\theta+\cos\theta)\cos^2\theta}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{(\sin\theta+\cos\theta)\cos^2\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^2\theta}{\sin\theta-\cos\theta} = R.H.S.$$

12. $\frac{\cos^2\theta}{\sin\theta} + \sin\theta = cosec\,\theta$

Solution:

$$L.H.S. = = \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= cosec\,\theta = R.H.S.$$

13. $\sec\theta - \cos\theta = \tan\theta\sin\theta$

Solution:

$$L.H.S. = \sec\theta - \cos\theta$$
$$= \frac{1}{\cos\theta} - \cos\theta$$
$$= \frac{1-\cos^2\theta}{\cos\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{\sin^2\theta}{\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta$$
$$= \tan\theta\sin\theta = R.H.S.$$

14. $\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

Solution:

$$L.H.S. = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{1}{\cos\theta}$$
$$= \sec\theta = R.H.S.$$

15. $\tan\theta + \cot\theta = \sec\theta\,\mathrm{cosec}\,\theta$

Solution:

$$L.H.S. = \tan\theta + \cot\theta$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$
$$= \sec\theta\,\mathrm{cosec}\,\theta = R.H.S.$$

16. $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \sec\theta + \mathrm{cosec}\,\theta$

Solution:

$$L.H.S = (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$$
$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) + (\cos\theta + \sin\theta)$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right)(\cos\theta + \sin\theta)$$

$$= \left(\frac{1}{\cos\theta \sin\theta}\right)(\cos\theta + \sin\theta)$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta}$$

$$= \frac{\cos\theta}{\cos\theta \sin\theta} + \frac{\sin\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$$

$$= \sec\theta + cosec\,\theta = R.H.S.$$

17. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

Solution:

$$L.H.S. = \sin\theta(\tan\theta + \cot\theta)$$

$$= \sin\theta\left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]$$

$$= \sin\theta\left[\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right]$$

$$= \frac{(\sin\theta)(1)}{\cos\theta \sin\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = R.H.S.$$

18. $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2cosec\,\theta$

Solution:

$$L.H.S. = \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2 + (\sin\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 + \cos^2\theta + 2\cos\theta + \sin^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{1 + 2\cos\theta + (\sin^2\theta + \cos^2\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 + 2\cos\theta + 1}{(\sin\theta)(1+\cos\theta)} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2 + 2\cos\theta}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\,cosec\,\theta = R.H.S.$$

19. $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2cosec^2\theta$

Solution:

$$L.H.S. = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$$

$$= \frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 2\,cosec^2\theta = R.H.S.$$

20. $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$

Solution:

$$L.H.S. = \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2 - (1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+2\sin\theta+\sin^2\theta) - (1-2\sin\theta+\sin^2\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1+2\sin\theta+\sin^2\theta - 1 + 2\sin\theta - \sin^2\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta} \qquad \{\because 1-\sin^2\theta = \cos^2\theta\}$$

$$= 4 \times \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}.$$

$= 4\tan\theta\sec\theta = R.H.S.$

21. $\boldsymbol{\sin^3\theta = \sin\theta - \sin\theta\cos^2\theta}$

Solution:

$$R.H.S. = = \sin\theta - \sin\theta\cos^2\theta$$
$$= \sin\theta\,[1 - \cos^2\theta]$$
$$= (\sin\theta)(\sin^2\theta) \quad \{\because 1 - \cos^2\theta = \sin^2\theta\}$$
$$= \sin^3\theta = L.H.S.$$

22. $\boldsymbol{\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)}$

Solution:

$L.H.S.$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$(1) \quad (\cos^2\theta - \sin^2\theta) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$(\cos^2\theta - \sin^2\theta) = R.H.S.$ *Proved*

23. $\boldsymbol{\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta}}$

Solution: $L.H.S. = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$$= \sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1)^2-(\cos\theta)^2}{(1-\cos\theta)^2}}$$

$$= \frac{\sqrt{1-\cos^2\theta}}{1-\cos\theta}$$

$$= \frac{\sqrt{\sin^2\theta}}{1-\cos\theta} \qquad \{\because 1 - \cos^2\theta = \sin^2\theta\}$$

$$= \frac{\sin\theta}{1-\cos\theta} = R.H.S.$$

24. $\sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = \frac{\sec\theta+1}{\tan\theta}$

