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MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ کاپٹ کر دوں گا۔
 کچھ کی اجازت نہیں ہے۔ لید پینسل کا استعمال ممنوع ہے۔

Answer Sheet No. _____

ہر سوال کے سامنے دیے گئے، کریکولم کے مطابق درست دائرہ کو پر کریں۔
 Invigilator Sign. _____

Fill the relevant bubble against each question according to curriculum:

Candidate Sign. _____

Question	A	B	C	D	A	B	C	D
1. For what value of a the vectors $2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $-4\mathbf{i} - 8\mathbf{j} + a\mathbf{k}$ are parallel?	-10	8	-8	10	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2. What is the value of $f(-1)$ if $f^{-1}(x) = \frac{8-x}{2}$?	$\frac{7}{2}$	10	6	$\frac{9}{2}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3. What is the value of $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$?	2	$\frac{1}{4}$	$\frac{1}{2}$	1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4. Which of the following is equal to y_2 if $xy = 2$?	$-\frac{4}{x^3}$	$\frac{2}{x^3}$	$-\frac{2}{x^3}$	$\frac{4}{x^3}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5. What is the value of $\frac{d}{dx}[f \circ g(x)]$ if $f(x) = e^x$ and $g(x) = \sin x$?	$\cos x e^{\sin x}$	$\sin(e^x)$	$\cos(e^x)$	$e^{\sin x}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6. Which of the following is the derivative of x^x w.r.t. x ?	$\frac{x^x}{1 + \ln x}$	$x^x \ln x$	$x^x(1 + \ln x)$	$\frac{x^x}{\ln x}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7. The derivative of odd function is always?	Both even and odd function	An even function	An odd function	Neither even nor odd function	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8. Which of the following is equal to $\int (\operatorname{Cosec}^2 x - 1) dx$?	$-\cot x + x + C$	$\cot x - x + C$	$-\cot x - x + C$	$\cot x + x + C$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9. For what value of x the function $f(x) = x^2 - 3x + 7$ has a critical point?	$\frac{3}{2}$	$-\frac{2}{3}$	$-\frac{3}{2}$	$\frac{2}{3}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10. If $\int_a^{\frac{\pi}{4}} \cos 2x dx = \frac{1}{2}$ then what should be the value of a ?	$\frac{\pi}{3}$	0	1	$-\frac{\pi}{4}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Question	A	B	C	D	A	B	C	D
11. Which of the following is the evaluation of $\int \frac{e^x}{e^x - 3} dx$?	$(e^x - 3)^{-2} + C$	$\ln(e^x) + C$	$\ln(e^x - 3) + C$	$e^x \ln(e^x - 3) + C$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
12. The slope and y -intercept of the line $2x + y - 3 = 0$ are:	Slope = -2 y-intercept = -3	Slope = 2 y-intercept = 3	Slope = 2 y-intercept = -3	Slope = -2 y-intercept = 3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
13. Perpendicular distance of the line $x + 2y + 4 = 0$ from the point $(1, 0)$ is:	1	2	5	$\sqrt{5}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
14. If line l_1 is making angle 45° with x -axis and line l_2 is perpendicular to y -axis then the angle between l_1 and l_2 is:	$\frac{\pi}{2}$	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
15. Which one is the solution of inequality $-20 > -2x$?	$10 > x > 2$	$10 < x < +\infty$	$10 > x > -\infty$	$-10 > x > -\infty$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
16. If $y^2 - y = \sin x$ then which one is equal to $\frac{dy}{dx}$?	$\frac{\sin x}{1 - 2y}$	$\frac{\cos x}{2y - 1}$	$\frac{\cos x}{1 - 2y}$	$\frac{\sin x}{2y - 1}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
17. What is the length of latus rectum of parabola $(x + 1)^2 = 8y + 8$?	4	$\frac{1}{8}$	8	$\frac{1}{4}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
18. What is the centre of ellipse $\frac{(2x+1)}{9} + \frac{(y-1)}{5} = 1$?	$(\frac{1}{2}, -1)$	$(1, -1)$	$(-1, 1)$	$(-\frac{1}{2}, 1)$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
19. If θ is the angle between two vectors \underline{a} and \underline{b} , and $\sin \theta = \frac{1}{2}$, $ \underline{a} = \underline{b} = \sqrt{2}$ then what is the value of $ \underline{a} \times \underline{b} $?	$\frac{3}{2}$	1	$\frac{1}{2}$	2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
20. When a force $\underline{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ is applied to an object a displacement $\underline{d} = 4\underline{i} + \alpha\underline{j} + 4\underline{k}$ occurred. For what value of α work done would be 6 units?	9	3	5	7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

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**MATHEMATICS HSSC-II**

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

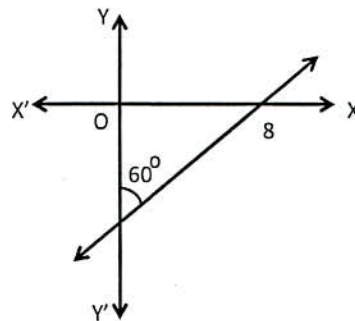
NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C'. Write your answers neatly and legibly.**SECTION – B (Marks 48)****Q. 2** Attempt any TWELVE parts. All parts carry equal marks.

