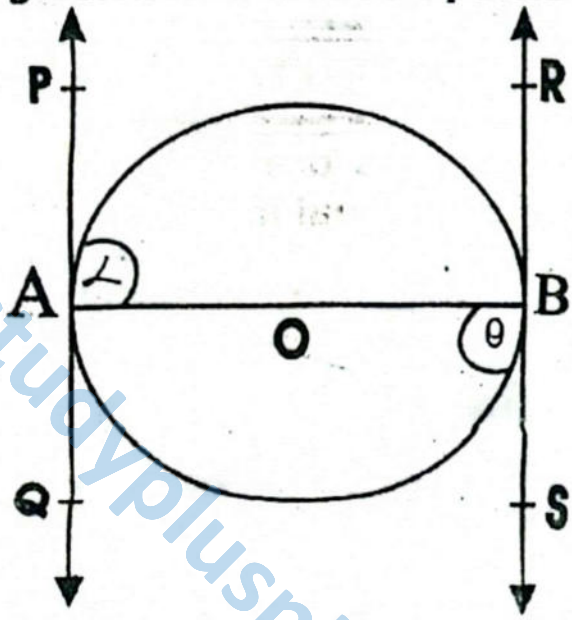
Exercise 10.1

Q.1 Prove that the tangents drawn at the end of a diameter in a given circle must be parallel.



Given:

(i) *A circle with centre O, has $\overleftrightarrow{AB}$ as diameter.*

(ii) *$\overleftrightarrow{PAQ}$ is tangent at point A.*

(iii) *$\overleftrightarrow{RBS}$ is tangent at point B.*

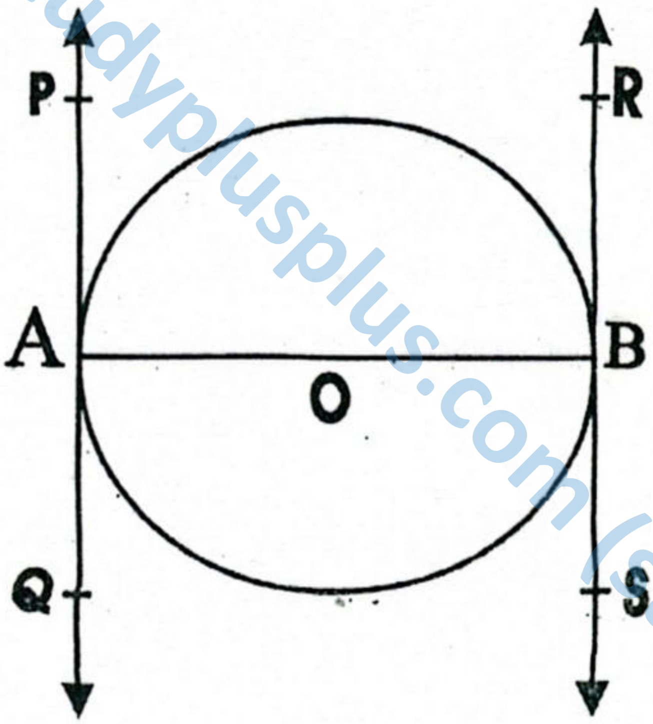
To prove:

$\overleftrightarrow{PAQ} \parallel \overleftrightarrow{RBS}$

Proof:

Statements	Reasons
$\overleftrightarrow{PA} \perp \overleftrightarrow{AB}$	*Tangent*
$\therefore \quad m\angle\alpha = 90^\circ \quad (i)$	
$\overleftrightarrow{BS} \perp \overleftrightarrow{AB}$	*Tangent*
$\therefore m\angle\theta = 90^\circ \quad (ii)$	
Thus, $m\angle\alpha = m\angle\theta$	*from* (i), (ii)
Therfore, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	*Alternate angles are equal in measurement*

Conversely:



Given:

A circle with centre O. $\overline{AB}$ *is diameter of the circle.*
$\overleftrightarrow{PQ}$ *touches the circle at A and* $\overleftrightarrow{RS}$

To prove:

$\overline{PQ}$ *and* $\overline{RS}$ *are tangents.*

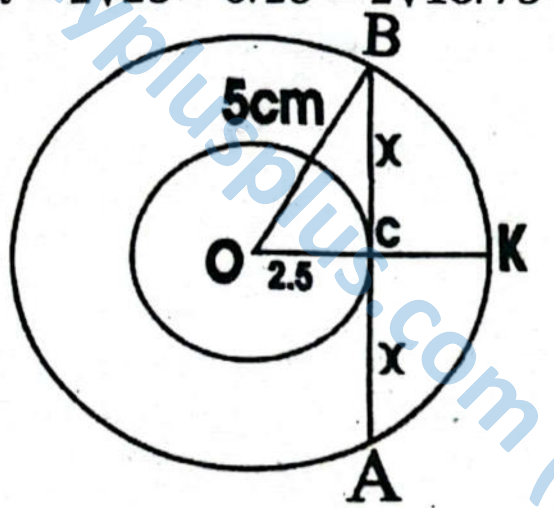
Proof:

