



Roll No. \_\_\_\_\_ to be filled in by the candidate

(For All Sessions)

Time: 30 Minutes

Marks: 20

**Mathematics (Objective)** *Lwp-12-1-23*

(Group-I)

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = ?$  (A) 1 (B) 2 (C) 0 (D) -2
2.  $\cos hx + \sin hx = ?$  (A)  $e^x$  (B)  $e^{-x}$  (C)  $e^{2x}$  (D)  $2e^x$
3.  $\frac{d}{dx} [\ln(2^x)] = ?$  (A)  $\ln 2$  (B)  $2^x$  (C)  $\frac{1}{2^x}$  (D)  $\frac{\ln 2}{2^x}$
4.  $\frac{d}{dx} (\cos hx) = ?$  (A)  $-\sin hx$  (B)  $\sin hx$  (C)  $\operatorname{sech} hx$  (D)  $\operatorname{cosech} hx$
5. If  $f(x) = \sqrt{x}$ , then  $f^{-1}(0) = ?$  (A) 0 (B) 1 (C) Undefined (D)  $\frac{1}{2}$
6.  $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = ?$  (A) 1 (B) 0 (C) -1 (D) 2
7.  $\int dx = ?$  (A)  $x$  (B)  $\frac{1}{x}$  (C)  $x^2$  (D)  $\frac{1}{x^2}$
8.  $\int e^x(x+1) dx = ?$  (A)  $xe^x + c$  (B)  $e^x + c$  (C)  $x + c$  (D)  $x^2 + c$
9.  $\int_0^{\pi/2} \cos x dx = ?$  (A) 0 (B) -1 (C) 1 (D) 2
10.  $\int \frac{\sin 2x}{\sin x} dx = ?$  (A)  $2 \cos x + c$  (B)  $2 \sin x + c$  (C)  $\frac{1}{2} \sin x + c$  (D)  $\frac{1}{2} \cos x + c$
11. The slope of a line  $x = 5$  is: (A) 0 (B) 1 (C) -1 (D) Infinite
12. Midpoint of (0, -2) and (-2, 0) is: (A) (0, 0) (B) (-1, -1) (C) (-2, -2) (D) (0, -1)
13. Distance between (-1, 2) & (7, 5) is: (A)  $\sqrt{73}$  (B) 7 (C)  $2\sqrt{73}$  (D) 73
14. The solution of inequality  $x + 2y < 6$  is: (A) (1, 4) (B) (1, 3) (C) (1, 1) (D) (1, 5)
15. Equation of Tangent to  $x^2 + y^2 = 4$  at (2, 0) is: (A)  $x = 1$  (B)  $y = 1$  (C)  $y = 2$  (D)  $x = 2$
16. Slope of tangent to parabola  $y^2 = 4ax$  at (a, 2a) is: (A) 2 (B) -1 (C) 1 (D) 3
17. Eccentricity  $e$  of a circle is: (A)  $e = 0$  (B)  $e = 1$  (C)  $0 < e < 1$  (D)  $e > 1$
18. Radius of a circle  $x^2 + y^2 = 2$  is: (A) 2 (B) 1 (C)  $\frac{1}{2}$  (D)  $\sqrt{2}$
19. If  $P = (2, 3)$ ,  $Q = (6, -2)$ , then  $|\overline{PQ}|$  is: (A)  $\sqrt{40}$  (B)  $\sqrt{42}$  (C)  $\sqrt{41}$  (D)  $\sqrt{43}$
20. For a vector  $\vec{v} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ , then  $\cos \beta = ?$  (A)  $\frac{3}{7}$  (B)  $\frac{2}{7}$  (C)  $-\frac{6}{7}$  (D)  $-\frac{3}{7}$

Mathematics (Subjective)

(For All Sessions)  
(GROUP-I)

Time: 2:30 hours

SECTION-I

Rwp-12-1-23

2. Write short answers of any eight parts from the following:

(8x2=16)

- Express perimeter  $P$  of a square as a function of its area  $A$ .
- Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- Define even function with example.
- If  $y = x^4 + 2x^2 + 2$ , prove that  $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Find derivative by definition  $\frac{1}{\sqrt{x}}$
- Find  $\frac{dy}{dx}; xy + y^2 = 2$
- Differentiate w.r.t  $x$ ,  $y = x^2 \sec 4x$
- Differentiate w.r.t  $x$ ,  $y = \cot^{-1} \left( \frac{x}{a} \right)$
- Find  $\frac{dy}{dx}$  if  $y = x\sqrt{\ln x}$
- Apply the Maclaurin series to prove that:  $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
- Graph the solution set of  $2x + y \leq 6$ .
- Define feasible region.