(12 x 4 = 48)

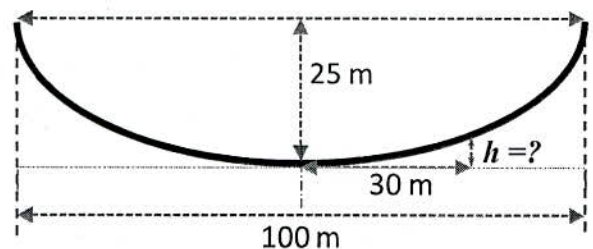
(i) Let $f(x) = x^2 + 2$ and $g(x) = x + 1$ then(a) Find $g \circ f(x)$ (b) Find x for which $f \circ g(x) = 11$ (ii) For the function $f(x) = \begin{cases} 3x + 4, & 0 \leq x < 3 \\ 16 - x, & 3 \leq x < 12 \\ x, & 12 \leq x < 14 \end{cases}$ (a) Explain whether $\lim_{x \rightarrow 3} f(x)$ exists(b) Discuss the continuity of the function $f(x)$ at $x = 12$ (iii) If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$ show that $a \frac{dy}{dx} + b \tan \theta = 0$ (Chain Rule)(iv) If $e^x + e^y = e^{x+y}$ then find the value of $\frac{dy}{dx}$ at $(1, 1)$ (v) Show that $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots \infty$

(vi) Find two positive integers whose sum is 9 and product of one with square of the other is maximum.

(vii) The side of a cube is measured to be 20cm with a maximum error of 0.12cm in its measurement. Find the maximum error in the calculated volume of the cube.

(viii) Evaluate $\int \frac{\cos x}{\sin x \ln \sin x} dx$ (ix) Find the equation of the curve for which $\frac{dy}{dx} + \frac{2xy}{2y+1} = x$. The curve passes through the point $(2, 1)$ (x) Find the equation of a straight line whose x -intercept is 8 and making angle of 60° with the positive direction of y -axis(xi) If $\int_{-3}^3 (x^3 + kx^2) dx = 54$ find the value of k (xii) Find the area of a triangle with one vertex as point of intersection of the lines $x + y - 5 = 0$ and $2x - y + 2 = 0$, and points $(2, -3)$ and $(3, 4)$ are other two vertices.(xiii) Find the equation of a circle concentric with the circle $x^2 + y^2 - 8x + 4 = 0$ and is tangent to the line $x + 2y + 6 = 0$

(xiv) A parabolic arch has a 100m base and height 25m. Find the height of the arch at the point 30m from the centre of the base.

(xv) Verify that \underline{a} and \underline{b} are perpendicular to $\underline{b} \times \underline{a}$ where $\underline{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ and $\underline{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$ (xvi) Prove that in any triangle ΔABC $b = c \cos A + a \cos C$

SECTION – C (Marks 32)

Note: Attempt any FOUR questions. All questions carry equal marks.

(4 x 8 = 32)

Q. 3 Let

$$f(x) = \begin{cases} 2-x, & 0 \leq x < 1 \\ k, & x = 1 \\ 2x-1, & 1 < x \leq 2 \end{cases}$$

then

- a. What are domain and range of $f(x)$
- b. Find the value of k for which $f(x)$ is continuous at $x = 1$
- c. State value of x where only left-hand limit and only right-hand limit exist.
- d. Sketch the graph of $f(x)$ (Use of Graph Paper is not necessary)

Q. 4 A box has square base and the sum of one side of the base and height of the box is 12. If the length of one side of box is x cm

- a. Express volume V of the box in terms of x
- b. Find $\frac{d}{dx}(V)$ and $\frac{d^2}{dx^2}(V)$
- c. Find the dimensions of the box for which the volume of the box is maximum. Hence find maximum volume of the box.

Q. 5 For a curve $f(x) = x^2 - 2x, x \in [-1, 3]$

- a. Find $\int_{-1}^3 f(x) dx$
- b. Sketch the graph of $f(x)$ and shade the area bounded by x-axis and the curve $f(x)$ (Use of Graph Paper is not necessary)
- c. Find the area bounded by x-axis and the curve $f(x)$

$$7x - y - 10 = 0$$

Q. 6 If the sides of a triangle ΔABC are $10x + y - 41 = 0$
 $3x + 2y + 3 = 0$

then

- a. Find the vertices A, B, C of the triangle
- b. Find the equations of altitudes of ΔABC
- c. Show that the altitudes of ΔABC are concurrent
- d. Find area of ΔABC

Q. 7 A car detailing company performs two types of detailing: deluxe and ordinary. The deluxe detailing requires 1 hour inspection and 3 hours maintenance time. While an ordinary detailing requires 1 hour inspection and 2 hours maintenance time. There are 100 hours of inspection and 240 hours of maintenance time during coming two weeks. How many detailing(s) of each type should the company perform to get maximum profit if the company gets a profit of Rs 2000 on deluxe detailing and Rs 1500 in ordinary detailing.

