# Project Tachyon: Endoscaling Convergence Problem

Tal Derei

October 20th, 2025

## 1 Geometric Series Convergence

In a curve cycle, endoscalars are scalar multiplications where the scalar is from the foreign field. Our cyclefold-inspired design exhibits a kind of reflexive cost: when we add circuits to handle endoscaling operations on one curve, each new circuit creates commitments that require endoscaling on the other curve. We'd like to determine how many total circuits the process converges to, given that each circuit can only handle $\mathbf{k}$ endoscaling operations ($\mathbf{k} = \mathbf{4}$ without overflowing the recursion threshold of $2^{11}$ constraints). The convergence can be modeled a **geometric series**.

Let $\mathbf{M}$ denote the base endoscalings needed on Pallas, $\mathbf{N}$ the base endoscaling needed on Vesta, and $\mathbf{k}$ the endoscaling capacity per circuit. For simplicity, we assume $\mathbf{M} = \mathbf{N}$ as the initial conditions. In the first round, we have $\frac{M}{k}$ circuits on Pallas and $\frac{N}{k}$ circuits on Vesta. In the next round, each circuit from the previous round introduces one additional endoscaling operation on the opposite curve. Consequently, Pallas requires an additional $\frac{\frac{M}{k}}{k}$ and Vesta $\frac{\frac{N}{k}}{k}$ circuits. This continues recursively across rounds.

## 2 Blowup Factor

We can model this generalized sequence as:

**Pallas circuits per round:** $\frac{M}{k}$, $\frac{M}{k^2}$, $\frac{M}{k^3}$, ...,

**Vesta circuits per round:** $\frac{N}{k}$, $\frac{N}{k^2}$, $\frac{N}{k^3}$, ...,

These are *cumulative* total across all rounds. This forms a two-geometric series with ratio $\frac{1}{k^2}$. If we expand this geometric series out for K = 4:

(Pallas circuits) $M_{total} = \frac{M}{4} * (1 + \frac{1}{16}) + \frac{1}{256} + ...) + \frac{N}{16} * (1 + \frac{1}{16}) + \frac{1}{256} + ...) = \frac{M}{4} * \frac{16}{15} + \frac{N}{16} * \frac{16}{15} = \frac{4M+N}{15}$.

This derivation becomes clearer if we dilineate, for the Pallas curve specifically, where the work comes from:

1. M Terms (Pallas's own work bouncing back):

   Round 0: $\frac{M}{4}$, Round 2: $\frac{M}{64}$, Round 3: $\frac{M}{1024}$

   ...

2. N Terms (Vesta work coming to Pallas):

   Round 0: $\frac{N}{4}$, Round 2: $\frac{N}{16}$, Round 3: $\frac{N}{256}$ ...

which, when cumulatively combined, equates to $M_{total}$. We can do the same thing, but in reverse for Vesta, to get the same $N_{total}$.

(Vesta circuits) $N_{total} = \frac{N}{4} * (1 + \frac{1}{16}) + \frac{1}{256} + ...) + \frac{M}{16} * (1 + \frac{1}{16}) + \frac{1}{256} + ...) = \frac{N}{4} * \frac{16}{15} + \frac{M}{16} * \frac{16}{15} = \frac{4M+N}{15}$.

In the symmetric case, where M = N, the base circuits required for each curve are $\frac{M}{4}$. To get the total number of circuits needed for each side, then we take the intermediate term $\frac{M}{4} * \frac{16}{15} + \frac{N}{16} * \frac{16}{15}$ and factor out the common terms and combine the fractions:

- Factor out $M * \frac{16}{15}$: $M * \frac{16}{15} * (\frac{1}{4} + \frac{1}{16})$
- Combine denominators: $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$.

This means $M * \frac{16}{15} * \frac{5}{16} = \frac{M}{3}$ represents the *total* number of circuits required. $\frac{M}{3}$ is the simplified version of $\frac{4M+N}{15}$ when $M = N$. Therefore, the blowup factor for K = 4 is: $\frac{M}{3}$ / $\frac{M}{4}$ = **1.33x**. The same math works out for if K = 8, since the base circuits would be $\frac{M}{8}$, so the total circuits requires would be $\frac{M}{7}$ and the blowup factor is $\frac{M}{8}$ / $\frac{M}{7}$ = **1.14x**.