3. Write short answers of any eight parts from the following:

(8x2=16)

- Evaluate  $\int \tan^2 x dx$ .
- Evaluate  $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- Evaluate  $\int x \sin x dx$ .
- Evaluate  $\int_{\pi/6}^{\pi/3} \cos t dt$
- Solve the differential equation  $y dx + x dy = 0$
- Evaluate  $\int \frac{x^2}{4+x^2} dx$
- Find the areas between the  $x$ -axis and the curve  $y = x^2 + 1$  from,  $x = 1$  to  $x = 2$
- Find a unit vector in the direction of  $\underline{V} = \frac{1}{2} \underline{i} + \frac{\sqrt{3}}{2} \underline{j}$
- Find direction cosines of  $\underline{V} = 4\underline{i} - 5\underline{j}$
- Find  $\alpha$ , so that vector  $\underline{u} = 2\alpha \underline{i} + \underline{j} - \underline{k}$ ,  $\underline{v} = \underline{i} + \alpha \underline{j} + 4\underline{k}$  are perpendicular.
- Find the area of parallelogram whose vertices are:  $A(0, 0, 0)$   $B(1, 2, 3)$   $C(2, -1, 1)$   $D(3, 1, 4)$
- A force  $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$  is applied at  $p(1, -2, 3)$ . Find its amount about the point  $Q(2, 1, 1)$

4. Write short answers of any nine parts from the following:

(9x2=18)

- Is  $(\sqrt{176}, 7)$  at a distance of 15 units from the origin?
- By means of slopes, show that the point  $(-4, 6)$ ,  $(3, 8)$ ,  $(10, 10)$  lie on the same line.
- Find  $K$  so that the line joining  $A(7, 3)$ ;  $B(k, -6)$  and the line joining  $C(-4, 5)$ ,  $D(-6, 4)$  are parallel.
- Find the equation of the line having  $y$ -intercept  $-7$  and slope  $-5$ .
- Find the point of intersection of the lines  $x - 2y + 1 = 0$  and  $2x - y + 2 = 0$
- Find equation of lines represented by  $2x^2 + 3xy - 5y^2 = 0$
- Find the measure of the angle between the lines represented by  $9x^2 + 24xy + 16y^2 = 0$
- Find an equation of the circle with ends of diameter at  $(-3, 2)$  and  $(5, -6)$
- Show that the line  $2x + 3y - 13 = 0$  is tangent to the circle  $x^2 + y^2 + 6x - 4y = 0$
- Check the position of the point  $(5, 6)$  with respect to the circle  $x^2 + y^2 = 81$ .
- Find focus and directrix of the parabola  $x^2 = -16y$
- Find an equation of ellipse if foci  $(-3\sqrt{3}, 0)$  and vertices  $(\pm 6, 0)$ .
- Find equation of hyperbola with given data foci  $(0, \pm 9)$ , directrices  $y = \pm 4$

SECTION-II

Note Attempt any three questions. Each question carries equal marks:

(10x3=30)

- (a) Evaluate:  $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$  (b) If  $y = \tan(2 \tan^{-1} \frac{x}{2})$ , then show that  $\frac{dy}{dx} = 4 \left( \frac{1+y^2}{4+x^2} \right)$
- (a) Evaluate:  $\int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x}$  (b) Find equation of line through intersection of  $x + 2y + 3 = 0$ ,  $3x + 4y + 7 = 0$  and making equal intercepts on the axes.
- (a) Find the area bounded by the curve  $f(x) = x^3 - 2x^2 + 1$  and  $x$ -axis in the 1<sup>st</sup> quadrant.  
(b) Minimize  $Z = 3x + y$  subject to the constraints  $3x + 5y \geq 15$   $x + 6y \geq 9$   $x \geq 0$ ,  $y \geq 0$
- (a) If  $y = a \cos(\ln x) + b \sin(\ln x)$  prove that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$   
(b) Find the coordinates of the points of intersection of the line  $2x + y = 5$  and the circle  $x^2 + y^2 + 2x - 9 = 0$ , also find the length of intercepted chord.
- (a) Find the centre foci, eccentricity and vertices of the ellipse  $x^2 + 16x + 4y^2 - 16y + 76 = 0$



Roll No. \_\_\_\_\_ to be filled in by the candidate

HSSC-(P-II)-A/2023

Paper Code 8 1 9 6

(For All Sessions)

(Group-II)