Q. 8 Find the centre, foci, eccentricity, vertices and directrices of the given conic $9x^2 - y^2 - 36x - 6y + 18 = 0$



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MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed.

Do not use lead pencil.

Do not use lead pencil.

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر نام مرتکز کے حوالے کریں۔ کاٹ کر دوہاں لکھنے کی اجازت نہیں ہے۔ لیسڈ پینل کا استعمال ممنوع ہے۔

Answer Sheet No. _____

ہر سوال کے سامنے دیے گئے، کریکولم کے مطابق درست دائرہ کو پر کریں۔ Invigilator Sign. _____

Fill the relevant bubble against each question according to curriculum: Candidate Sign. _____

Question	A	B	C	D	A	B	C	D
					○	○	○	○
1. What is the value of $f(-4)$ if $f^{-1}(x) = \frac{1-x}{2}$?	$\frac{5}{2}$	$-\frac{3}{2}$	9	-7	○	○	○	○
2. If the distance (x) travelled by a car in time (t) is given by $x = 2t^3 - 24 \ln(t+1)$. What will be the velocity of the car for $t = 5$? (Velocity = $\frac{dx}{dt}$)	128	134	140	146	○	○	○	○
3. What is the value of $\lim_{x \rightarrow 0} (x \sin x)$?	∞	-1	1	0	○	○	○	○
4. What is the value of $\frac{d}{dx}(g \circ f(x))$ if $f(x) = \sin x$ and $g(x) = \ln x$?	$\sin(\ln x)$	$\tan x$	$\cot x$	$\ln(\sin x)$	○	○	○	○
5. If $y = \ln \sqrt{\tan x}$ then what is the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$?	∞	1	0	$\frac{1}{2}$	○	○	○	○
6. If $y = \left(1 - x^{\frac{1}{2}}\right) \left(1 + x^{\frac{1}{2}}\right)$ then what is the value of $\frac{dy}{dx}$?	1	-1	x	\sqrt{x}	○	○	○	○
7. Which of the following is the evaluation of $\int (\sec^2 x - 1) dx$?	$\tan x + x + C$	$\tan x - x + C$	$-\tan x + x + C$	$-\tan x - x + C$	○	○	○	○
8. For which of the following intervals functions $f(x) = 2x^2 - 8x + 4$ is increasing.	$(-\infty, 2]$	$[0, +\infty)$	$[2, +\infty)$	$[0, 2]$	○	○	○	○
9. If $\int_{-\pi/2}^a \cos x dx = 2$ then what should be the value of a ?	$\frac{\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	○	○	○	○



Question	A	B	C	D	A	B	C	D
10. Which of the following is the evaluation of $\int \frac{e^x}{e^x + 2} dx$?	$\ln(e^x + 2) + C$	$\ln(e^x - 2) + C$	$x - 2e^{-x} + C$	$x + 2e^{-x} + C$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
11. The x - intercept and y - intercept of the line $3x - y + 6 = 0$ are:	x - intercept = 2 y - intercept = -6	x - intercept = -2 y - intercept = -6	x - intercept = -2 y - intercept = 6	x - intercept = 2 y - intercept = 6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
12. For what value of k the distance from the point $(0, 1)$ to line $3x - 4y + k = 0$ is 3 ?	15	-19	19	11	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
13. If the slope of line l_1 is 2 and it makes angle 45° with line l_2 . What is the slope of line l_2 ? $\left(\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2} \right)$	0	1	$-\frac{1}{2}$	-3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
14. For which of the following inequalities, $(3, 3)$ is NOT the solution?	$x + y > 2$	$3x + 5y < 7$	$3x + 5y > 7$	$3x - 7y < 3$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
15. Which of the following is the slope of normal to curve $x^2 - y^2 - 12 = 0$ at $P(4, 2)$?	2	-2	$\frac{1}{2}$	$-\frac{1}{2}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
16. What are the vertices of an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?	$(\pm a, 0)$	$(0, \pm a)$	$(\pm b, 0)$	$(\pm c, 0)$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
17. Eccentricity of an ellipse 'e' is:	$e > 1$	$e < 1$	$e = 0$	$e = 1$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
18. If $ \underline{a} = \underline{b} = \sqrt{2}$ and $ \underline{a} \times \underline{b} = 2$ then what is the angle between \underline{a} and \underline{b} ?	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
19. A displacement $\underline{d} = -\underline{i} + 3\underline{j} + 8\underline{k}$ occurred when a constant force $\underline{F} = \alpha\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object. If the work done is 45, then what would be the value of α ?	-1	4	-4	5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
20. What is the volume of a tetrahedron determined by the vectors \underline{i} , $2\underline{j}$ and $3\underline{k}$?	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

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MATHEMATICS HSSC-II

34

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C'. Write your answers neatly and legibly.