Statements	Reasons
$\overline{PQ}$ touches the circle at A, therefore all the points A on the $\overline{PQ}$ lie out side the circle. Hence $\overline{OA} \perp \overline{PQ}$ Thus $\overline{PQ}$ is tangent to the circle. Similarly $\overline{RS}$ is tangent to the circle.	Theorem

Q.2 The diameter of two concentric circles are 10 cm and 5 cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

(Hint) From the figure

$m\overline{AB} = 2x = 2\sqrt{25 - 6.25} = 2\sqrt{18.75} \approx 8.7cm$



Solution:

Let $\overline{AB}$ be any chord of the outer circle that touches inner circle.

Join O to C.

Now OCB is a right angled triangle at angle C.

In BOC triangle.

$$m\overline{OB} = \frac{10}{2} = 5\ cm$$

$$m\overline{OC} = \frac{5}{2} = 2.5$$

Applying Pythogorus Theorm

$$(mBC)^2 = (mOB)^2 - (mOC)^2$$

$$m\overline{BC} = \sqrt{(5)^2 - (2.5)^2}$$

$m\overline{BC} = \sqrt{25 - 6.25}$

$m\overline{BC} = \sqrt{18.75}$

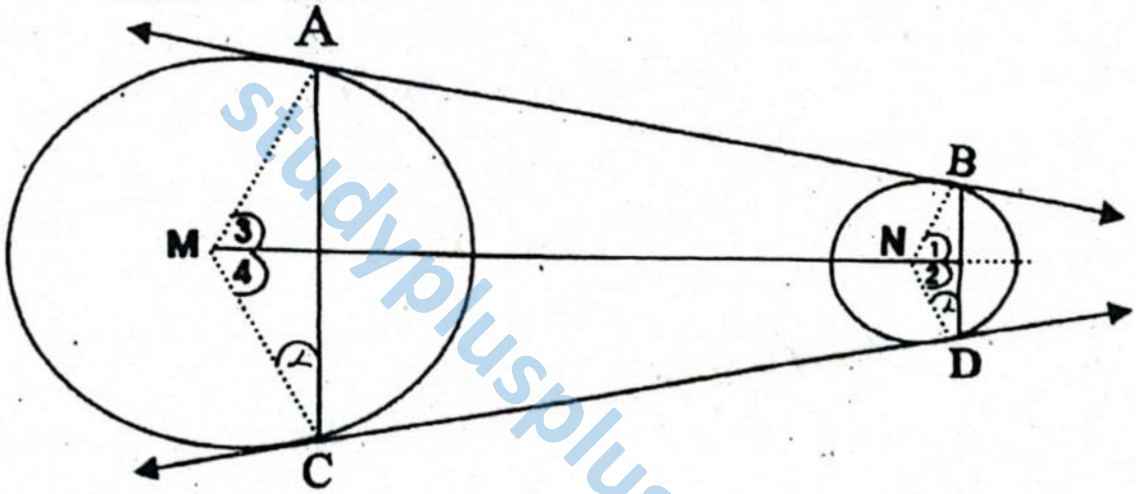
$= 4.33\ cm$

Now $m\overline{AB} = 2(m\overline{BC})$

$= 2(4.330$

$= 8.66\ cm$

3. $\overleftrightarrow{AB}$ **and** $\overleftrightarrow{CD}$ **are the common tangents drawn to the pair of circles . If A and C are the points of tangency of** 1^{st} **circle where B and D are the points of tangency of** 2^{nd} **circle, then prove that** $\overline{AC} \parallel \overline{BD}$.



Given:

Two circles with centre M and N.

$\overline{AB}$ *and* $\overline{CD}$ *their common is joined with D.*

A is joined with C and B is joined with D.

To prove:

Prove that $\overline{AC} \parallel \overline{BD}$

Construction:

(i) *Join M with A, C*

(ii) *Join N with B, D*

Proof:

Statements	Reasons
$\overline{MA} \perp \overline{AB}$	*Theorm*
$\overline{NB} \perp \overline{AB}$	*Theorm*
Therefore, $\overline{MA} \parallel \overline{NB}$	*From(i), (ii)*
Also $m\angle 3 = m\angle 1$ (iii)	*Corresponding angles*
Similarly $m\angle 4 = m\angle 2$ (iv)	
$m\angle 3 + m\angle 4 = m\angle 1 + m\angle 2$	*From (iii) + (iv)*

or $m\angle AMC = m\angle BND$ Now in $\Delta^s AMC, BND$ $\frac{m\overline{MA}}{m\overline{NB}} = \frac{m\overline{MC}}{m\overline{ND}}$ and included angles $\angle AMC$, $\angle BND$ are Congruent, therefore, Δ^s AMC, BND are similar. Thus $m\angle MCA$ i.e., $\propto$ and $m\angle NDB$ are equal. Hence $\overline{AC} \parallel \overline{BD}$.	