Time: 30 Minutes

Marks : 20

# Mathematics (Objective)

Rwp-12-2-23

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 Midpoint of A(1,2) & B(3,8) is: (A) (2, 5) (B) (4, 10) (C) (2, 6) (D) (2, 8)
2. (1, -3) is in the solution of \_\_\_\_\_ (A)  $x + y \geq 1$  (B)  $x + y \leq 0$  (C)  $x + y = 0$  (D)  $x - y = 0$
3. Centre of circle  $x^2 + y^2 - 6x + 4y + 13 = 0$  (A) (3, 2) (B) (-3, 2) (C) (3, -2) (D) (-3, -2)
4. Focus of parabola  $x^2 = 4ay$  is: (A) (-a, 0) (B) (0, -a) (C) (a, 0) (D) (0, a)
5. Eccentricity e for hyperbola is: (A) e = 1 (B) e = 0 (C) e < 1 (D) e > 0
6. Length of major axis of  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  (A) 03 (B) 06 (C) 02 (D) 04
7. Which one is not scalar quantity: (A) Work (B) Time (C) Magnetic field (D) Speed
8.  $[k \ i \ j]$  (A) 2 (B) 0 (C) 1 (D) -1
9.  $\lim_{x \rightarrow 2} \sqrt{x^3 + 1} - \sqrt{x^2 + 5}$  (A) -1 (B) 0 (C) 2 (D) -2
10. Area of circle of unit radius is: (A)  $\pi$  (B)  $2\pi$  (C)  $\pi^2$  (D)  $2\pi^2$
11.  $\frac{d}{dx}(3^x) =$  \_\_\_\_\_ (A)  $3^x \ln x$  (B)  $3^x \ln 2$  (C)  $3^x \ln 3$  (D)  $x 3^{x-1}$
12. Lagrange used \_\_\_\_\_ notation for derivative. (A)  $D f(x)$  (B)  $f^1(x)$  (C)  $\frac{d}{dx} f(x)$  (D)  $f'(x)$
13.  $\frac{d}{dx} \cos 7x =$  \_\_\_\_\_ (A)  $7 \sin 7x$  (B)  $-7 \sin 7x$  (C)  $7 \cos 7x$  (D)  $-7 \cos 7x$
14. Minimum value of function  $f(x) = x^2 + 2x - 3$  is at  $x =$  \_\_\_\_\_ (A) -3 (B) -2 (C) 0 (D) -1
15.  $\int \frac{1}{1+x^2} dx =$  \_\_\_\_\_ (A)  $\sin^{-1} x + c$  (B)  $\cos^{-1} x + c$  (C)  $\tan^{-1} x + c$  (D)  $\cot^{-1} x + c$
16.  $\int \frac{1}{x^2} dx =$  \_\_\_\_\_ (A)  $-\frac{1}{x} + c$  (B)  $\frac{1}{x} + c$  (C)  $\frac{2}{x} + c$  (D)  $-\frac{2}{x} + c$
17. Solution of  $\frac{dy}{dx} = 1$  is \_\_\_\_\_ (A)  $y = x^2 + c$  (B)  $y = e^x + c$  (C)  $y = \ln x + c$  (D)  $y = x + c$
18.  $\int_0^1 3x^2 dx =$  \_\_\_\_\_ (A) 3 (B) 1 (C) 2 (D) 0
19. Equation of line through origin with slope 2: (A)  $2x - y = 0$  (B)  $2x + y = 0$  (C)  $x + 2y = 0$  (D)  $x - 2y = 0$
20. Slope of line parallel to y-axis: (A) -1 (B) 0 (C)  $\infty$  (D) 1

**ematics (Subjective)**

(For All Sessions)

**(GROUP-II)**

Time: 2:30 hours

**SECTION-I**

*Rwp-12-2-23*

Write short answers of any eight parts from the following:

(8x2=16)

- i. Express perimeter P of a square as a function of its area A.
- ii. If  $f(x) = (-x + 9)^3$ , find  $f^{-1}(x)$
- iii. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- iv. Differentiate w.r.t "x"  $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$
- v. If  $y = \sqrt{x + \sqrt{x}}$  find  $\frac{dy}{dx}$
- vi. Find  $\frac{dy}{dx}$  if  $x = y \sin y$
- vii. Find  $f'(x)$  if  $f(x) = x^3 \cdot e^{1/x}$
- viii. If  $y = x^2 \cdot \ln(\frac{1}{x})$ , find  $\frac{dy}{dx}$
- ix. If  $y = \sin h^{-1}(\frac{x}{2})$ , Find  $\frac{dy}{dx}$
- x. Apply the Maclaurin series to prove that:  $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$
- xi. Graph the solution set of linear inequality in  $xy$  - plane,  $2x + y \leq 6$
- xii. What is a feasible solution?