SECTION - B (Marks 48)

Q. 2 Attempt any TWELVE parts. All parts carry equal marks.

(12 x 4 = 48)

(i) Let $f(x) = 1 + 2x$ and $g(x) = \frac{x}{x-1}$ then

(a) Find $f \circ g(x)$

(b) Find x for which $g \circ f(x) = 7$

(ii) For $f(x) = \begin{cases} 7-4x & x < 1 \\ x^2+2 & x \geq 1 \end{cases}$

(a) Explain whether $\lim_{x \rightarrow 1} f(x)$ exists

(b) Discuss the continuity at $x = 1$

(iii) Differentiate w.r.t. x $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

(iv) If $\sqrt{x} + \sqrt{y} = 4$ then find value of $\frac{dy}{dx}$ at $x = 1$

(v) Show that $e^{x+h} = e^x \left\{ 1 + h + \frac{h^2}{2} + \frac{h^3}{3} + \dots \infty \right\}$

(vi) Find the dimensions of a rectangular garden having perimeter 80 meters, if its area is to be maximum.

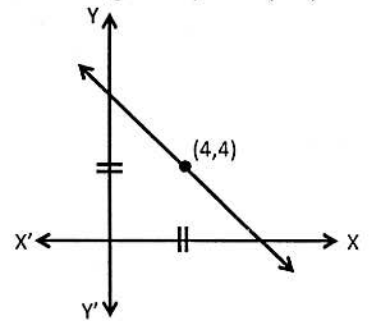
(vii) Find the approximate increase in the volume of a cube if the length of each edge changes from 5 to 5.02 cm

(viii) Evaluate $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$

(ix) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \cos\left(\frac{x}{2}\right) dx$

(x) Find the equation of the curve for which $x^2(2y+1)\frac{dy}{dx} = 1$. The curve passes through the point (2,1)

(xi) Find the equation of a straight line making equal intercepts on the axes and passes through the point (4,4)



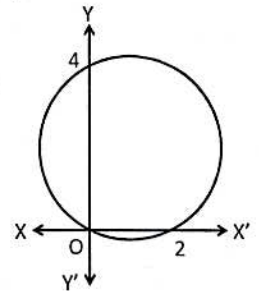
$$x + ky - 2 = 0$$

$$2x - y + 4 = 0$$

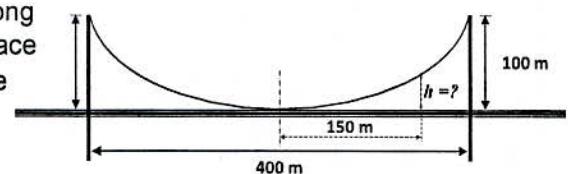
$$x - 11y + 14 = 0$$

(xii) For what value of k the following lines are concurrent

(xiii) Find the equation of circle which passes through origin and whose intercepts on coordinate axes are 2 and 4



(xiv) A suspension bridge with weight uniformly distributed along the length has two towers 100m high above the road surface and 400m apart. The cables are parabolic in shape and are tangent to road surface at the centre of the bridge. Find the height of the cable at a point 150m from the centre.



(xv) Find a unit vector perpendicular to a plane containing $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$ and $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$. Also verify.

(xvi) Prove that in any triangle ΔABC , $c^2 = a^2 + b^2 - 2ab \cos C$

SECTION – C (Marks 32)

Note: Attempt any FOUR questions. All questions carry equal marks.

(4 x 8 = 32)

Q. 3 Let

$$f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$$

then

- Find $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$
- Find the values of a and b for which $f(x)$ is continuous at $x = 1$
- Sketch the graph of $f(x)$ after finding values of a and b (Use of Graph Paper is not necessary)

Q. 4 A solid rectangular block has a base of length $2x \text{ cm}$ and width $x \text{ cm}$. Height of the block is $y \text{ cm}$ and volume of the block is 72 cm^3

- Express total surface area (A) of the block as a function of x
- Find $\frac{d}{dx}(A)$ and $\frac{d^2}{dx^2}(A)$
- Find the dimensions of the block for which its surface area is minimum.

Q. 5 For a curve $f(x) = x^2 - 4, x \in [0, 4]$

- Find $\int_0^4 f(x) dx$
- Sketch the graph of the curve and shade the region bounded by x-axis and the curve $f(x)$ (Use of Graph Paper is not necessary)
- Find the shaded area bounded by the curve $f(x)$ and x-axis

Q. 6 If ΔABC is a triangle with vertices $A(0,0), B(8,6), C(12,0)$ then

- Find mid-points and slopes of three sides of the triangle
- Find the equations of right bisectors of the sides of ΔABC
- Show that the right bisectors of ΔABC are concurrent
- Find area of ΔABC

Q. 7 A farmer plans to mix two types of food to make a low cost feed for the animals in his farm. A bag of food P costs Rs. 40 and contains 5 units of proteins and 4 units of vitamins. A bag of food Q costs Rs. 50 and contains 4 units of proteins and 8 units of vitamins. How many bags of food P and Q should be consumed by the animals each day in order to meet the minimum daily requirement of 120 units of proteins and 144 units of vitamins at a minimum cost?