Solution: $L.H.S. = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

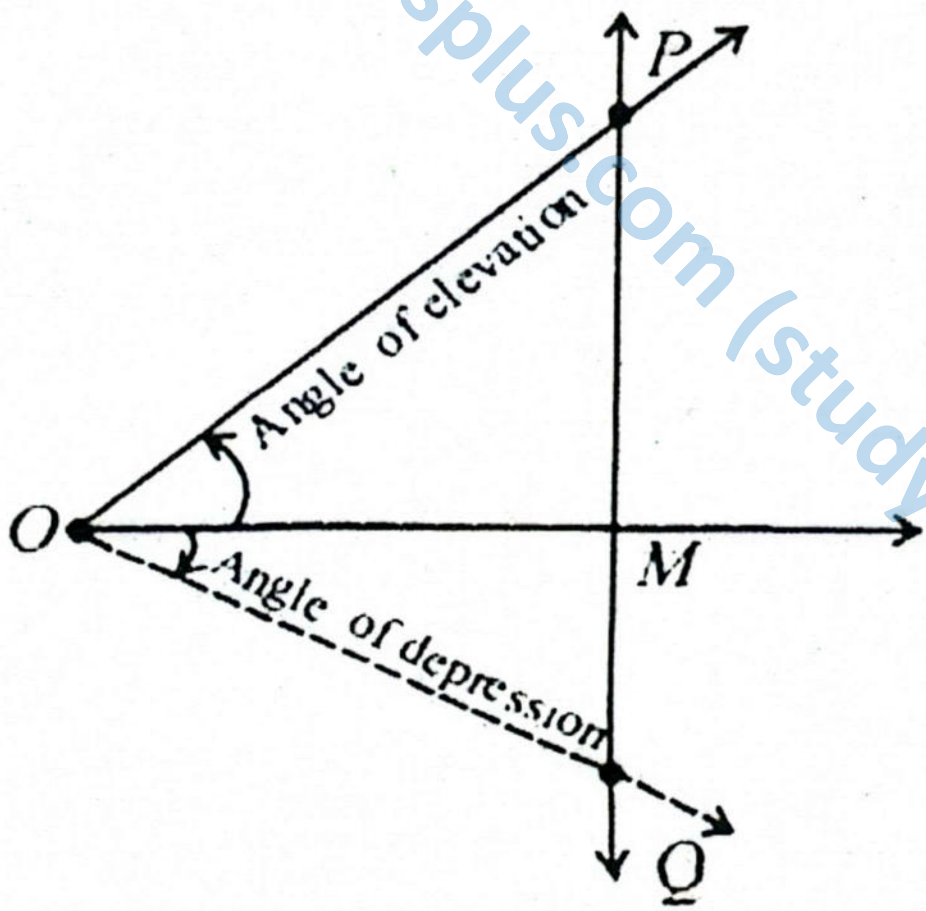
$= \sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$

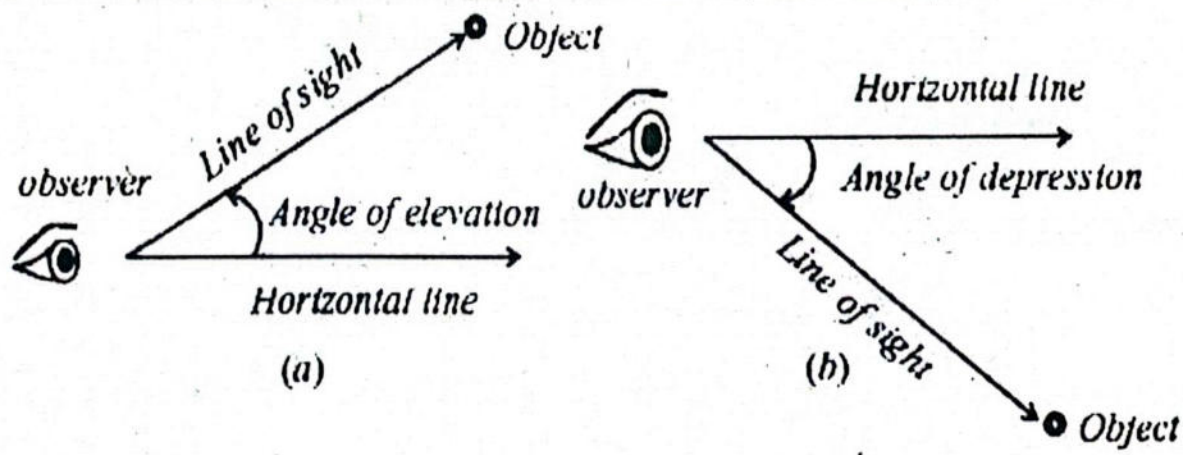
$= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$

$= \frac{\sec\theta+1}{\sqrt{\tan^2\theta}}$ $[\because\ 1+\tan^2\theta = \sec^2\theta]$

$= \frac{\sec\theta+1}{\tan\theta} = R.H.S.$

Important Figures of the Chapter




Object
Line of sight
observer
Angle of elevation
Horizontal line
(a)
Horizontal line
observer
Angle of depression
Line of sight
(b)
Object