3. Write short answers of any eight parts from the following:

(8x2=16)

- i. Using differentials find  $\frac{dy}{dx}$  and  $\frac{dx}{dy}$  for  $x^2 + 2y^2 = 16$
- ii. Evaluate:  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- iii. Evaluate:  $\int \frac{x+2}{\sqrt{x+3}} dx$
- iv. Evaluate:  $\int \tan^{-1} x dx$
- v. Evaluate:  $\int \frac{5x+8}{(x+3)(2x-1)} dx$
- vi. Evaluate:  $\int_{-2}^0 \frac{1}{(2x-1)^2} dx$
- vii. Solve the differential equation  $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
- viii. Find sum of  $\overline{AB}$  and  $\overline{CD}$  where  $A(1, -1)$ ,  $B(2, 0)$ ,  $C(-1, 3)$  and  $D(-2, 2)$
- ix. Find direction Cosines of vector  $\underline{V} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
- x. Find  $\alpha$  so that  $\underline{U} = 2\alpha \mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\underline{V} = \mathbf{i} + \alpha \mathbf{j} + 4\mathbf{k}$  and perpendicular.
- xi. Compute  $\underline{a} \times \underline{b}$  and  $\underline{b} \times \underline{a}$  for  $\underline{a} = \mathbf{i} + \mathbf{j}$ ,  $\underline{b} = \mathbf{i} - \mathbf{j}$
- xii. Find volume of parallelepiped determined by  $\underline{U} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $\underline{V} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  and  $\underline{w} = \mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$

4. Write short answers of any nine parts from the following:

(9x2=18)

- i. The point  $C(-5, 3)$  is the center of the circle and  $P(7, 2)$  lies on the circle. What is the radius of the circle.
- ii. Show that the points  $A(0, 2)$ ,  $B(\sqrt{3}, -1)$  and  $C(0, -2)$  are vertices of a right triangle.
- iii. The points  $P(-2, 6)$  and  $O(-3, 2)$  are given in  $xy$  - coordinate system. Find the  $XY$  - Coordinate of P referred to the translated axes  $OX'$  and  $OY'$ .
- iv. Find an equation of the line through  $(-5, -3)$  and  $(9, -1)$ .
- v. Convert  $4x + 7y - 2 = 0$  in slope-intercept form.
- vi. Find the lines represented by  $3x^2 + 7xy + 2y^2 = 0$
- vii. Find the point of intersection of the lines  $3x + y + 12 = 0$  and  $x + 2y - 1 = 0$
- viii. Find center and radius of circle  $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- ix. Find focus and vertex of parabola  $y^2 = -12x$
- x. Find foci of an ellipse  $9x^2 + y^2 = 18$
- xi. Find eccentricity of hyperbola,  $\frac{y^2}{4} - x^2 = 1$
- xii. Write parametric equations of hyperbola.
- xiii. Write down equation of tangent to the circle  $x^2 + y^2 = 25$  at  $(4, 3)$ .

**SECTION-II**

(10x3=30)

Note Attempt any three questions. Each question carries equal marks:

- 5. (a) Evaluate:  $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
- (b) Find  $\frac{dy}{dx}$  if  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ .
- 6. (a) Evaluate:  $\int \frac{x}{x^4 + 2x^2 + 5} dx$
- (b) Find equation of the line through  $(5, -8)$  and perpendicular to the join of  $A(-15, -8)$  and  $B(10, 7)$ .
- 7. (a) Solve the differential equation  $(y - x \frac{dy}{dx}) = 2(y^2 + \frac{dy}{dx})$
- (b) Graph the feasible region of the following system of linear inequalities and find the corner points.  
 $2x + y \leq 10, x + 4y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$
- 8. (a) Show that  $y = \frac{\ln x}{x}$  has maximum value at  $x = e$ .
- (b) Write an equation of the circle that passes through the given points  $A(4, 5)$ ,  $B(-4, -3)$ ,  $C(8, -3)$
- 9. (a) Find the focus, vertex and directrix of the parabola  $x^2 - 4x - 8y + 4 = 0$