Q. 8 Find the centre, foci, eccentricity, vertices and directrices of the given conic $x^2 + 16x + 4y^2 - 16y + 76 = 0$

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Answer Sheet No. _____

Sign. of Candidate _____

Sign. of Invigilator _____

Section - A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

MATHEMATICS HSSC-II
SECTION - A (Marks 20)
Time allowed: 25 Minutes

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ گات کردہ بارہ گنتے کی اجازت نہیں ہے۔ لیدر پینسل کا استعمال ممنوع ہے۔

ہر سوال کے سامنے دیے گئے درست دائرہ کو پر کریں۔

Fill the relevant bubble against each question:

- A function $f: X \rightarrow Y$ defined by $f(x) = a, \forall x \in X, a \in Y$ is called:
 - Linear function
 - Constant function
 - Identity function
 - Implicit function
- If $f(x) = \sqrt{x^2 - 1}$ then Domain of f is:
 - $(-\infty, \infty)$
 - $[1, \infty)$
 - $[0, \infty)$
 - $(-\infty, -1] \cup [1, \infty)$
- What result occurs in Evaluating $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{3}-\sqrt{x}}$
 - $-2\sqrt{3}$
 - $2\sqrt{3}$
 - $3\sqrt{2}$
 - $3\sqrt{3}$
- If $f(x) = \cos x$, then what is the value of $f'(\sin^{-1} 3x)$ is:
 - $-\sin 3x$
 - $-3x$
 - $\frac{-3}{\sqrt{1-9x^2}}$
 - $\frac{3}{\sqrt{1-9x^2}}$
- If $f(x) = \ln x^2$ then what is the value of $f''(\sqrt{5})$ is:
 - $-\frac{1}{5}$
 - $\frac{1}{5}$
 - $-\frac{2}{5}$
 - $\frac{2}{5}$
- $(1+x^2) \frac{d}{dx} (\tan^{-1} x + \cot^{-1} x) =$
 - 2
 - $\frac{2}{1+x^2}$
 - 0
 - $\frac{-2}{1+x^2}$
- The integral $\int \frac{dx}{x \ln x}$ is equal to:
 - $\ln x + c$
 - $\frac{1}{x} + c$
 - $\ln(\ln x) + c$
 - $\frac{(\ln x)^2}{2} + c$
- What is the value of k if $\int_0^1 (3x+k) dx = 2$
 - $\frac{1}{2}$
 - $-\frac{3}{2}$
 - $-\frac{1}{2}$
 - $\frac{2}{3}$
- What is the area between the x-axis and curve $y = \cos \frac{x}{2}$, from $-\pi$ to π ?
 - 1
 - 2
 - 4
 - 8

10. The equation of a line $\frac{x}{P \sec \alpha} + \frac{y}{P \operatorname{cosec} \alpha} = 1$ is called: Symmetric equation of a line Two intercept form of a line Slope intercept form of a line Normal form of a line
11. For what value of k the lines $kx - 2y + 5 = 0$ and $x - 2ky + 3 = 0$ are parallel line? 3 ± 1 2 ± 2
12. The equation of the vertical line through $(-6, 5)$ -5 -6 5 6
13. Which one satisfies the inequality $x + 2y < 6$ $(4, 1)$ $(1, 3)$ $(1, 4)$ $(3, 1)$
14. What is the length of tangent from $(1, 1)$ to the circle $x^2 + y^2 - 2x + 3y + 6 = 0$? 1 2 3 4
15. What is the eccentricity of an ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ $\frac{1}{\sqrt{3}}$ $\frac{2}{\sqrt{3}}$ $\frac{\sqrt{3}}{2}$ $\sqrt{3}$
16. What is the length of latus rectum of the hyperbola whose equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$? $\frac{2}{9}$ $\frac{9}{2}$ $\frac{4}{9}$ $\frac{9}{4}$
17. What is the Directrix of Parabola with vertex at origin and focus at $(8, 0)$? $x + 8 = 0$ $x - 8 = 0$ $x + 4 = 0$ $x + 2 = 0$
18. What is the projection of vector $-2\hat{i} + 3\hat{j} + 7\hat{k}$ on $2\hat{j} + \hat{k}$? $\frac{\sqrt{13}}{5}$ $\frac{13}{\sqrt{5}}$ $\frac{5}{\sqrt{13}}$ $\frac{\sqrt{5}}{13}$
19. What is the angle between the vectors $2\vec{i} + \vec{j} + \vec{k}$, $-\vec{i} + 2\vec{j}$ are? $\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{\pi}{6}$ $\frac{\pi}{2}$
20. For what value of α the vectors $2\vec{i}$, $\vec{j} + \vec{k}$ and $\vec{i} + \alpha\vec{j} + 2\vec{k}$ are coplanar? -2 2 3 -3

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MATHEMATICS HSSC-II

32

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet-B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION - B (Marks 48)

(12 x 4 = 48)

Q. 2 Attempt any TWELVE parts. All parts carry equal marks.

- (i) For the real valued function, $f(x)$ is defined by $f(x) = \sqrt{x^3 + 4}$ find $f^{-1}(x)$. Also verify $f(f^{-1}(x)) = x$
- (ii) Evaluate $\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x}$
- (iii) If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$ prove that $(2y - 1) \frac{dy}{dx} = \cos x$
- (iv) Show that $\sin(x+h) = \sin x + h \cos x - \frac{h^2}{2!} \sin x - \frac{h^3}{3!} \cos x + \dots$ (by Taylor's Series)
- (v) If $y = \sin^{-1} \frac{x}{a}$ then show that $y_2 = x(a^2 - x^2)^{-\frac{3}{2}}$
- (vi) Evaluate $\int \frac{dx}{3x(\ln 3x)^4}$
- (vii) Evaluate $\int_0^3 \frac{x^3 + 9x + 3}{x^2 + 9} dx$
- (viii) Solve the differential equation $\frac{dy}{dx} + \frac{4xy}{4y+2} = x$
- (ix) Find an equation of the perpendicular bisector of a line joining the points $A(5,6)$ and $B(8,4)$.
- (x) Find the value of k such that the lines $2x - 2y + 2 = 0$, $3x - 5y - 1 = 0$ and $2x + ky + 8 = 0$ meet at a point.
- (xi) Graph the feasible region of the system of linear inequalities by shading
 $5x + 7y \leq 35$, $-x + 3y \leq 3$, $x \geq 0$, $y \geq 0$
- (xii) Find the equation of a circle passing through the points $A(2,3)$, $B(0,2)$ having centre at $3x + 2y - 3 = 0$
- (xiii) Find the equation of Parabola with focus $(3,2)$ and directrix $2x - y + 5 = 0$.
- (xiv) Find the equation of tangent to hyperbola $9x^2 - 4y^2 = 36$ parallel to the line $3x + 2y + 7 = 0$
- (xv) Find the scalar ' α ' so that vectors $3\bar{i} + \alpha\bar{j} + 4\bar{k}$ and $4\bar{i} + 5\bar{j} + \alpha\bar{k}$ are perpendicular to each other.
- (xvi) Find the volume of the tetrahedron whose vertices are $A(-2,1,4)$, $B(3,2,5)$, $C(-3,-5,0)$, $D(5,8,9)$

SECTION – C (Marks 32)

Note: Attempt any **FOUR** questions. All questions carry equal marks.

(4 x 8 = 32)

Q. 3 Let $f(x) = \begin{cases} mx+3 & \text{if } x < 3 \\ m+n & \text{if } x = 3 \\ -x+9 & \text{if } x > 3 \end{cases}$

- a. Find $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$
- b. Find the $\lim_{x \rightarrow 3} f(x) = f(3)$
- c. Find the value of m and n such that $f(x)$ is continuous at $x = 3$
- d. After finding the values of m and n , sketch the graph of the function

Q. 4 The perimeter of a triangle is 18 centimetres. If one side is of length 8 cm. What are lengths of the other sides for maximum area of a triangle?

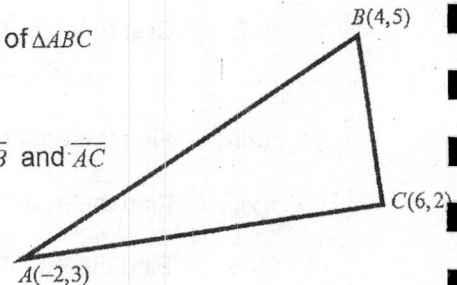
- a. Find function $f(x)$
- b. Find $f'(x)$ and $f''(x)$
- c. Find the values of $f(x)$ for which has maximum or minimum values?
- d. Find the sides of triangle ABC

Q. 5 Evaluate the integral $\int \frac{2x^2 + 5x + 3}{(x-2)^2(x^2 + x + 1)} dx$

- a. Resolve $\frac{2x^2 + 5x + 3}{(x-2)^2(x^2 + x + 1)}$ into Partial fraction
- b. After Partial Fraction Integrate the result $\int \frac{2x^2 + 5x + 3}{(x-2)^2(x^2 + x + 1)} dx$

Q. 6 The diagram shows a triangle ABC where $A(-2,3)$, $B(4,5)$, $C(6,2)$ are vertices of $\triangle ABC$

- a. Find the slopes of side \overline{AB} , \overline{BC} and \overline{AC}
- b. Find the angle between the sides \overline{AB} and \overline{BC} and angle between \overline{AB} and \overline{AC}
- c. Find the equations of sides \overline{AB} and \overline{BC}
- d. Find the area of triangle ABC check these three points are collinear



Q. 7 An agent wishes to purchase a number of chairs and tables. He has only Rs. 12000 to invest and has space at most for 28 items. A chair costs him Rs. 480 and a table costs Rs. 300. His expectation is that he can sell a chair at a profit of Rs. 200 and table at a profit of Rs. 150. Assuming that he can sell all the items that he can buy. How should he invest his money in order to maximize his profit?