**Mathematics** (Objective Type)

Time: 30 Minutes

RwP22

Marks: 20

**Note:** You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

- 1.1.  $\int \sec x \, dx =$  \_\_\_\_\_
- (A)  $\ln |\sec x - \tan x| + c$  (B)  $\ln |\sec x + \cot x| + c$   
 (C)  $\ln |\sec x + \operatorname{cosec} x| + c$  (D)  $\ln |\sec x + \tan x| + c$
2.  $\int_0^1 |x| \, dx =$  \_\_\_\_\_
- (A) 1 (B) 2 (C) 0 (D)  $\frac{1}{2}$
3. Solve  $\frac{1}{y} dy = \frac{1}{x} dx$
- (A)  $y = xc$  (B)  $y = -xc$  (C)  $y = x^2 + c$  (D)  $xy = c$
4. Distance between A(-1, 2) and C(2, -6) is \_\_\_\_\_
- (A)  $\sqrt{73}$  (B)  $\sqrt{70}$  (C) 7 (D) 8
5. If  $m_1 = m_2$  then lines are \_\_\_\_\_
- (A) perpendicular (B) not parallel  
 (C) parallel (D) neither parallel nor perpendicular
6. Slope of  $12x + 35y - 7 = 0$  is \_\_\_\_\_
- (A)  $\frac{12}{35}$  (B)  $-\frac{12}{35}$  (C)  $\frac{1}{35}$  (D) 12
7. Normal form of  $x + y = 1$  is \_\_\_\_\_
- (A)  $x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} = \frac{1}{\sqrt{2}}$  (B)  $x \cos \frac{\pi}{2} + y \sin \frac{\pi}{2} = 1$   
 (C)  $x \cos \frac{\pi}{4} + y \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$  (D)  $x + y = 2$
8. If  $P(x, y) = 40x + 50y$  then  $P(1, -1) =$  \_\_\_\_\_
- (A) 10 (B) 40 (C) 50 (D) -10
9. Centre of  $5x^2 + 5y^2 + 24x + 36y + 10 = 0$  is \_\_\_\_\_
- (A) (-12, -18) (B)  $\left(\frac{-12}{5}, \frac{-18}{5}\right)$  (C) (12, 18) (D) (-12, 18)
10. Axis of  $y^2 = -4ax$  is \_\_\_\_\_
- (A)  $y = 0$  (B)  $y = a$  (C)  $x = 0$  (D)  $x = a$
11. Vertices of  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is \_\_\_\_\_
- (A)  $(\pm b, 0)$  (B)  $(a, b)$  (C)  $(\pm a, 0)$  (D)  $(-a, -b)$
12. Scalar triple product of coplaner vectors is \_\_\_\_\_
- (A) 1 (B) 0 (C) 2 (D) -1
13.  $2\mathbf{i} \times 2\mathbf{j} \cdot 2\mathbf{k} =$  \_\_\_\_\_
- (A) 4 (B) 2 (C) 8 (D) 16
14. Which one is even function
- (A)  $\sin x$  (B)  $\cos x$  (C)  $\tan x$  (D)  $x^{101}$
15. If  $f(x) = \sqrt{x^2 - 9}$ ; then range of  $f(x)$  is \_\_\_\_\_
- (A)  $(0, -\infty)$  (B)  $(-\infty, \infty)$  (C)  $(-5, 5)$  (D)  $(0, +\infty)$
16.  $\frac{d}{dx} 2^x =$  \_\_\_\_\_
- (A)  $2^x \ln 2$  (B)  $2^x \ln e$  (C)  $2^x \ln 4$  (D)  $x 2^{x-1}$
17. Leibniz used \_\_\_\_\_ notation for derivative.
- (A)  $f'(x)$  (B)  $f'(x)$  (C) D f(x) (D)  $\frac{dy}{dx}$
18.  $\frac{d}{dx} (\operatorname{cosec} 7x) =$  \_\_\_\_\_
- (A)  $\operatorname{cosec} 7x \cot 7x$  (B)  $-\operatorname{cosec} x \cot x$  (C)  $-7 \operatorname{cosec} 7x \cot 7x$  (D)  $\operatorname{cosec} 7x \tan 7x$
19. Which one is decreasing function
- (A)  $2 - 4x$  (B)  $4x - 2$  (C)  $4x$  (D)  $4x + 5$
20.  $\frac{d(xy)}{dx} =$  \_\_\_\_\_
- (A)  $x dx + y dy$  (B)  $(x + y) dx$  (C)  $x dy + y dx$  (D)  $x dy - y dx$

SECTION - I

RWP-22

## 2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Search the domain and range from the real numbers of  $g(x) = \sqrt{x^2 - 4}$   
 ii- The real valued functions  $f$  and  $g$  are defined below. Find (a)  $f^2(x)$  (b)  $g^2(x)$ ,

$$f(x) = \frac{1}{\sqrt{x-1}}; \quad x \neq 1, \quad g(x) = (x^2 + 1)^2$$

iii- Evaluate  $\lim_{x \rightarrow \infty} \frac{5x^4 - 10x^2 + 1}{-3x^3 + 10x^2 + 50}$

iv- Evaluate  $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$

v- Give any example and sketch graphically discontinuous function.

vi- Differentiate w.r. to 'x';  $\frac{(1 + \sqrt{x})(x - x^{3/2})}{\sqrt{x}}$

vii- Find  $\frac{dy}{dx}$  if  $y = \sqrt{\frac{a^2 + x^2}{a^2 - x^2}}$

viii- Find the derivative w.r.t. variable involved  $\cos \sqrt{x} + \sqrt{\sin x}$

ix- Find  $f'(x)$  if  $f(x) = \ln(\sqrt{e^{2x} + e^{-2x}})$

x- Produce  $y_2$  from  $y = e^{ax} \sin bx$

xi- Determine the intervals in which  $f$  is increasing or decreasing;

$$f(x) = \cos x \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

xii- The perimeter of a triangle is 16 centimeters. If one side is of length 6 cm, what are lengths of the other sides for maximum area of the triangle?