Q. 8 Find the Centre, Foci, Eccentricity, Vertices and Equation of directrices of the conic $25x^2 + 4y^2 - 250x - 16y + 541 = 0$

33



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Answer Sheet No. _____

Sign. of Candidate _____

Sign. of Invigilator _____

Section - A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

MATHEMATICS HSSC-II
SECTION - A (Marks 20)
Time allowed: 25 Minutes

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ گات کر دوبارہ لکھنے کی اجازت نہیں ہے۔ لیڈ پینسل کا استعمال ممنوع ہے۔

ہر سوال کے سامنے دیے گئے درست دائرہ کو پر کریں۔

Fill the relevant bubble against each question:

- $x = a \cos \theta$, $y = b \sin \theta$ are parametric equations of: Circle Parabola Ellipse Hyperbola
- Which of the following represents $f^{-1}(5)$ if $f(x) = x^{\frac{1}{3}} + 2$ 1 3 9 27
- In which of the following intervals, $f(x) = 4x - 2x^2$ is increasing? $0 \leq x \leq 1$ $0 < x < 1$ $1 \leq x \leq 2$ $0 < x < 2$
- What result will occur, in evaluating $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{3n}$ e e^2 e^3 e^6
- For a function $f(x) = a \sin 3x$ and $f'\left(\frac{\pi}{3}\right) = 6$ then what is the value of a ? -2 2 3 6
- $\frac{d}{dx}(\sec^{-1} x + \operatorname{cosec}^{-1} x) =$ -1 0 1 2
- $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx =$ $-\ln(\sqrt{x}+1) + c$ $\ln(\sqrt{x}+1) + c$ $2\ln(\sqrt{x}+1) + c$ $-2\ln(\sqrt{x}+1) + c$
- Which one of the following results occurs of the integral $\int_0^2 \frac{dx}{x^2+4}$ $\frac{\pi}{6}$ $\frac{\pi}{8}$ $\frac{\pi}{4}$ $\frac{\pi}{2}$
- If $\int_0^2 f(x) dx = 3$ then what is the value of k if $\int_0^2 (3f(x) + 4) dx = k$ 11 17 20 23

10. The points $A(2,5)$ and $B(3,-2)$ are the ends of a diameter of a circle, what is the radius of a circle? $2\sqrt{5}$ $5\sqrt{2}$ $\frac{5}{\sqrt{2}}$ $\frac{2}{\sqrt{5}}$
11. A line cuts the x-axis at $(2,0)$ and y-axis at $(0,-4)$, then equation of a line is: $2x - y - 4 = 0$ $2x - y + 4 = 0$ $x + 2y - 4 = 0$ $x - 2y + 4 = 0$
12. Pair of lines represented by Homogeneous equation $ax^2 + 2hxy + by^2 = 0$ through origin will be real and coincident if: $h^2 > ab$ $h^2 < ab$ $h^2 = ab$ $a + b = 0$
13. The solution set of $2y + 5 > 4y - 3$ $y > -4$ $y > 8$ $y < -4$ $y < 4$
14. The line $y = mx + c$ will be tangent to a circle $x^2 + y^2 = a^2$ if: $c = \pm m\sqrt{1+a^2}$ $c = \pm a\sqrt{1+m^2}$ $c = \pm m\sqrt{1-a^2}$ $c = \pm a\sqrt{1-m^2}$
15. What is the Length of Latus Rectum of Parabola $x^2 = 5y$ 5 20 $\frac{5}{4}$ 10
16. Which one of the following represents the graph of $9x^2 - 18x + 4y^2 + 8y - 23 = 0$? Circle Parabola Ellipse Hyperbola
17. The co-vertices of hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ are: $(0, \pm 4)$ $(\pm 2, 0)$ $(\pm 4, 0)$ $(0, \pm 2)$
18. The area of the triangle whose adjacent sides are $3\bar{i} + 4\bar{j}$ and $12\bar{i} + 9\bar{j}$ is: $\frac{45}{2}$ $\frac{21}{2}$ $\frac{55}{2}$ $\frac{25}{2}$
19. If vectors $\bar{v} = \bar{i} - 3\bar{j} + 4\bar{k}$ and $\bar{w} = \lambda\bar{i} + 9\bar{j} - 12\bar{k}$ are parallel then what is the value of λ ? -3 3 -9 9
20. What is the volume of a parallelepiped if its edges are $2\bar{i} - 4\bar{j} + 5\bar{k}$, $2\bar{i} - 3\bar{j} + 6\bar{k}$, $-\bar{j} - \bar{k}$? 0 3 15 24