## 3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

i- Use differential find  $\frac{dy}{dx}$ ;  $x^4 + y^2 = xy^2$

ii- Evaluate  $\int \frac{e^{2x} + e^x}{e^x} dx$

iii- Evaluate  $\int \sec x dx$

iv- Evaluate  $\int \sin^{-1} x dx$

v- Evaluate  $\int e^x \left(\frac{1}{x} + \ln x\right) dx$

vi- Evaluate  $\int \frac{5x + 8}{(x + 3)(2x - 1)} dx$

vii- Evaluate  $\int_1^2 \frac{x}{x^2 + 2} dx$

viii- Find the area between the x-axis and the curve  $y = \sin 2x$  from  $x = 0$  to  $x = \frac{\pi}{3}$

ix- Show that the points A (-1, 2); B(7, 5) and C(2, -6) are vertices of a right triangle.

x- Find an equation of vertical line through (-5, 3)

xi- Convert  $15y - 8x + 3 = 0$  in slope-intercept form.

xii- Find the lines represented by;  $x^2 - 2xy \sec \alpha + y^2 = 0$

4. Write short answers to any NINE questions:

RWP-22

(2 x 9 = 18)

- i- Indicate the solution set of inequality  $3x - 2y \geq 6$
- ii- What is objective function?
- iii- Write an equation of circle with centre at  $(\sqrt{2}, -3\sqrt{3})$  and radius  $2\sqrt{2}$
- iv- Check the position of the point (5, 6) with respect to the circle  $x^2 + y^2 = 81$
- v- Find an equation of parabola with focus  $(-3, 1)$  and directrix  $x = 3$
- vi- Determine the equation of ellipse having foci  $(\pm 3, 0)$  and minor axis of length 10.
- vii- Calculate the eccentricity of  $\frac{y^2}{16} - \frac{x^2}{49} = 1$
- viii- Find an equation of the normal line to  $y^2 = 4ax$  at  $(at^2, 2at)$
- ix- If O is origin and  $\vec{OP} = \vec{AB}$ , find the point P when A and B are  $(-3, 7)$  and  $(1, 0)$  respectively
- x- Write the direction cosines of  $\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$
- xi- Prove that in any triangle ABC,  $a^2 = b^2 + c^2 - 2bc \cos A$
- xii- If  $\underline{a} + \underline{b} + \underline{c} = 0$ , then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
- xiii- A force  $\vec{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$  is applied at a point  $(1, -1, 2)$ . Find the moment of  $\vec{F}$  about the point  $(2, -1, 3)$

### SECTION - II

Note: Attempt any three questions from the following.

10 x 3 = 30

- 5- (a) Show that  $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$
- (b) Show that  $y = x^x$  has maximum value at  $x = \frac{1}{e}$
- 6- (a) Integrate  $\int \frac{4 + 7x}{(1+x)^2(2+3x)} dx$
- (b) Find the point which is equidistant from the point  $A(5, 3)$ ,  $B(-2, 2)$  and  $C(4, 2)$ .  
What is radius of circumcircle of triangle ABC.
- 7- (a) Find  $\int_{\pi/6}^{\pi/4} \cos^2 \theta \cot^2 \theta d\theta$
- (b) Minimize  $Z = 2x + y$  subject to constraints  $x + y \geq 3$ ,  $7x + 5y \leq 35$ ,  $x \geq 0$ ,  $y \geq 0$
- 8- (a) Find the area of the triangular region. Whose vertices are  $A(5, 3)$ ,  $B(-2, 2)$ ,  $C(4, 2)$
- (b) Find the length of the chord cut off from the line  $2x + 3y = 13$  by the circle  $x^2 + y^2 = 26$
- 9- (a) Find equations of the common tangents to the two conics  $\frac{x^2}{16} + \frac{y^2}{25} = 1$  and  $\frac{x^2}{25} + \frac{y^2}{9} = 1$
- (b) Use vectors, prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long.

Roll No. 754990 to be filled in by the candidate.

(For all sessions)

Paper Code	8	1	9	1
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**Mathematics** (Objective Type)

RWP-21

Time: 30 Minutes

Marks:20

NOTE: Write answers to the questions on objective answer sheet provided. Four possible answers A,B,C & D to each question are given. Which answer you consider correct, fill the corresponding circle A,B,C or D given in front of each question with Marker or pen ink on the answer sheet provided.

1-1. If  $g(x) = \frac{1}{x^2}, x \neq 0$  then  $g \circ g(x)$  equals.

- (A)  $x$  (B)  $x^2$  (C)  $x^4$  (D)  $x^3$

2.  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$  equals.

- (A) zero (B) 1 (C) 2 (D) 3

3. The derivative of  $\sqrt{x}$  at  $x = 1$  is:

- (A)  $\frac{1}{2}$  (B) 2 (C) 1 (D)  $-\frac{1}{2}$

4.  $\frac{d}{dx} \left[ \frac{1}{g(x)} \right]$  equals.

- (A)  $\frac{1}{g^2(x)}$  (B)  $\frac{-g'(x)}{(g(x))^2}$  (C)  $-g(x)$  (D)  $\frac{1}{g(x)}$

5. If  $y = 5e^x$  then  $y_3$  equals.

- (A)  $25e^x$  (B)  $75e^x$  (C)  $15e^x$  (D)  $5e^x$

6. If  $f(x+h) = \cos(x+h)$  then  $f'(x)$  equals.

- (A)  $\cos x$  (B)  $-\cos x$  (C)  $-\sin x$  (D)  $\sin x$

7. Inverse of  $\int \dots dx$  is:

- (A)  $\frac{d}{dy}$  (B)  $\frac{d}{dx}$  (C)  $\frac{dy}{dx}$  (D)  $\frac{dx}{dy}$

8.  $\int_a^b f(x) dx$  equals:

- (A)  $-\int_b^a f(x) dx$  (B)  $\int_{-b}^a f(x) dx$  (C)  $\int_b^{-a} f(x) dx$  (D)  $\int_a^{-b} f(x) dx$

9. The general solution of  $\frac{dy}{dx} = \frac{-y}{x}$  is:

- (A)  $xy = c$  (B)  $x^2 y^2 = c$  (C)  $\frac{x}{y} = c$  (D)  $\frac{y}{x} = c$

10.  $\int e^{-x} (\cos x - \sin x) dx$  equals:  
 (A)  $-e^{-x} \sin x + c$  (B)  $e^{-x} \cos x + c$  (C)  $e^{-x} + c$  (D)  $e^{-x} \sin x + c$
11. The distance of point (3,7) from x-axis is:  
 (A) 3 (B) 7 (C) -3 (D) -7
12. Slope of Y-axis is:  
 (A) zero (B) 1 (C) 2 (D) undefined
13. Equation of horizontal line through (7,-9) is:  
 (A)  $y = -9$  (B)  $y = 7$  (C)  $x = -9$  (D)  $x = 7$
14. (0,2) is solution of inequality.  
 (A)  $3x + 5y > 7$  (B)  $3x + 5y < 7$  (C)  $x < 0$  (D)  $x > 0$
15. Centre of circle  $x^2 + y^2 + 2gx + 2fy + c = 0$  is:  
 (A)  $(g, f)$  (B)  $(-g, -f)$  (C)  $(0, 0)$  (D)  $(-g, -f)$
16. Equation of Latus rectum of parabola  $x^2 = 4ay$  is:  
 (A)  $y = -a$  (B)  $y = a$  (C)  $x = -a$  (D)  $x = a$
17. Vertices of  $\frac{x^2}{16} - \frac{y^2}{25} = 1$  are:  
 (A)  $(0, \pm 4)$  (B)  $(0, \pm 5)$  (C)  $(\pm 4, 0)$  (D)  $(\pm 5, 0)$
18. The non zero vectors  $\underline{a}$  and  $\underline{b}$  are parallel if  $\underline{a} \times \underline{b}$  is:  
 (A) zero (B) 1 (C) 2 (D) 3
19.  $\cos \theta$  equals:  
 (A)  $\underline{a} \cdot \underline{b}$  (B)  $\underline{a} \times \underline{b}$  (C)  $|\underline{a} \times \underline{b}|$  (D)  $\hat{a} \cdot \hat{b}$
20. If any two vectors of scalar triple product are equal then its value is:  
 (A) -1 (B) zero (C) 1 (D) 2

Roll No. \_\_\_\_\_ to be filled in by the candidate.

(For all sessions)

**Mathematics** (Essay Type)

Time: 2:30 Hours

**RWP-21**  
Section -I

Marks: 80

2. Write short answers of any eight parts from the following.

2x8=16

- i. If  $f(x) = x^2 - x$ , find (a).  $f(-2)$  (b).  $f(x-1)$
- ii. Find  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 + x - 6}$ .
- iii. Find  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$ .
- iv. Differentiate w.r.t "x".  $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ .
- v. Find  $\frac{dy}{dx}$  if  $3x + 4y + 7 = 0$ .
- vi. Differentiate w.r.t "x"  $\cos \sqrt{x} + \sqrt{\sin x}$ .
- vii. Differentiate w.r.t "x"  $\cot^{-1}\left(\frac{x}{a}\right)$ .
- viii. If  $y = \log_{10}(ax^2 + bx + c)$ , then find  $\frac{dy}{dx}$ .
- ix. If  $y = x^2 \cdot e^{-x}$ , then find  $\frac{d^2y}{dx^2}$ .
- x. Apply Maclaurin series, Prove that  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- xi. If  $f(x) = \sqrt{x+1}$  and  $g(x) = \frac{1}{x^2}$ , then find (a).  $(f \circ g)(x)$  (b).  $(g \circ f)(x)$ .
- xii. Find the intervals in which  $f(x)$  is increasing or decreasing  $f(x) = \cos x$ ,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ .