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MATHEMATICS HSSC-II

34

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet-B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION - B (Marks 48)

(12 x 4 = 48)

Q. 2 Attempt any TWELVE parts. All parts carry equal marks.

- (i) Let the real valued function, f and g defined by $f(x) = 4x + 1$ and $g(x) = 2x^2 + 5x$ obtain the expression for:
- a. $f((g))$ b. $g(f(x))$ c. $f(f(x))$ d. $g(f(x))$
- (ii) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$
- (iii) Find $\frac{dy}{dx}$ if $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$
- (iv) If $y = \tan(4 \tan^{-1} \frac{x}{4})$ show that $\frac{dy}{dx} = \frac{16(1+y^2)}{16+x^2}$
- (v) Use implicit rule to find the second derivative of the function $y = x + \tan^{-1} y$
- (vi) If $x = \cos \theta$; $y = \cos n\theta$ show that $(1-x^2)y_2 - xy_1 + n^2y = 0$
- (vii) Find the area between the x-axis and the curve $f(x) = x^2 - 2x$ from $x = 0$ to $x = 3$
- (viii) Evaluate $\int x^3 \sqrt{1+x^2} dx$
- (ix) Find the point two-fifth of the way along the line segment $A(-3,5)$ to $B(5,3)$.
- (x) Find the angle θ form the lines L_1 and L_2 : $L_1: 7x + 3y - 9 = 0$
 $L_2: 5x - 2y + 2 = 0$
- (xi) Graph the feasible solution region of the system of linear inequalities by shading, also find the corner points. $3x + 7y \leq 21$, $x - y \leq 3$, $x \geq 0$, $y \geq 0$
- (xii) Find the equation of parabola with focus $(1,3)$ and vertex $(4,3)$.
- (xiii) Find the equation of parabola, with Directrix, $y = 3$ and vertex $(2,2)$.
- (xiv) Write the equation of ellipse with vertices at $(-1,2)$ and $(7,2)$ and 2 is the length of semi minor axis whereas major axis is horizontal.
- (xv) Prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
- (xvi) Find constant α so that vectors are coplaner $\bar{i} - \alpha \bar{j} - k$, $\bar{i} + \bar{j} + 2\bar{k}$ and $\alpha \bar{i} - \bar{j} + \bar{k}$

SECTION – C (Marks 32)

Note: Attempt any FOUR questions. All questions carry equal marks.

(4 x 8 = 32)

Q. 3 If θ is measured in radian then prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

- Draw the figure and give explanation.
- Find area of triangles in figure.
- From figure, see the inequalities of area and prove the theorem.

Q. 4 Consider the function $f(x) = \sin x + \frac{1}{\sqrt{2}} \cos 2x$ where $x \in (0, 2\pi)$

find the extreme values of the functions in the interval $x \in (0, 2\pi)$

- Find function $f'(x)$
- Find $f''(x)$
- Find the values of $x \in (0, 2\pi)$ for which $f(x)$ has maximum or minimum values
- Find possible extreme values of $f(x)$

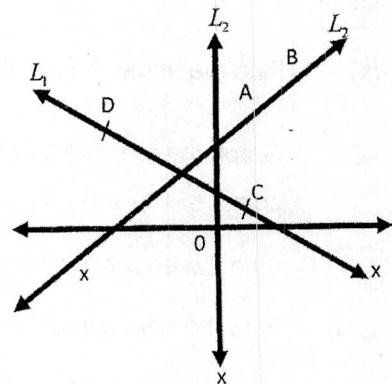
Q. 5 Integrate $\int \frac{2x+5}{(x-3)^2(x^2-x+5)} dx$

- Resolve $\frac{2x+5}{(x-3)^2(x^2-x+5)}$ into Partial fraction
- After Partial Fraction Integrate the result $\int \frac{2x+5}{(x-3)^2(x^2-x+5)} dx$

Q. 6 The diagram shows two Lines L_1 and L_2 passing through points:

L_1 : joins $A(2,7)$ and $B(7,10)$ L_2 : joins $C(1,1)$ and $D(-5,3)$

- Find the slope of lines L_1 and L_2
- Find the angle between the lines L_1 and L_2
- Find the equations of line L_1 and L_2
- Find the point of contact where line L_1 and L_2 intersect



Q. 7 Find the maximum and minimum values of f and g defined as $f(x) = 3x + 5y$ and $g(x) = 6x + 8y$ under the constraints. $2x - 3y \leq 6$, $2x + y \geq 2$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$

Q. 8 Find the equations of tangent and normal lines at a point $\left(3, \frac{12}{5}\right)$ to ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ For what value of C the line

$x + y + c = 0$ will touch the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$