3. Write short answers of any eight parts from the following.

2x8=16

- i. Using differential find  $\frac{dy}{dx}$ , if  $x^2 + 2y^2 = 16$ .
- ii. Evaluate  $\int x\sqrt{x^2-1} dx$ .
- iii. Evaluate  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$ .
- iv. Evaluate  $\int \sin^2 x dx$ .
- v. Evaluate  $\int \frac{ax+b}{ax^2+2bx+c} dx$ .
- vi. Evaluate  $\int e^{3x} \left(\frac{3 \sin x - \cos x}{\sin^2 x}\right) dx$ .
- vii. Solve  $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$ .
- viii. Find an equation of the vertical line through (-5,3).
- ix. Find an equation of the line through (-5,-3), (9,-1)
- x. Convert  $4x + 7y - 2 = 0$  in normal form.
- xi. Find the area below the curve  $y = 3\sqrt{x}$  and above the x-axis between  $x = 1$  and  $x = 4$ .
- xii. Find the mid point of the line segment joining the points A(3,1), B(-2,-4).

4. Write short answers of any nine parts from the following.

2x9=18

- i. Graph the solution set by shading of inequality  $5x - 4y \leq 20$ .
- ii. Find equation of circle with centre at  $(\sqrt{2}, -3\sqrt{3})$  and radius  $2\sqrt{2}$ .
- iii. Write equation of tangent to the circle  $3x^2 + 3y^2 + 5x - 13y + 2 = 0$  at  $\left(1, \frac{10}{3}\right)$ .

## RWP-21

- iv. Find vertex of  $x^2 - 4x - 8y + 4 = 0$ .
- v. Find point of intersection of conics  $3x^2 - 4y^2 = 12$  and  $3y^2 - 2x^2 = 7$ .
- vi. Find equation of parabola whose focus is  $F(-3,4)$  and directrix is  $3x - 4y + 5 = 0$ .
- vii. Find the unit vector in the same direction of vector  $\underline{V} = [3, -4]$ .
- viii. If  $\overline{AB} = \overline{CD}$  find the co-ordinate of the point A when points B,C,D are  $(1,2)$ ,  $(-2,5)$  and  $(4,11)$  respectively
- ix. Find  $|\underline{3v} + \underline{w}|$  if  $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$ ,  $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$ ,  $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$ .
- x. Find a vector of length 5 in the direction opposite that of  $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$ .
- xi. Compute  $\underline{b} \times \underline{a}$  if  $\underline{b} = \underline{i} - \underline{j} + \underline{k}$ ,  $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$ .
- xii. Find the work done if the point at which the constant force  $\overline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$  is applied to an object, moves from  $p_1(3,1,-2)$  to  $p_2(2,4,6)$ .
- xiii. If  $\underline{a} + \underline{b} + \underline{c} = 0$  then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ .

### Section -II

Note: Attempt any three questions from the following.

10x3=30

5. (a) If  $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ , then show  $f(x)$  is continuous at  $x = 1$ .

(b) If  $x = \frac{a(1-t^2)}{1+t^2}$ ,  $y = \frac{2bt}{1+t^2}$ , then find  $\frac{dy}{dx}$ .

6. (a) Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02.
- (b) Determine the value of P such that the lines  $2x - 3y - 1 = 0$ ,  $3x - y - 5 = 0$  and  $3x + py + 8 = 0$  meet at a point.

7. (a) Evaluate  $\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$ .

(b) Minimize  $z = 2x + y$  subject to the constraints  $x + y \geq 3$ ,  $7x + 5y \leq 35$ ,  $x \geq 0$ ,  $y \geq 0$ .

8. (a) Write equations of two tangents from  $(2,3)$  to the circle  $x^2 + y^2 = 9$ .

(b) Prove by vector method  $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ .

9. (a) Show that  $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$

(b) Show that an equation of the parabola with focus at  $(a \cos \alpha, a \sin \alpha)$  and

directrix  $x \cos \alpha + y \sin \alpha + a = 0$  is  $(x \sin \alpha - y \cos \alpha)^2 = 4a(x \cos \alpha + y \sin \alpha)$ .